HEAT, LIGHT AND SOUND

BY THE SAME AUTHOR

A MODERN SCHOOL ELECTRICITY AND MAGNETISM
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HEAT AND LIGHT
With Diagrams.

HEAT, LIGHT AND SOUND

BY

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PREFACE

THE aim of this book is to give a general introductory course in Heat, Light and Sound suitable for students working up to the standard of a First School Examination, for technical students who are reading these sections of Physics for their professional examinations for the qualification of A.M.I.E.E., A.M.I.M.E., etc., and also for those taking the National Certificate in Chemistry.

The author has again adopted the practice followed in his *Modern School Electricity and Magnetism:* the basic principles of the subjects are stressed in as clear and instructive a manner as possible. Whenever a suitable opportunity arises, an account has been given of the historical development of the ideas and of the discovery of facts; not, however, with such wealth of detail that the student is unable to "see the wood for the trees." In addition, attention has been directed to many practical applications.

In view of the publication of the work in two bindings —Heat and Light, and Heat, Light and Sound—an introductory study of wave-motion has been given as part of the subject of "Radiation of Energy," and not as an introduction to Sound. It is important that students who study only Heat and Light should acquire an elementary conception of the theory of the propagation of light by wave-motion.

For much of the matter in the book the author is indebted to many different sources too numerous to mention; in particular, the authority for many of the historical facts, especially those in dispute, is the standard *History of Physics*, by F. Cajori.

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HEAT AND LIGHT

CHAPTER I

MATTER—GENERAL PROPERTIES—FLUID PRESSURE—BAROMETERS

IN commencing a study of a particular branch of Science it is well to consider the aims and methods of Science in general. In early scientific training there is a concentration on facts—the characteristics and properties of objects. This entails careful observation and realisation of the details of objects, and phenomena associated with them. But man's interest is not confined to facts alone; the human mind is also interested in the development of ideas regarding the facts—in the way facts are interpreted. This interpretation is essential to the growth of Science, and so, in this book, stress will be laid on the development of man's ideas regarding the observed facts, as well as on the facts themselves. It is the fashion to build up the beginnings of Science from the works of Greeks—chiefly because of a lack of knowledge of civilisation before their pre-eminence.

Natural Sciences deal with the substance, or stuff, of which things are made, *i.e.* with *matter*. One of the earliest Greek philosophers, Thales (640–546 B.C.), considered that material things were all, in the end, composed of the same primary matter, probably water. A little later came the idea of 4 simple substances, or *elements*—earth, water, air, and fire. So gradually has come the modern idea that there are 92 elements, which ordinarily remain intact, whether by themselves or in combination. The knowledge regarding matter, and of

the phenomena associated with it, is now so vast that its study has, of necessity, fallen into sections. For example, Biology deals with living things, Chemistry deals with elements and their combination, whilst Physics deals with the properties of matter, various phenomena associated with matter and changes which occur without involving any change in composition.

Properties of Matter.—There are certain definite facts about matter in general, and it is important that they should be recognised.

(1) *Matter occupies space*, and usually affects one or more of our senses. There are many substances which we can both see and feel. Many gases, *e.g.* coal-gas, affect our sense of smell but not of sight. If one moves a hand vigorously to and fro, the air is felt; in a wind, of course, the touch of the air is obvious. The space occupied by a substance, its volume, is measured in terms of units derived from the units of length. They are cubic feet, etc., in the English system of units, and cubic centimetres, cubic metres, etc., in the Continental system.

(2) *Matter can exist in different forms*, known respectively as *solids*, *liquids*, and *gases* (or vapours). Solid substances differ from the other two forms in that they retain their shape. Liquids and gases do not—they yield to the slightest pressure, and flow readily; they are thus called *fluids*. The essential difference between the two kinds of fluids is that the volume of a liquid is practically a constant for the ordinary air conditions, despite pressure, whilst a gas, or vapour, occupies the whole of the space in which it is confined and will readily spread out to occupy further space, if given the opportunity. Thus a liquid has one free surface, which is usually horizontal, whilst a gas, or vapour, has no free surface. The distinction between solid and liquid is not always obvious—a so-called *viscous* state often exists. Pitch, chewing-gum and dough are common examples of this.

(3) *Matter cannot be destroyed, neither can it be created by us*. This is known as the *conservation of matter* (L. *con*, together; *servare*, keep) and was shown by

accurate weighing at the beginning of the nineteenth century. When ice is changed to water, and to steam, if the latter is condensed to water again, exactly the same amount of matter is left as at the commencement, provided none has been permitted to escape. In any chemical change the quantity of matter at the end of the change is the same as before. **Quantity of matter is known as mass,** and is measured in terms of the pound, in English units, or the gram, in Continental units. The mass of a piece of matter is the same wherever it is taken.

(4) *Matter by itself is inert, or inactive,* i.e. it cannot change its own state of rest, or of motion. When in a train to which the brakes are suddenly applied, one is jerked forward because the part of one's body in contact with the train is slowed up with it, whilst the rest of one's body tends to move on with its previous motion. If the train restarts suddenly, one feels jerked backwards owing to the part of the body not in contact with the train tending to remain where it is. Many examples of the effect of inertia of matter will come readily to the mind of the reader. To overcome the inertia of a body, and so make it move, or cause it to change its motion, a *force* must be applied to it. But it must not be thought that motion, or change in motion, *must* take place when a force is being exerted. When two tug-of-war teams are pulling, and neither moves, two forces are being exerted, one by each team, but they neutralise one another. It is thus better to say that **a force is that which changes, or tends to change, the state of rest or motion of a body.**

(5) *Matter is divisible,* possibly to a limited extent. It has already been stated that there are 92 simple substances, called elements, which exist. Many of these react, chemically, with other elements, and when two or more elements are so united we call the product a *compound.*

The nature of matter has been a subject of speculation from the time of the early Greeks, and about 400 B.C. Leucippus and Democritus propounded an *atomic theory* — that substances are made up of unchangeable,

indivisible bodies which they called *atoms* (Gk. *a*, not; *tomos*, cut; *i.e.* not to be cut up or divided). These were considered to be too small to affect the senses, and different atoms to be of different sizes and masses. The atoms were regarded as in a state of continual motion in the space which separated them. In the seventeenth century this idea of the atom was quite commonly believed by such people as Sir Isaac Newton (1642–1727).

In 1803, *John Dalton* put forward his *Atomic Theory* to explain chemical combination of elements into compounds. He had observed that the same chemical compound always contained the same elements in the same fixed proportions, and further that when two elements combined together to form more than one compound the different masses of the one element which combined with a fixed mass of the other were in a simple proportion. For example, 12 units of mass of carbon always unite with 32 units of mass of oxygen to form carbon dioxide, and with 16 units of mass of oxygen to form carbon monoxide. The ratio of the parts of oxygen combining with the 12 parts of carbon is the simple one of 2 to 1.

Thus Dalton put forward the following points in his theory:

(*a*) Matter is divisible; the smallest portions obtainable, called atoms, are indestructible.

(*b*) All atoms of one element are the same in size and mass, but are different from those of other elements.

(*c*) Compounds are formed by the combination of simple numbers of elementary atoms.

Avogadro very soon realised the need to extend the idea of an indivisible portion to compounds. The smallest portion of anything which could exist and show the properties of the substance was called a *molecule*. The molecule was divisible into atoms, which might be similar or different. Common salt can be divided into small particles and these into smaller ones. This division could be carried on, if it were possible to see to do it, to a certain extent; the particles obtained, the smallest possible which would have the properties of salt, would be molecules. These molecules, however, could be split

up, each molecule giving one atom of sodium and one atom of chlorine.

Avogadro also put forward a *hypothesis* which has been of vital importance in the development of molecular theory and knowledge. It has never been found to lead to conclusions which prove to be incorrect in practice and so is often recognised as a law. It is that equal volumes of all gases, under the same conditions, contain the same number of molecules.

The atomic theory still holds, even though atoms have been shown to be divisible by special methods. The atom is the smallest portion of any element, however, which has been found to take part in chemical combination.

(6) *Matter is porous* and *compressible* to varying extents. This follows from a development of the atomic theory which considered the atoms to be in motion (to be dealt with later), for there must be space between the atoms.

(7) *All particles of matter attract one another.* This is known as the *Universal Theory of Gravitation*, the force of attraction being called a gravitational force. We are all conscious of the fact that, if a stone be thrown up into the air, it reaches a certain height, is then momentarily at rest, and finally falls back to the earth. The problem of " Why does the stone return to earth ? " is said to have been solved by Sir Isaac Newton. His attention was drawn to it, in 1665, by the fall of an apple from a tree under which he was sitting. He came to the conclusion that the earth pulls everything towards it—that it exerts a force of attraction, called the force of gravity. From this he put forward the general theory stated above. This has been probably the most fundamental theory ever expressed, and, by virtue of it, the motions of heavenly bodies can be calculated, etc. It also accounts for the fact that a collection of atoms, separated from one another, behave and appear to us as a rigid piece of matter. The particles are very close together, and though they are small, the gravitational force of attraction between them is very great. This force between similar particles is called *cohesion*. The force of attraction holding unlike particles together (*i.e.* water sticking to glass) is called *adhesion.*

(8) *Matter opposes either a change of shape, or of volume, or of both*, and is said to be *elastic*. This opposition is realised when a force is exerted to stretch a piece of rubber, and is then removed. Immediately, the rubber recovers its original shape and size, unless it has been overstrained. Metals, etc., are more elastic, *i.e.* they offer a greater opposition force to a change. When the applied stretching force is removed the elastic force of opposition serves as a force of restitution, *i.e.* causes the substance to recover its original shape or size.

Movement of Bodies and the Unit of Force.—When a body is moving, *i.e.* changing its position, we consider the rate at which it does so, thus introducing the question of time. If we ignore the direction of motion, and only consider the rate of movement, we speak of that as the *speed*. We say, for example, that a motor-car is moving at a speed of 20 miles per hour. But we may also consider the direction of motion, and sometimes it is important to do so. When we do, we speak of the *velocity* of movement, *e.g.* the velocity of the wind is 4 miles per hour west, or an aeroplane is flying with a velocity of 90 miles per hour S.S.E., etc. In common talk this difference between speed and velocity is not always recognised.

When a body is changing its position regularly, *i.e.* at a constant rate, its velocity is said to be uniform ; if the velocity is changing, we say the body is being accelerated, or undergoing *acceleration*. If the velocity is increasing, we say the acceleration is positive ; if the velocity is decreasing, we say the acceleration is negative (in ordinary conversation we speak of negative acceleration as retardation).

Acceleration is measured by the change of velocity in a second in a given direction. Thus if the velocity of a train entering a station decreases 2 ft. per sec. in a second, we say its acceleration is —2 ft. per sec. per sec. (or —2 f.s.s.).

In scientific work we use the centimetre, gram, second, system of units (or C.G.S. system), measuring distances in centimetres, masses in grams, and times in

seconds. Unit acceleration on the C.G.S. system is a change of velocity of 1 cm. per sec. in a second; *i.e.* unit acceleration is 1 cm. per sec. per sec.

Since a force will change the motion of a body, *i.e.* accelerate it, we have a ready means of defining unit force. But we must realise that the mass of the body affects the definition. It is much easier to move an empty tar barrel than a full one; it requires less force to kick a tennis ball than a football. **Thus we define our unit of force as that force which, acting on unit mass, gives it unit acceleration.** This unit of force which will accelerate a mass of 1 grm. by 1 cm. per sec. per sec. is called the *dyne*.

Mass and Weight.—We have seen that the earth exerts a force on bodies—pulls them towards it. By virtue of this force a body has weight. When we speak of the weight of a body we really mean the pull of the earth on it. At the top of a mountain this pull would be different from that in the valley below, owing to the difference in distance from the centre of the earth, from where the earth's force acts. Thus, if we measure the weight of a body by its extension of a spring, in the spring balance, its weight will record differently over the earth, which is not a perfect sphere. To weigh substances we balance the pull of the earth on the substance against the pull of the earth, *at the same place*, on some standard lump or lumps of metal. The amount of metal in the standard lump does not vary from place to place; its *quantity of matter*, or its *mass*, is constant. Thus the amount of sugar which balances a pound mass (*i.e.* we say weighs 1 lb.) will do so wherever the same standard of mass is used. We use standard masses of 1 lb. and multiples or fractions of it. Unfortunately, too, these are now called weights.

We often use the earth's pull on standard masses for known forces at any place. Thus if a mass of 1 lb. is tied to a spring, the pull of the earth on the mass is a force of 1 lb. weight, and a force of 1 lb. weight is necessary to support the mass.

It is found that the earth's pull on a body which can move freely vertically downwards gives it an acceleration of 981 cms.

per sec. per sec. (this is often expressed as g)—*i.e.* it will pull a mass of 1 grm. towards it with an acceleration of 981 cms. per sec. per sec.

The force which produces in a mass of 1 grm. an acceleration of 1 cm. per sec. per sec. is called a dyne. Thus, 981 dynes will produce an acceleration of 981 cms. per sec. per sec. on unit mass.

Thus the pull of the earth on 1 grm. is equivalent to a force of 981 dynes; or a force of 1 dyne is equivalent to the pull of the earth on $\frac{1}{981}$ grm. (We say loosely, that 1 dyne $=\frac{1}{1{,}000}$ grm. weight approx., or $=0{\cdot}001$ grm. weight.)

Thus it is seen that the force due to the pull of the earth on any body is its mass in grams, multiplied by 981 (or 1,000 approximately), *i.e.* the pull of the earth on 1 grm.=981 dynes, and on 0·001 grm.=0·981 dyne (or 1 dyne approx.).

Density.—The old problem, "Which is the heavier, a pound of lead or a pound of feathers?" is well known. The answer to it is, of course, "Neither," for a pound is a unit of mass, and the nature of the substance does not affect the mass: the pull of the earth, at a place, on each one pound mass is the same. But the purpose of the problem is to cause confusion between ideas of mass and volume. It is a very common observation that different substances, of the same mass or weight, occupy different volumes, *e.g.* a pound of flour occupies more space than a pound slab of chocolate. The difference in bulk is due to the varying degrees of closeness with which the particles of different substances are packed, and also to differences in the respective masses and volumes of the particles. We compare different substances by the masses of a known volume, for convenience, unit volume. The value of the mass of unit volume of a substance, *i.e.* the total mass of the substance divided by its volume, is called its *density*. In the units usually employed in scientific work, density (D) in grms. per c.c. $=\frac{\text{Mass in grms. (M)}}{\text{Volume in c.c. (V)}}$, and thus the mass of a substance in grms. or weight in grms. weight, is equal to its volume in c.c. $\times$ its density in grms. per c.c.

The unit of mass, the gram, is the mass of 1 c.c. of water, and so the density of water is 1 grm. per c.c. The density of iron is 7·8 grms. per c.c., and that of mercury 13·6 grms. per c.c.

Because of the unitary value for water, it is common to compare the density of other substances with that of water.

Thus the *relative density of a substance*

$$= \frac{\text{the density of the substance}}{\text{the density of water (1 grm. per c.c.)}}$$

and thus
$$= \frac{\text{the mass of a volume of the substance}}{\text{the mass of an equal volume of water}}.$$

Relative density is called *specific gravity*, and is obviously a ratio, *i.e.* it states by how many times the substance is as dense as water. This number is, of course, the same as the value of the density, *e.g.* the density of paraffin is 0·8 grm. per c.c. and its specific gravity is thus 0·8.

Pressure.—The particles of the book which you are now reading are all attracted towards the earth by the gravitational force of the earth. This force of attraction is exerted when the book is at rest on a table, exactly as when it is held above the table and released. In the latter case, however, the book falls on to the table as there is not sufficient support for it; in the former the table exerts an upward force to balance the gravitational force (the force exerted by the table is called a *reaction*). The force continuously exerted on the table by the book, whilst it is lying there, is called a *pressure*. The pressure is expressed by the force per unit area, *i.e.* by the weight per unit area, since the force is due to, and *is*, the weight of the book. Thus the *total pressure* exerted on a surface =pressure × area over which the pressure is exerted.

Pressure in Liquids.—A liquid is attracted towards the earth just as is a solid, and so there is a pressure downwards on the base of a vessel containing a liquid. The total pressure over the whole base is equal to the weight of the whole liquid, and so the pressure (force per

unit area) at the base $= \dfrac{\text{weight of the liquid in grms. weight}}{\text{area of the base in sq. cms.}}$

i.e. it is the weight of the column of liquid which is 1 sq. cm. area of cross-section. This obviously depends on (1) the height of the column above the base, (2) the density of the liquid (for weight=volume×density). Thus the pressure of a liquid is measured in grms. weight per sq. cm. and is obtained by multiplying the height of the liquid, above the 1 sq. cm. area where the pressure operates, by its density.

Thus the *deeper below the surface, the greater the downward pressure due to the weight of the liquid*, and it is, of course, at a maximum at the bottom of the liquid. This can be shown by the arrangement as in Fig. 1 (using a manometer tube AB).

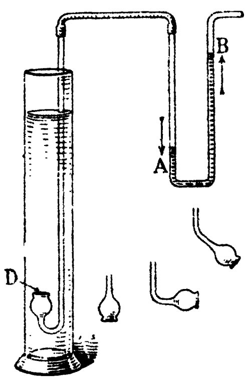


FIG. 1.

A thistle-funnel is bent as indicated and across its mouth a piece of light water-proof fabric, such as a piece of toy balloon, is securely fastened. The tube of the funnel is joined by rubber tubing to a small U-tube containing coloured water. Levels A and B are the same, but on lowering the thistle-funnel into a deep jar full of water, level A falls and level B rises. This shows that (*a*) the water exerts a pressure on the surface D, and (*b*) this downward pressure increases with the depth below the surface of the water. By lowering the thistle-funnel successively in different liquids such as methylated spirit, paraffin, copper sulphate solution, and saturated brine, and by observing the depression of level A when D is immersed to the same level in these liquids, it can be shown that the pressure depends on the density of the liquid.

The denser the liquid the greater is the pressure. By rotating the thistle-funnel, or moving it to and fro, at the same level in any of the liquids, it can be shown, by the maintenance of level A, that the *pressure at the same level in the same liquid is the same*.

By using a series of similarly fitted thistle-funnels,

bent at different angles (see Fig. 1) and joined in turn to the same tube of coloured water, and lowering them into the jar of a liquid, it can be shown that a *pressure is exerted in a liquid in all directions*. The pressure is exerted downwards, upwards, and sideways in all directions. At all points at the same level all these pressures are equal, but the pressure varies with depth.

Some further experiments illustrating the above results are given.

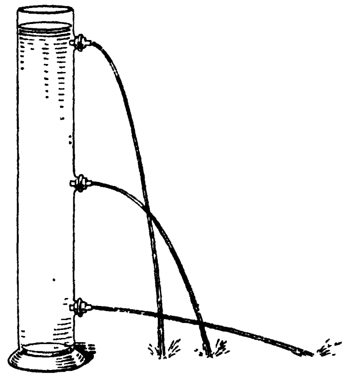


FIG. 2.

(1) A tall metal vessel has three holes with corks in them at different levels. Through these holes are fitted glass tubes, with short rubber connections at the ends, and these are closed by spring clips. The vessel is filled with water and then, at the same instant, all the clips are removed. Water emerges from the three horizontal tubes simultaneously, the different pressures at the different depths being indicated by the difference in the jets, as shown in Fig. 2.

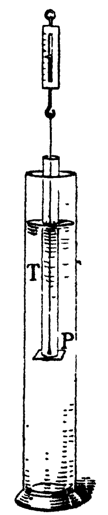


FIG. 3.

(2) A straight chimney lamp-glass is ground at one end to fit on a glass plate, by rubbing them together using moistened emery powder. A piece of fine wire is attached to the centre of the glass plate by means of sealing-wax. The whole is then arranged as shown in Fig. 3, and water is poured into the lamp-glass T, on to plate P which is supported by a spring balance. The height of the latter must be carefully adjusted so that the downward pressure of the water on P is balanced by the upward pull of the spring balance, which indicates the force. By varying the amount of water in T, and adjusting the balance, holding plate P during the adjustment, it is shown that the pressure increases with the depth of liquid.

Another experiment is performed with this tube and plate

to show that the upward pressure in a liquid is equal to the downward pressure. The tube is fitted on the plate and the latter is held tightly by means of the wire. The whole is lowered gradually into a liquid, and, once the plate is underneath the surface, the upward pressure on it supports it and the wire can be released. Tube T is clamped with the plate well below the surface of the liquid. Some of the same kind of liquid is now poured into T until it nearly approaches the level of the liquid outside it. Plate P still remains in position. When a little more liquid is poured into T and the levels inside and outside are the same, the slightest addition of liquid into T causes the plate P to fall. Thus the upward pressure on P and the downward pressure at the same depth are equal. This is found to hold good at any depth.

Pressure of the Air.—The earth's atmosphere is, of course, attracted towards the earth, and, as the latter rotates, moves round with it. Thus air exerts a pressure, as is shown by the following experiments:

(1) Into a tall cylindrical can made of thin metal, and with a small hole at the top, a little water is placed. The can is heated and the water boiled vigorously. Thus the air is driven out of the can by the steam generated, and, after a time, a rubber bung is firmly fixed in the hole at the top, and the can allowed to cool. When the steam, which displaced the air, has condensed on cooling, the sides of the tin collapse, because the pressure of the air outside is not balanced by a pressure on the inside. The same effect can be obtained by removing the air from such a can by means of an air suction pump.

(2) If an inverted beaker, or gas-jar, is gradually pushed down into water in a large glass vessel it is seen that only a little water rises into the beaker or jar. The air inside pushes down on the water and prevents its entrance. The volume of air in the beaker or jar becomes a little smaller—it is compressed owing to the extra water pressure.

(3) A well-known experiment is the "Magdeburg Hemispheres" experiment, which uses two well-fitting, hollow metal hemispheres. One has let into it a metal tube which can be opened or closed by a stop-cock. The pair are fitted together, a little vaseline being used to make them air-tight, and then attached to an air-pump, the stop-cock being opened. Much of the air inside the sphere is removed by the pump,

the stop-cock is closed, and the joined pair of hemispheres removed. It is found that a very strong pull is required to separate the hemispheres owing to the fact that the pressure of the air on the outside of the sphere is not balanced by the pressure of air inside.

(4) Over the connection to an air-pump and on its metal table is placed a special glass cylinder, on the top of which is securely fastened an air-tight indiarubber disc. As the air is removed from the vessel the greater pressure of the air outside causes the covering to bulge inwards. Another experiment is to place a partially inflated toy balloon inside a receiver from which the air is then gradually removed. The balloon swells out and eventually bursts, owing to the unequal pressure of the air outside and inside the balloon.

The fact that air exerts a pressure was first definitely shown by Torricelli in 1643, but before the time of Torricelli and Galileo (1564–1642) there is evidence of the prevalence of an idea that air had weight. Galileo was apparently the first to record a test of it; he observed that there was a difference in the weight of a vessel under ordinary conditions and of the same vessel with much air compressed into it by means of a pump. Previous to this there was much speculation regarding the vacuum, the existence of which was considered impossible. This doctrine went so far as to attribute to Nature the abhorrence of a vacuum. The action of a simple suction water-pump (Fig. 4) was considered to be due to this fact. When the piston P, fitting tightly in the cylinder C, was pushed down, the hinged metal plate, or valve *a*, closed up the opening in the pipe T which went down into the water. At the same time valve *b* in the piston was forced open, and so air in E was forced out to D and the open air. When the piston reached the bottom, valve *b* was found to drop down. On lifting up the piston, water was found to rise up into E, the valve *a* opening. This was attributed to Nature's abhorrence of

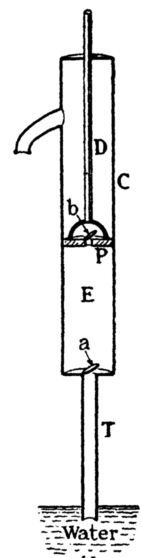


FIG. 4.
Simple water pump.

the vacuum apparently first formed in E, and its desire to fill the empty space.

Later it was shown that if the water W was more than 33 ft. below the pump, it would not work, and water only rose 33 ft. in the tube T. Galileo could not rid his mind of the above-mentioned idea, and so said that Nature's objection was a restricted one and apparently could be measured—it being equivalent to a column 33 ft. high.

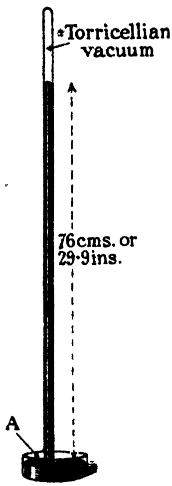


FIG. 5.—Simple barometer tube.

In 1643, the year after Galileo's death, his friend and pupil, Torricelli, carried out the famous Torricellian experiment which showed that air exerted a pressure which could be measured ; it was this pressure on the water in a well which pushed the water into a pump when the air in the pump was ejected. A tube, about a yard long, and closed at one end, was filled with mercury and closed up. The tube was inverted, lowered vertically into mercury in a bowl, and then the open end left free. Mercury was found to remain in the tube to a height of about 29·9 ins. (or 76 cms.). The space in the tube above the mercury obviously contained no air and so was called the *Torricellian vacuum*.

The air pressure on the surface of the mercury in the trough was balanced by an upward pressure in the liquid, a pressure which must be the same all along the same mercury surface level. It thus operated at A (Fig. 5) and so supported mercury in the Torricellian tube. Thus the height of the mercury in that tube was a measure of the pressure of the air. This pressure is called *Atmospheric Pressure*, and was thus shown to be 29·9 ins. or 76 cms. of mercury.

Torricelli obviously understood this before making his experiment, for he carefully calculated the length of tube necessary. He knew that water would rise 33 ft. up the pipe of a water pump. He must have argued that, since mercury is 13·6 times as dense as water, it

would only rise up $\frac{33}{13\cdot6}$ ft., which$=2\frac{1}{2}$ft. approximately. Such a tube is called a *barometer tube* (Gk. *baros*, weight; *metron*, measure), since it measures the weight of the air.

News of the Torricellian experiment (carried out at Florence) spread slowly and eventually reached Paris. Blaise Pascal heard of it and reasoned that if the earth's atmosphere did exert a pressure, then higher up in the air the pressure should be less owing to the shorter column of air above. He asked his brother-in-law in the South of France to experiment on the Puy-de-Dôme mountain. One Torricellian tube was fitted up and left at the bottom of the mountain, whilst another was set up half-way, and then again at the top of the mountain. Pascal's theory was justified, for whereas the mercury kept at constant level in the tube at the bottom of the mountain, the other tube showed a lower reading half-way up, and a still lower one at the top of the mountain.

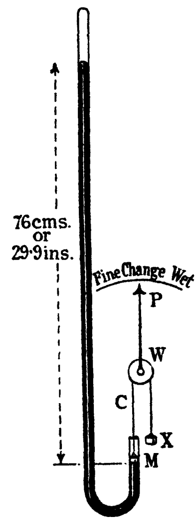


FIG. 6.—Hooke's siphon or wheel barometer.

Otto von Guericke set up a water barometer and observed the rise and fall of the level of the water; he explained that it was due to variations in the air pressure with variations in the amount of water-vapour in the atmosphere, and used the barometer to predict weather changes. Water in the form of a gas (or vapour) is less dense than dry air. 1·1 grms. of air occupy 1,000 c.c., whereas 1 grm. or c.c. of water when turned into steam occupies 1,600 c.c. approximately. Thus when the air has much water vapour in it, the pressure falls below the normal atmospheric pressure of 67 cms. of mercury, whereas when the air is very dry the pressure rises above that value.

Robert Hooke set up the first registering barometer —the wheel barometer—a form often seen in public parks. It consists of a siphon tube, as shown in Fig. 6. As the air

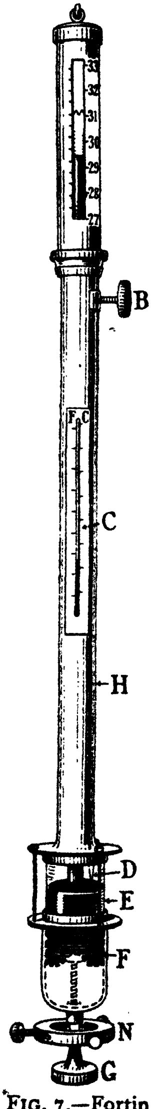


FIG. 7.—Fortin barometer.

pressure alters, the metal float M rises and falls with the mercury level, and the compensating weight X keeps the cord C stiff. Thus the pulley-wheel W is rotated and the pointer P indicates the probable changes in the weather.

When using a simple barometer tube for a precise measurement of atmospheric pressure, the readings of the levels of the mercury in the tube and the trough would need to be read, as both change together. To avoid this, and, at the same time to obtain accurate readings, Fortin introduced a form known as the *Standard* (or *Fortin*) *Barometer*, which has a fixed zero, and a vernier scale for accurately measuring the height of the barometer. The mercury trough is a glass cylinder E with a chamois-leather bag F attached at the bottom (Fig. 7). Fastened to the outside of the bag is a metal plate and a screw G which is held by the fixed nut N and can be moved up and down, altering the bag and hence the mercury level in it. Above the mercury is a fixed ivory pin D, and the process of using the barometer consists of adjusting the level of the mercury so that it just touches the point of D. This level corresponds with the position of the zero of the scale by which the height of the mercury in the Torricellian tube is read, but only part of the scale is fixed on the barometer. Such an instrument is permanently fixed in a laboratory and requires careful use.

A barometer which is portable, because it is small and needs no liquid, is the *aneroid barometer* (Gk. *a*, not; *neros*, wet). It consists of an air-tight metal box A (Fig. 8) from which some air has been extracted. Thus there is an excess of air pressure on the thin metal diaphragm B fixed in the top of the box, and so it is supported by a spring C.

Small movements of B consequent upon changes in atmospheric pressure are magnified by a system of levers, a pointer indicating the corresponding pressure on a scale. The whole is enclosed in a metal box with a glass face. A *barograph* is a self-recording aneroid barometer. An inked pen is attached to the pointer and it traces its movement on a cylinder which rotates, by clockwork mechanism, once in 24 hours. Aneroid barometers are also used as *altimeters* to indicate the height of a place above sea-level by the atmospheric pressure there.

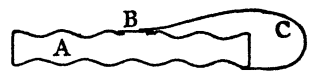


FIG. 8.—Aneroid barometer principle.

Atmospheric Pressure expressed in C.G.S. and F.P.S. Units.—We have seen that the pressure of the atmosphere, when normal, supports a column of mercury 76 cms. high; *i.e.* the pressure per sq. cm. is equivalent to the weight of a column of mercury 76 cms. high, and 1 sq. cm. area of cross-section. Such a column would have a mass of $76 \times 13\cdot6$ grms. at 0° C., since the density of mercury at 0° C.$=13\cdot6$ grms. per c.c.

Thus atmospheric pressure$=76 \times 13\cdot6$ grms. weight per sq. cm.

$$=76 \times 13\cdot6 \times 981 \text{ dynes per sq. cm.}$$
$$=1\cdot01396 \times 10^6 \text{ dynes per sq. cm.}$$

A pressure of 10^6 dynes per sq. cm. is called a *bar*, and this equals 1,000 millibars. On meteorological maps the air pressures are often expressed in millibars—the normal pressure being thus approximately 1,014 millibars.

In English units, the atmospheric pressure of $76 \times 13\cdot6$ grms. weight per sq. cm. is equivalent to

$$\frac{13\cdot6 \times 76 \times (2\cdot54)^2}{454} \text{ lbs. weight per sq. in.}$$

since $2\cdot54$ cms.$=1$ inch and 454 grms.$=1$ lb.,

or to $14\cdot7$ lbs. per sq. in. ($14\frac{1}{2}$ lbs./sq. in. approx.)

Pressure in Gases.—So far air pressure has been considered as a downward pressure, but gases behave like liquids; fluids, in general, transmit pressure equally in all directions. Indeed, this is the distinction between

fluids and solids (which do not so transmit pressure). Thus there is an upward pressure in the air in addition to the downward pressure. If it were not so the roof of a building could hardly be supported, for the pressure per sq. ft. due to the atmosphere $=144\times14\frac{1}{2}$ lbs. weight $=0{\cdot}94$ ton weight approx.

Obviously the air near the earth's surface is withstanding, or resisting, the air of the atmosphere above it, and is therefore exerting a large upward pressure. As we shall see later, under such a large downward pressure the air near the surface of the earth becomes squeezed into a smaller space, *i.e.* it is compressed, and so is more dense than that higher up. Since air resists, or opposes, the pressure of the air above it, it should be realised that it is elastic in that it objects to a change of volume. Very important consequences of this property of air will be studied later.

Other gases, too, behave like air, *i.e.* they exert a gas pressure. This can be shown by joining up a manometer, the instrument described on p. 10, to the gas supply. The molecules of a gas are obviously much more separated than those of its solid or liquid form. It has been stated, for example, that 1 c.c. of water becomes approximately 1,600 c.c. of steam. The molecules of a material are now considered to be in a state of motion which is very rapid in a gas, where there is great freedom of movement. Thus the molecules are constantly colliding with the sides of the containing vessel, giving the effect of a continuous push, or pressure, which we call the pressure of the gas.

Energy.—One of the most fundamental conceptions in physical science during the last century was that of *energy*. The development of the idea and the experimental work which led up to it are treated in Chapter VII.

It has already been stated that matter is inert, or incapable of doing anything by itself, but a force acting on a body can move it, and, only if it does so, we say that *work* has been done on the body. This conception of work is held because then it is measurable, depending as it does upon (*a*) the magnitude of the force F exerted,

which, of course, has to be sufficient to overcome the force-resisting motion, and (*b*) the distance, *d*, through which the force moves in moving the body. Thus work is measured by the product of F and *d* in appropriate units.

> If the force is in lbs. wt. and the distance in ft., work is in foot-pounds ;
> if the force is in dynes and the distance in cms., work is in dyne-cms., or *ergs* ;

e.g. if a force of 1,000 dynes (approximately the pull of the earth on 1 grm.) acts through 15 cms., the work done =15,000 ergs.

Energy is the name given to the capacity, or power, of doing work, and obviously anything which can produce motion possesses energy, *e.g.* a wound clock-spring, when released, moves the cog-wheel system of a watch. Stored-up energy of this kind is called *potential energy.* Water, when lifted above the ground-level, possesses energy due to its position, for in falling it can set a wheel in motion. Energy of this type is also called potential energy. When the water is falling, *i.e.* it is losing its position but gaining motion, its energy gradually changes to energy of movement. When the water reaches ground-level all its energy is energy of motion, which we call *kinetic energy* (cf. kinema for moving pictures). During the course of study planned in this book it will be realised that energy is continually being transferred from matter to other matter—whenever work is done energy is transferred, and often released in another form. It will be understood, too, that heat, light, sound, electricity, etc., are all forms of energy, whilst energy is also stored in a form we call chemical energy. Animals eat food and so convert some of the energy stored in it into such a form that muscular action is possible, with a consequent production of motion, *i.e.* kinetic energy. It is found that whenever energy is utilised to do work it is immediately released in another form ; energy is never destroyed. Thus on a parallel with the theory of conservation of matter is that of *Conservation of Energy,* i.e. energy can neither be created nor destroyed. But as has been stated, it can be

transferred, and life consists essentially of a continuous transference of energy.

Circular Measure of Angles and Trigonometrical Ratios for Small Angles. In work in Physics it is often useful to use the trigonometrical value of the sine or tangent in lieu of its circular measure. This is quite legitimate for angles up to approximately 15°. *E.g.* consider an angle of circular measure 0·25 radian.

$$\text{Its value in degrees} = \frac{180}{\pi} \times 0{\cdot}25, \text{ since } \pi \text{ radians} = 180^\circ;$$

$$\text{\textit{i.e.} it} = 14^\circ\ 20' \text{ approx.}$$

From tables it is seen that

$$\left.\begin{array}{ll} & \sin 14^\circ\ 20' = 0{\cdot}2476 \\ \text{and} & \tan 14^\circ\ 20' = 0{\cdot}2555 \end{array}\right\} \begin{array}{l}\text{and the circular} \\ \text{measure is } 0{\cdot}25.\end{array}$$

Thus the error in taking the trigonometrical ratio of the sine or tangent *for the circular measure* is not 1 per cent. In particular, the use of the tangent instead of the circular measure value will be made in the work in Light.

Exercises on Chapter I

1. State and explain some of the chief properties of matter.

2. Discuss the question of the divisibility of matter, giving some details of our ideas regarding the existence of molecules and atoms.

3. Differ between mass and weight of a body. Would there be any difference in the weight of a 1 lb. mass when weighed (*a*) at the equator, (*b*) in London, (*c*) at the North Pole, (*d*) 20,000 ft. above the earth's surface, (*e*) at the bottom of a deep coal-mine? Give reasons for your statements.

4. Differ between the density and the specific gravity of a substance. Explain "pressure."

5. What are the factors affecting the pressure in a liquid? Give experimental evidence to support your statements.

6. Give experimental evidence in favour of the belief that the air around us exerts a pressure. Give any details you can regarding atmospheric pressure.

7. State the principle of the mercurial barometer and describe a good form of barometer for laboratory use. A barometer reads 75·82 cms. of mercury at the foot of a hill and 73·60 cms. at the summit. Calculate the height of the hill in metres, taking the density of mercury as 13·6 grms./cm.^{3}, and the average density of air as 0·00125 grm./cm.^{3} What is the difference between the pressures in the two positions, expressed as dynes per sq. cm.? [*J.M.B.* 1926.]

8. Describe fully how you would set up a simple mercury barometer. Two barometers are set up in the same trough. The diameters of the tubes are the same at the level of the mercury in the trough, but the bore of one is uniform, while that of the other widens gradually from the level upwards. Explain how the following quantities differ, if at all, in the two cases: (*a*) the weight of mercury above the free level, (*b*) the pressure due to the mercury at the free level, (*c*) the height of the mercury above the free level.

[*C.W.B.* 1929.]

9. Describe a good form of standard barometer. Express the atmospheric pressure in (*a*) dynes per sq. cm., (*b*) lbs. per sq. in., when the barometric height is 75 cms. given the density of mercury$=13{\cdot}6$ and $g=981$, both in C.G.S. units.

CHAPTER II

HEAT AND TEMPERATURE

QUITE early in life we realise the feeling of warmth we get when the sun is shining and what a difference there is when it is hidden from us. Soon we learn such phrases as "hot dinner," "cold ice-cream," "the heat from the fire," etc., our senses thus appreciating differences between bodies with regard to hotness. Standing before a fire, we feel ourselves getting "hotter." Why do we become hotter? In the days of Newton, when scientists endeavoured to explain phenomena they had observed, the feeling of warmth we experience when in the sun was said to be due to the sun giving out—continuously and in all directions—a material fluid which they called *Caloric.* Some of the early Greeks—*e.g.* Democritus who lived 460–370 B.C.—had the idea that heat (received from the sun or fires) was matter. The acceptance of a material nature for heat by Newton, and other scientists of his time, was greatly due to the "Phlogiston Theory of Combustion (or burning)" put forward by Stahl. His theory was that a burning body gave off a substance called *phlogiston.* Since burning was accompanied by the production and giving off of heat, it is easy to see how these two theories went together. Thus heat was considered to be a material fluid—caloric—the particles of which were said to repel one another, but were attracted to other matter. Thus hot bodies gave off caloric and other bodies received it. The discovery by Lavoisier, a French scientist (1743–1794), that burning caused a gain, not a loss, in weight resulted in the collapse of the Phlogiston Theory, and about the same time the Caloric Theory

lost favour. A very simple argument against a material theory of heat is that a piece of matter, in air, weighs the same, hot or cold (unless a chemical action takes place when heated).

We now consider that heat is a form of energy (this will be treated in Chapter VII)—a body which is giving out heat is losing energy and so becoming colder, whilst one which is receiving heat is gaining energy and so becoming hotter. Our sense of touch enables us to differ between the hotness and coldness of bodies, but most of us have already realised that " opinions differ," that our own sensations are not always reliable. When we enter a crowded room we may remark on its extreme hotness, only for the occupants to disagree with us. A simple experiment with three bowls of water—one containing cold water, one very hot water, and the third a mixture of the two—shows how unreliable is our sense of touch. If one hand is held in the hot water and the other hand in the cold water, and then, after a little time, both hands are simultaneously transferred to the third bowl of water (lukewarm), one hand feels cold and the other warm *i.e.* the brain receives two distinctly different impressions regarding the same water. It is obviously a question of contrast, or standard of comparison, and it follows that to compare the hotness of bodies we need a standard of comparison, and we call **the degree of hotness of a body, compared with some standard, the temperature of the body.**

To measure the temperature of a body we now use instruments called *thermometers* (Gk. *thermé*, heat+ *metron*, a measure) and we must consider

(*a*) the nature of the fixed standard, or standards,

(*b*) the method by which the change in hotness indicates itself on the instrument.

In studying the growth of thermometer knowledge we shall realise (*a*), whilst to understand (*b*) it is necessary to consider the general *effects of heat* and see which can be utilised for indicating a change in hotness, or temperature.

The *effects of heat* are several.

(1) **Change of State.**—We know that when a kettle containing water (a liquid) is left on a hot fire, the water

changes into steam (a vapour). When a candle is burning, the heated tallow or wax near the flame changes from solid to liquid, and the liquid fat rises up the wick and is burned at the top. When a substance is cooled, *i.e.* heat taken away from it, a reverse change of state often takes place. As the liquid fat runs out of a hot leg of mutton it cools down, and eventually a layer of solid fat is seen in the dish. No doubt you have all noticed the formation of drops of water on the inside of the window in a warm room or railway carriage. This is due to the cooling (and condensation into liquid) of water vapour in the air near the window which is kept cold by the cold air outside.

(2) **Change of Temperature** (without change of state).

(3) **Change of Dimension**—expansion on heating and contraction on cooling.

(*a*) *Gravesande's Ball and Ring Experiment* (Fig. 9) to show the expansion of a solid on heating. A small iron retort stand A has a ring R. Above this is a clamp C, supporting, by a wire chain, an iron ball B which is minutely smaller than the inside of the ring, and so will just pass through it when cold. The iron ball is heated in a bunsen flame till nearly red-hot and is then fixed so that it falls on to the ring R. It is now found that the ball will not pass through the ring owing to the expansion on heating. If, however, the ball is left on the ring, it gives heat to the ring, which expands and so allows the ball to drop through.

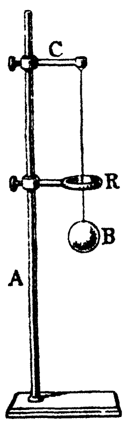


FIG. 9.

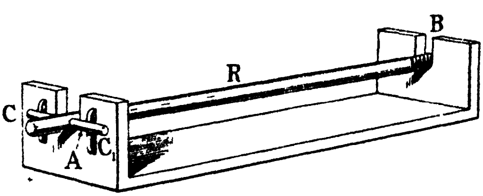


FIG. 10.

(*b*) *Breaking cast iron by a contraction bar.* The latter is a special bar R (Fig. 10) which is heated till it is nearly red-hot. It is then fixed in the clamp and screwed tightly at B, whilst a

small piece of cast iron A, fixed through a hole in the rod, and pivoting on CC_1, holds it at the other end. As the rod cools and tends to contract, it is held firmly by the screw, but the great contractile force causes the cast iron A, holding the iron at CC_1, to break under the strain.

(*c*) *The expansion of a liquid* on heating can be shown by means of a round-bottomed flask (A in Fig. 11) with cork to fit. Through a hole in this is fixed a long piece of glass tubing B of small bore. The flask is filled with coloured water (or other suitable liquid), and then the cork pushed in so that water rises about two or three inches up the tube above the cork. The level of the liquid in B is now carefully watched as the flask is plunged into boiling water. The level is seen first to fall a little. This is due to the fact that the glass flask is heated first and expands, so that the liquid does not fill it to the same level. As the liquid receives heat, from the water through the glass, it expands and rises up the tube B, showing that the expansion of the liquid is much greater than that of the glass. If the flask be taken out and allowed to cool down to its original temperature, the level of the liquid in B will be found to be at its first height.

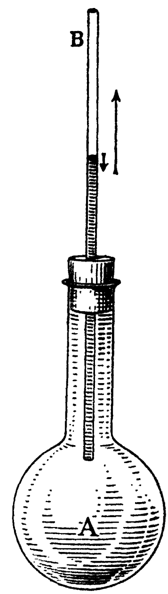


FIG. 11.

(*d*) *The expansion of a gas* on heating, and contraction on cooling, can be shown simply with air (Fig. 12). Take a dry flask and tube, similar to that used in the previous experiment, and heat the flask by holding it in boiling water for a time. Then invert the flask and hold the end of the tube in water. The contraction of the air as it cools is readily seen from the rise of water up the tube, and it seems to be much greater than the expansion of the liquid in the previous experiment.

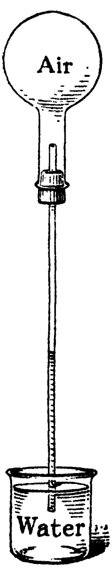


FIG. 12.

(4) **Changes in Composition—Chemical Effect.—** Many substances when heated become changed into other

substances. If sugar, for example, be heated in a test-tube water is given off and condenses at the top of the tube, whilst carbon is left at the bottom of the tube.

The subject of Chemistry deals with such changes in composition.

(5) **Electrical Effect.**—One common method of obtaining an electric current in a wire is by means of a cell—the chemical energy of the materials in the cell becoming converted into electrical energy. Seebeck in 1821 discovered a means whereby heat could be converted into electricity. Two dissimilar metals, *e.g.* iron and copper, though an antimony-bismuth combination is the best, are joined together at one end, the other ends being connected to a sensitive galvanometer (Fig. 13), an instrument which indicates when an electric current is flowing in a circuit. On heating the junction of the two metals an electric current is set up and the pointer on the galvanometer is seen to move. Such a combination of two dissimilar metals is known as a thermocouple and the current is called a thermo-electric current.

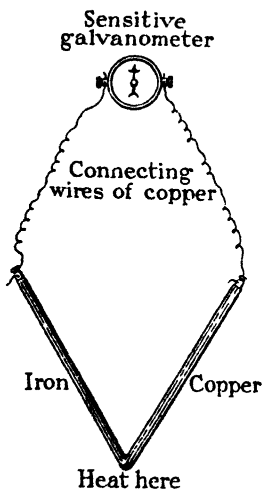


FIG. 13.

(6) **Internal Strains.**—After being heated many bodies show weaknesses possibly due to some internal rearrangement of their molecules, and most glass when heated becomes weakened. If a small piece of glass be heated and cooled suddenly, by dropping in water, it breaks up into many small fragments and is seen to have a peculiar frosted structure. Pyrex glass, the latest discovery in glass for cooking and laboratory utensils, does not weaken appreciably in use. Relative to this point, the behaviour of iron is interesting. When heated till white-hot and then allowed to cool, a piece of iron gradually changes from white to red and then becomes no longer

bright. A little later, at a certain temperature known as the temperature of *recalescence*, the iron suddenly glows again. This generation of heat is probably due to a rearrangement of the molecules at that temperature.

Historical Development of the Thermometer.—Of the changes due to heat just summarised, the most convenient for use in showing small changes of the degree of hotness, or temperature, is the change of volume produced in a fluid. If the fluid used has no peculiarities of behaviour it always expands, on being raised in temperature, and contracts, on being lowered in temperature, over the range of temperature at which it exists in the same state. The change in size can thus be used to indicate the nature of the change in temperature.

Records show that the first instrument to show changes in degree of hotness was set up by Galileo about 1593. This was really a thermoscope (Gk. *scopein*, to tell). Galileo took a glass bulb about the size of a hen's egg and with a long stem of the thickness of a straw, slightly warmed the bulb, and then inverted it and stood the end of the stem in a vessel of water (Fig. 12). As the air in the bulb cooled and contracted, water rose up the stem. The level of the water in the stem varied with the temperature (and thus volume) of the air in the bulb. The drawback to this instrument is that the level of the water in the stem also varies with changes in atmospheric pressure, and if an instrument is to be used for comparison from day to day the level must vary only with temperature.

The next development was due to Jean Rey, a Frenchman, who in 1631 inverted Galileo's bulb and filled it with water to about half-way up the stem, the level of the water varying with the temperature but being unaffected by variations in air pressure. But as the stem was open to the air, water gradually evaporated from the surface and so readings on the instrument gradually changed. The obvious remedy for such a fault was to seal the end, and this was carried out by scientists of Florence about 1654. But if in the stem above the liquid some air were left, it would exert a pressure on the liquid and oppose the expansion of the latter when heated. This opposition

would increase as the temperature, and hence expansion, increased, and so the thermometer would not be sensitive, *i.e.* it would not show small temperature changes, at higher temperatures. So evacuation of the air being necessary spirits of wine (alcohol) was used as the thermometric fluid. The bulb and part of the stem were filled with it and the bulb was immersed in boiling water. At that temperature alcohol vaporised and thus forced the air out of the stem. When sufficient alcohol was vaporised the open end of the stem was sealed by a hot flame. The level of the liquid in such a thermometer is only affected by temperature.

Further developments were in the nature of markings, to form a scale, for use in reading the thermometer Several attempts were made at fixing two temperatures (called the **fixed points**) and dividing the distance between into regular divisions, later called " degrees." Florentine scientists used for the cold temperature that of ice or snow in the severest frosts of winter, and for the hot temperature that obtained at middle summer, and by using a scale based on them they found that the temperature of melting ice was constant. The first thermometer scale which has survived, however, was due to *Fahrenheit* (1686–1736), who made experiments on the temperatures at which liquids boil, since previous observers had noted that a few liquids had constant boiling points. Fahrenheit found that all the liquids he dealt with had constant boiling points. Fahrenheit was chiefly responsible for the growth in use of mercury thermometers (mercury first being used about 1670), but in 1714, using alcohol thermometers, he used two fixed points : (1) the temperature of mixed ice, salt, and water which he called zero, and (2) the temperature of a healthy human being and which was known to be constant. He called the latter 24 degrees (represented 24°) and found that on the scale so obtained the temperatures of melting ice and boiling water were 8° and 53° respectively. With his mercury thermometer Fahrenheit multiplied the values by 4 and ignored the zero position, as he found it was not reliable owing to a variation with the relative proportions of the ice, salt, and water. Thus

he formulated the scale still used on *Fahrenheit Thermometers*, where the fixed points are

	Melting Point of Ice	32° F.
and	Boiling Point of Water . . .	212° F.,

the interval between the fixed points, and which equals 180° F., being called the *fundamental interval.*

Réaumur Thermometer.—This thermometer is little used now except in parts of Switzerland and of Eastern Europe. Réaumur, a Frenchman, used alcohol in preference to mercury, because of its greater expansion, bulk for bulk, and he observed that 1,000 parts of a mixture of 5 parts of alcohol with one of water expanded to 1,080 parts between the melting point of ice and the boiling point of water. He thus used a thermometer with these two temperatures as fixed points and marked 0° and 80° respectively, the fundamental interval thus being 80°.

Centigrade Thermometer (from L. *centum*, a hundred+*gradus*, step). In 1742 Celsius of Upsala, Sweden, introduced the idea of using 100 divisions on a mercury thermometer. He used the boiling point of water as the zero, but in 1750 Strömer, a colleague of Celsius, inverted the arrangement into the present form of the Centigrade thermometer, with the fixed points

	Melting Point of Ice	0° C.
and the	Boiling Point of Water . . .	100° C.

Comparison between the Fahrenheit and Centigrade Scales.—Fig. 14 shows clearly the difference between these two common scales of temperature, and the conversion of readings from one scale to the other should be readily understood. It is quite sufficient to remember the respective fixed points, whence it is seen that the fundamental interval is 180° F. or 100° C.

EXAMPLES OF CONVERSION.—(*a*) Convert the reading 55° C. to degrees Fahrenheit.

55° C. is 55° C. above the lower fixed point.
But 100° C. are equivalent to 180° F.

$\therefore$ 55° C. ,, ,, $\frac{180}{100} \times 55°$ F. $= 99°$ F.

But the melting point of ice, the lower fixed point on the Fahrenheit scale, is 32° F.

Hence 99° F. above the lower fixed point is $99+32 = 131°$ F.

Thus the reading 55° C. = 131° F.

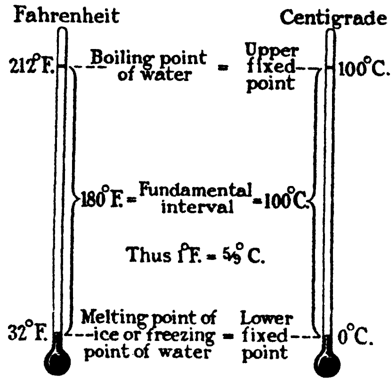


FIG. 14.

(*b*) Convert the reading 194° F. to degrees Centigrade.

194° F. is $(194-32)°$ F. or 162° F. above the lower fixed point.

But 180° F. are equivalent to 100° C.

$$\therefore\ 162°\text{ F.} \quad ,, \quad ,, \quad \frac{100\times 162}{180}=90°\text{ C.}$$

But the lower fixed point on the Centigrade scale = 0°.

Thus the reading 162° F. = 90° C.

The student is strongly advised to calculate interchanges on these scales from first principles, as shown above, and not to learn formulæ for conversion. When, however, a large number of conversions are to be made, time is saved by using a graphical method. A graph is drawn (Fig. 15) with the values of the two axes the equal ranges 0–100° C. (for OA) and 32–212° F. (for OD) and a rectangle AODB completed. Then O represents the lower fixed point on both scales, and B similarly

represents the upper fixed point. A straight line OB is thus a line of equivalent temperature values, assuming both thermometer scales are uniform. Then the Fahrenheit scale value corresponding to a Centigrade value

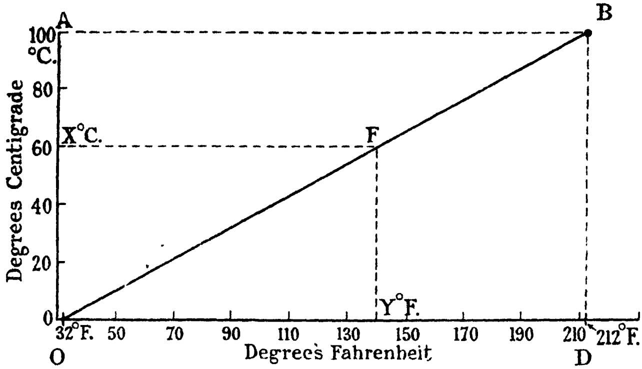


FIG. 15.

X° C. is found by drawing a line XF, parallel to OD, cutting OB in F. From F a line is drawn parallel to OA, cutting OD in Y. Then Y is the Fahrenheit scale value corresponding to X° C. For accuracy a large scale graph is drawn.

Making Thermometers.—To make a simple thermometer in the laboratory a bulb is blown on one end of a piece of glass tubing with a fine bore. A funnel is joined to the top of the tube by rubber tubing and some coloured alcohol is poured into the funnel. The bulb is immersed in hot water, the air inside expands and is driven out, rising above the liquid in the funnel. When the bulb is allowed to cool, alcohol is sucked back into the stem, replacing the air previously driven out. A little gentle tapping assists this liquid to fall down into the bulb. By repeating this process alcohol is made to fill the bulb and part of the stem, more liquid than is needed in the thermometer being allowed to enter, since, before sealing the top, this alcohol is boiled—the bulb being immersed in boiling water or an oil bath—(*a*) to drive off air in the stem above the alcohol, (*b*) to drive off any dissolved air

in the alcohol. Thus the excess alcohol is vaporised and expelled, and then a hot bunsen flame is directed on the top of the stem to seal it. Manufacturing methods are, of course, not so tedious, for the air is removed by a pump and the liquid carefully admitted little by little so that the thin glass bulb is not broken.

There are several points to be observed regarding thermometer construction.

(1) The size of the bulb and the bore of the tube depend on the required range of the thermometer (the number of degrees it is to register) and sensitivity. Obviously, a thermometer to read to 1/10th° must have a longer movement of the thread for 1° change in temperature than a thermometer only reading to 1°, and hence a longer tube with a finer bore.

(2) The quantity of liquid used should be a minimum, otherwise on being heated up it might take so much heat from the source whose temperature is being recorded as to result in a lowering of that temperature.

(3) The bulb of the thermometer should be thin so that the latter will be quick in action—heat quickly passing through to warm up the liquid.

Thermometric Liquids.—For ordinary ranges of temperature there are two liquids commonly used—mercury and alcohol—and a comparison of their properties gives important information regarding their respective merits. They are tabulated below.

Mercury.		*Alcohol.*
Boiling point 357° C.	This shows why mercury thermometers are used for general scientific work, and alcohol thermometers for low temperatures.	Boiling point 78° C.
Freezing point −39° C.		Freezing point −114·9° C.
Easily seen in glass.		Has to be coloured.
Mercury tends to stick to the glass when falling, and so, if possible, a mercury thermometer should be read when the thread is rising.		Alcohol does not stick, but it does tend to condense at the top of the tube.
Has a regular expansion in glass over ordinary temperature ranges.		Expands somewhat irregularly.

Alcohol is more sensitive as regards expansion than mercury, but mercury requires less heat to warm it up an does an equal volume of alcohol. Thus mercury id alcohol thermometers of equal sensitivity and quick-ss of action can be made by using less alcohol than ercury.

Mercury thermometers are used ordinarily up to o° C., but they can be used for higher temperatures if inert gas like nitrogen or argon is put in the tube ove the mercury. As will be explained in a later apter, this gas exerts a pressure on the mercury and prevents it boiling; it merely expands and can dicate temperatures to 600° C. or 700° C. For read-gs above 540° C. quartz must be used instead of glass, the latter softens above that temperature. For low mperature work alcohol or a special petroleum ether reezing point about −190° C.) is used.

For temperatures outside the limits of liquids in glass quartz, other methods of measurement have to be vised. These methods depend on electrical phenomena, d two forms are as follows:

(1) Thermometers (called pyrometers), using the inciples of the thermo-couple already explained on 26, are used for measuring the temperature of furnaces, c. A large number of thermo-couples are joined up, forming a *thermopile*, which is sensitive to a small antity of heat. It is thus a thermo-electric battery and suitable for measuring small quantities of heat received m a source of heat. By special methods we are able calculate the temperature of a source knowing the te at which it is emitting heat.

(2) *Platinum Resistance Thermometers* are used for easuring extremes of high and low temperature. When ctricity is being sent through a wire which is a con-ctor (*i.e.* allows the electricity to pass through it), ere is an opposition to the flow of electricity along the re, just as there is an opposition to the flow of water in pipe. The opposition offered by the wire to the electrical w is called the *resistance* of the wire, and it can be asured by special electrical methods. The electrical

resistance of most metallic wires increases as the wire gets hotter, the variation in resistance with change in temperature being fairly regular and so can be used to indicate temperature. The wire must have an extremely high melting point and, after much experimental work, a thin platinum wire was found to be suitable, if loosely wound on a mica insulating frame and not allowed to come into contact with fumes from the furnaces, etc., in which it was used. This form of thermometer is very sensitive.

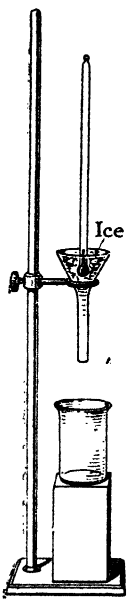


FIG. 16.

For the measurement of low temperatures, gas-thermometers are also used, the thermometric fluid being a gas. Such thermometers will be explained in Chapter V.

Determination of the Fixed Points of Ordinary Range Thermometers. — The lower fixed point (that of the melting point of ice) is obtained by standing the thermometer with its bulb immersed in crushed melting ice in a filter funnel, the thermometric liquid level, when constant, being marked by a scratch on the glass (Fig. 16).

The higher fixed point (that of the boiling point of water) is obtained by leaving the bulb and most of the stem in a current of steam in a vessel called a *hypsometer*, shown in Fig. 17. The manometer or water-gauge M is used to ensure that the steam is at atmospheric pressure when the level of the mercury in the thermometer stem is marked (the reason for this will be explained later in Chapter VIII).

The distance between the fixed points is marked off into equal divisions, each representing a degree, or fraction of a degree, on the scale used. This method of marking assumes that the bore of the tube is absolutely uniform and that the liquid expands uniformly. If a thermometer is required to give very accurate readings, then the divisions must be standardised against an exact thermometer. (This can be done at the National Physical Laboratory at Teddington.) For simple corrections of the readings

is sufficient to find the errors (if any) in the two fixed points, and then, by means of a simple graph, an approximate correction can be made for any reading of the thermometer. In Fig. 18 errors in the readings are shown, AB

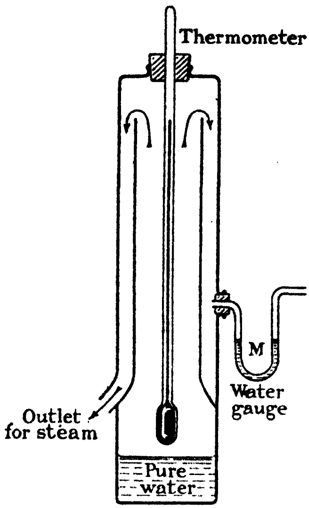


FIG. 17.—Hypsometer.

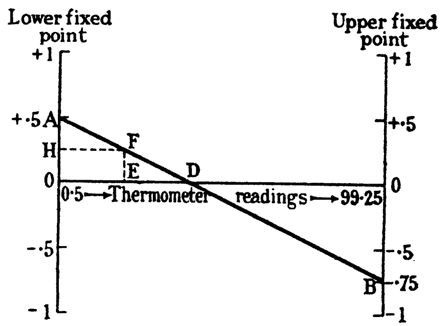


FIG. 18.

being a line joining the two points of the " fixed point " errors. Thus when the thermometer reads D°, that reading is correct. When the reading is E°, draw a line parallel to the error line to meet AB in F. Then EF, which equals OH, is the error and so the correct value for the reading E° is E° — OH°.

A graph of the form shown in Fig. 19 enables one to read off the corrected temperature values directly. In the example shown, the readings for the fixed points are supposed to be —1° C. and 101° C.

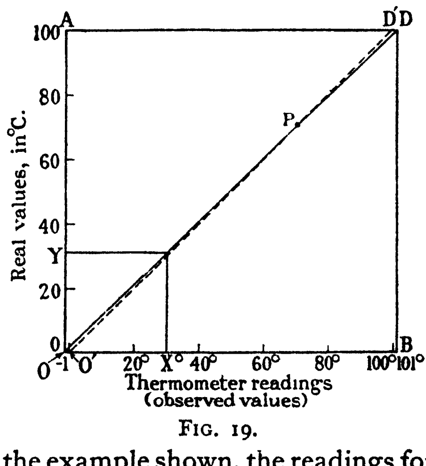


FIG. 19.

Thus OA represents real values and is divided 0° to 100°, and OB represents observed values and is divided −1° to 101°.

Complete the rectangle OADB. Then the line OD gives the relation between observed and corrected readings, *e.g.* the corrected value of a reading of the thermometer X° is Y°.

If the thermometer had no errors in the fixed points, then the corresponding line would be O′D′, where the positions are: O′ 0, 0; D′ 100, 100. Thus where OD and O′D′ intersect at P is the position where the thermometer reading *is* the correct reading. For this method a large piece of graph paper is required to give accuracy.

Other corrections are necessary if the highest degree of accuracy is required. These include—

(*a*) the "stem-correction" for the part of the thread of liquid which is not immersed in the liquid the temperature of which is being taken;

(*b*) a correction for the expansion of the scale, if it is a metal one, outside the thermometer;

(*c*) a correction for altitude;

(*d*) a correction for the latitude of the place.

Maximum and Minimum Thermometers.—In meteorological work the maximum temperature at day and the minimum temperature at night are often required. To obtain them, thermometers with special recording devices are used.

The common form, invented by Six in 1782, gives both maximum and minimum readings. The principles involved are also used in separate maximum and minimum thermometers.

In the *maximum* form, mercury, with a steel index on the top of the thread, is used. This steel index is pushed up as the mercury expands, but when the mercury contracts on cooling the steel index remains behind, owing to springs attached to it, and indicates the highest temperature reached (Fig. 20).

In the *minimum* form alcohol is used because of the fact that alcohol wets a steel index (whereas mercury

does not) and so passes by it, but, on contraction, sucks it back when the index is at the surface of the alcohol (see Fig. 21). Thus the index remains, attached to the bore by means of the springs, to indicate the lowest temperature reached. If the lowest temperature reached during a night was $(32-x)^\circ$ F., then we say there were x° of frost (*i.e.* x° F. below freezing point).

In both these types the steel index is returned to the liquid level by means of a magnet.

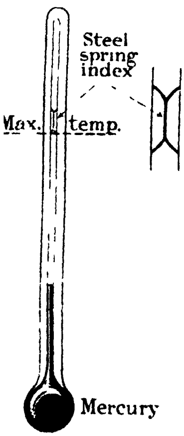


FIG. 20.—Maximum Thermometer.

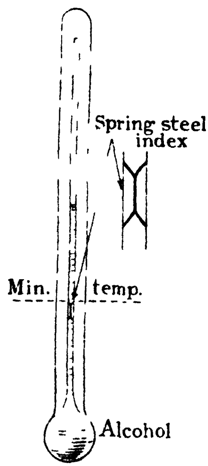


FIG. 21.—Minimum Thermometer.

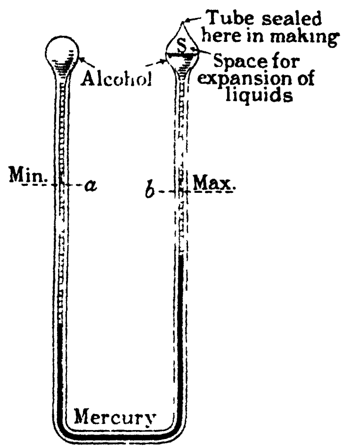


FIG. 22.—Six's Maximum and Minimum Thermometer.

Six's Maximum and Minimum Thermometer is shown in Fig. 22. During a rise in temperature the liquids expand into S, the mercury pushing index *b* up, and the alcohol on that side assisting to suck it back. On cooling and contraction of the liquids, *b* is left and its lower end records the maximum temperature reached, the mercury receding and the alcohol passing by it. The index *a* is then pushed back, and on a rise in temperature it is left for its lower end to record the minimum temperature reached, the mercury again receding from, and the alcohol passing by, the index.

Rutherford invented maximum and minimum thermometers in which a glass index is used instead of the steel index. These thermometers are always used in a horizontal position and are often seen in greenhouses, incubators, etc. They were invented a few years after Six's and were considered then to be an improvement.

Phillips introduced another idea for a maximum thermometer in which a small, narrow constriction (a very narrow bend) in the stem near the bulb is the main feature. On warming up, the mercury in the bulb expands into the stem. On cooling, the thread above the constriction remains, and the top of it indicates the maximum temperature.

This is applied in *clinical thermometers* (Fig. 23) used to find the temperature of the human body. They are small, sensitive, and quick-acting thermometers (and thus fragile since the bulb needs to be thin) with a range

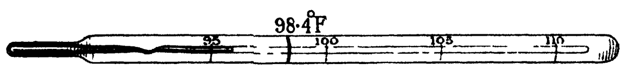


FIG. 23.—Clinical Thermometer.

usually from 95° F. to 110° F., the normal body temperature being 98·4° F. The thread is made to go back to the bulb by shaking, which explains why a doctor or nurse always shakes the clinical thermometer before using it to take a temperature. Better forms have the scale on milky glass and the front of the glass tube lens-shaped, so as to make it easier to observe the very thin mercury thread and the markings which are, of necessity, fine.

EXERCISES ON CHAPTER II

1. Explain the meaning of the term *temperature*. Describe the mode of construction and graduation of the ordinary mercury thermometer, and mention the chief corrections which have to be made in order to get an accurate reading. [*L.G.S.* 1920.]

2. Draw up a list of the essential steps in the construction of a mercury thermometer, indicating *briefly* how each step is carried out. How does

the maker arrange so that the thermometer reads over any desired range, *e.g.* from $-10°$ C. to $110°$ C.? [*L.G.S.* 1922.]

3. Convert 15, 30, 50, 70 and 90° C. to ° F., and 50, 77, 113, 149 and 167° F. to ° C.

4. How would you test the accuracy of the "fixed points" on a mercury thermometer? Convert 0° F. and 100° F. to Centigrade, and 32° C. and 212° C. to Fahrenheit. [*C.W.B.* 1928.]

5. Describe the Centigrade and Fahrenheit temperature scales. Rewrite the following statement, using the Fahrenheit scale: "The heater only raises the temperature of the water 25° C., so that, in winter, when the temperature of the tap water is usually below 10° C., a really hot bath cannot be obtained by means of it." [*L.M.* 1926.]

6. Describe experiments illustrating the effects of heat and discuss the possibility of the use of these effects to indicate the degree of hotness attained by a liquid.

7. Being given a mercury thermometer the bore of which is uniform, describe how you would determine any possible errors in its standard points, and show how, knowing these errors, you would construct a graph from which the error of any other reading within the range of the thermometer might be directly read off.

8. State clearly how you would construct a Centigrade thermometer for measuring temperatures between 0° C. and 100° C., giving the materials you would use and your reasons for using them in preference to other possible substances If the temperature of melting paraffin wax is 54 on a Centigrade scale, what will be the temperature on a Fahrenheit scale? [*J.M.B.* 1923.]

9. Describe the construction of a mercury thermometer. How would the construction of a thermometer intended to read from 100° C. to 250° C. differ from that of one reading from 0° C. to 100° C.? [*L.M.* 1921.]

10. A faulty Fahrenheit thermometer is handed to you for report as to the accuracy of its readings at the upper and lower fixed points. Describe, *in detail*, how you would conduct the necessary tests. If the readings you obtained were 215 and 30 respectively, what readings on this instrument would record the correct temperatures on the Fahrenheit scale? What are the *distinctive* features about a sensitive mercury thermometer which reads to 1/10th degree? [*L.G.S.* 1926.]

11. Describe and explain the action of a thermometer suitable for recording the highest and lowest temperatures attained during a given interval of time. If the greatest and least readings recorded by such a thermometer be 50° F. and 18° F. respectively, what are the corresponding temperatures on the Centigrade scale? [*L.G.S.* 1921.]

12. How are the "fixed points" of an ordinary thermometer obtained? Describe a clinical or any other form of maximum thermometer. Explain the conditions which contribute to its sensitiveness. [*J.M.B.* 1928.]

13. What are the fixed points of a mercury thermometer? Describe how they are determined. The freezing point of a Fahrenheit thermometer is correctly marked and the bore of the tube is uniform, but it reads 76·5° when a standard Centigrade thermometer records 25°. What is the reading of the boiling point on this Fahrenheit thermometer? [*L.M.* 1929.]

CHAPTER III

EXPANSION OF SOLIDS

TWO simple experiments illustrating the expansion of solids on heating and their contraction on cooling were described on pp. 24, 25. The increase in length, or the "linear expansion," of substances on heating is seen in the difference in tautness of copper telegraph wires in summer and winter. Railway lines, on account of their variation in length during the seasons, are fixed with small gaps about every 12 yards. Piano wires have been known to break on cold nights owing to the strain of their contraction. Expansible metals are being commonly used in structures whose temperature varies considerably during the year. It is therefore important to study the changes in length, due to changes in temperature, of these substances.

Measurement of the Linear Expansion of a Solid on Heating.—The use of the spherometer gives very accurate results. It is based on the principle of the micrometer screw

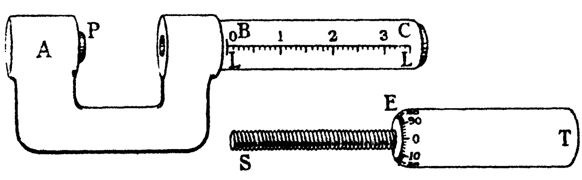


FIG. 24.—Micrometer Screw Gauge.

gauge. The latter consists of two parts. A rigid frame ABC (Fig. 24) has a hollow arm BC, on the outside of which is a scale as shown, and which is threaded inside. The second part is a screw S, which screws into the arm BC, and a tube T, which fits over the outside of BC. When conjoined and

the end of S just touches the post P, the end E of the tube T just reaches the zero line of the scale of BC. The circumference of E is divided into 100 equal parts; the one which meets the scale line LL is marked as the zero position and the others marked 0-99 accordingly. The pitch of the screw is such that the tube moves 1 mm. (or a known fraction of a cm. or an inch) per revolution, and so when tube T rotates x divisions, *i.e.* $x/100$ths of a revolution, this is equivalent to a lengthwise movement of $x/100$ths of a mm. If an object is just held between P and S, its thickness can be measured. For example, if on looking at E it is found to be between 1·3 and 1·4 cms. on scale BC, and at the same time the marking 37 divisions of scale E meets the line LL, then E is 37 hundredths of 1 mm. from the 1·3 cms. mark. Hence the thickness of the object

$$= 1{\cdot}3 \text{ cms.} + 37/100 \text{ of a mm.}$$
$$= 1{\cdot}337 \text{ cms.}$$

A spherometer (Fig. 25) is a frame, threaded through the middle, with three rigid legs, the points of which are in the same plane and at the corners of an equilateral triangle. A screwed rod M carries a circular plate P, which is usually divided at its edge into 100 equal divisions, and passes through the threaded middle of the rigid frame. One complete revolution of this plate causes it to rise or fall 1 mm. or some known fraction of an inch or cm. The frame carries a vertical scale S so that the plate rotates with its scale near S, and any up or down movement of the middle leg M can be measured.

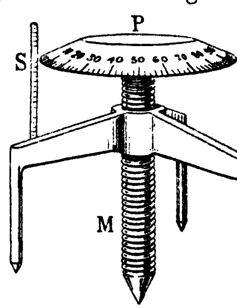


FIG. 25.—Spherometer.

The rod of substance AB (Fig. 26) is fixed by rubber bungs through the centre of a metal or glass water-jacket which has two outlets as shown. The bottom of the rod is wedged against a piece of porcelain at the base, so that, on heating, AB only expands upwards. The spherometer is rigidly clamped just above the top of the rod. Water is run in at D to fill the jacket, so that AB attains the temperature of the water (read on the thermometer T fixed into the jacket). The centre leg of the spherometer

is screwed down till it just touches the top A of the rod. An electrical device, as shown in the diagram, indicates when contact is just made. The spherometer reading is noted and the centre leg then screwed up to leave a space for the rod to expand. The water is run out of the jacket and then steam is passed in at C and out of D, the substance AB being heated up to the temperature of the

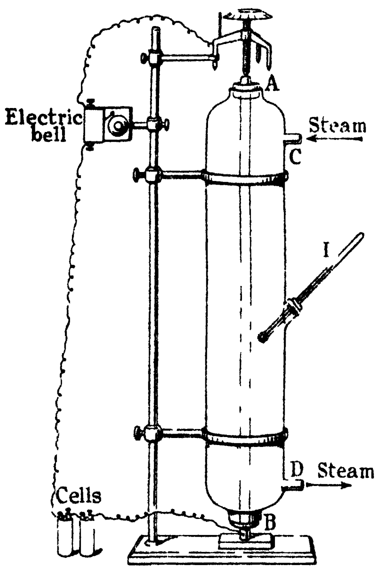


FIG. 26.—Measurement of Expansion of a Rod on Heating.

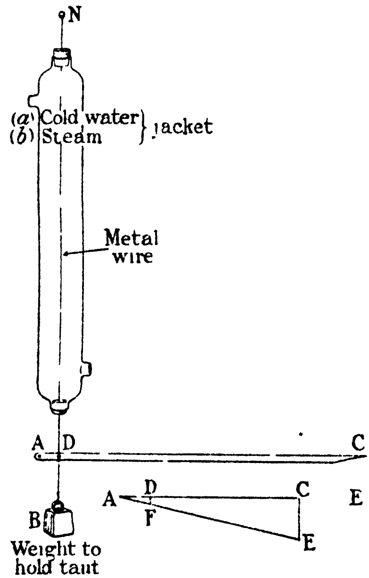


FIG 27 —Measurement of Expansion of a Metal Wire on Heating.

steam (read on T). After a time the spherometer is adjusted as before to just touch A and its reading noted. This process is repeated every 5 minutes until successive readings are the same, *i.e.* the rod has ceased to expand. The difference between the readings of the spherometer when the rod is surrounded by cold water and when by steam gives its increase in length.

Another method of measuring the increase in length of a material on heating is indicated in Fig. 27. The wire NB

is rigidly attached at D to a light index pointer AC pivoted at A. When the wire expands on heating, the point D moves to F and the end C of AC moves to E.

Then, since triangles ADF, ACE are similar,

$$\frac{\text{the expansion DF}}{\text{measured length CE}}=\frac{\text{length AD}}{\text{length of index AC}}$$

and so the expansion is calculated.

Many modifications of these two methods are used, and by many experiments it has been found that **the increase in length of a piece of substance on heating**

(1) **is proportional to the original length,** which we will suppose to be L_0 at 0° C. (that is, the greater the length of the bar the greater is its expansion for the same rise in temperature);

(2) **is proportional to the rise in temperature,** which we will suppose to be t° C. (that is, the same bar expands more and more as its temperature is raised).

Thus for any substance the increase in length (1) is proportional to L_0t, or

$l=\alpha L_0 t$, where α is a constant, depending on the nature of the material.

Suppose $t=1$° C. and $L_0=1$ unit of length, then $\alpha=1$, in the same units of length.

Thus α is the increase in length of unit length of the substance when raised in temperature by 1° C. and is called the **coefficient of linear expansion of the substance.**

Some values of this coefficient are :

Iron	0·000012	Platinum	0·0000089
Copper	0·0000167	Ordinary Glass	0·000009
Brass	0·0000189	Pyrex Glass (19–350° C.)	0·0000032
Aluminium	0·0000255	Silica (0–100° C.)	0·0000005
"Invar" Steel (36 per cent. Nickel)	0·0000009	Gas Carbon	0·0000054

From the relationship obtained above it is seen that $\alpha=\frac{l}{L_0t}$, or $=\frac{l}{L_0}$ when $t=1$° C., *and is thus the fractional increase in length of a bar when raised in temperature* by 1° C.

Also if L_t=the length of the substance at $t°$ C,

then $L_t=L_0+l$,

or $L_t=L_0+\alpha L_0 t$.

Thus $L_t=L_0(1+\alpha t)$.

This is a very useful expression and will be much used in later work.

It should now be realised that

(1) *the units of measurement of lengths are of no consequence in determining coefficients of linear expansion,*

(2) *the scale of temperature must be considered.*

The statement that the coefficient of expansion of iron is 0·000012 can be interpreted as:

1 cm. of iron raised in temperature by 1° C. expands by 0·000012 cm.,

1 yard of iron raised in temperature by 1° C. expands by 0·000012 yard,

1 mile of iron raised in temperature by 1° C. expands by 0·000012 mile,

or 1 foot of iron raised in temperature by 1° C. expands by 0·000012 foot, etc., etc.

It is usual, however, to state the coefficient of expansion per degree C., and tables give these coefficients. Obviously, the coefficient per degree F. must be smaller. Since 1° F.=$\frac{5}{9}$° C., it follows that a coefficient of linear expansion per degree F. is equal to $\frac{5}{9}$ of the corresponding coefficient of linear expansion per degree C.

Determination of the Linear Coefficient.—The methods given on pp. 41–43 can be used to determine α, the length of the substance used being measured. It should be of a good length (50–100 cms.), so that there is an appreciable expansion and thus less error in its measurement. In ordinary laboratory determinations it is not usual to work from 0° C., but to consider L_0 as the "original" length at the original temperature, which is

usually that of cold water. Thus the coefficient of linear expansion of the substance is measured by

$$\frac{\text{its increase in length}}{\text{its original length} \times \text{its rise in temperature in } ^\circ\text{C.}}$$

It is important to notice that the increase in length must be very accurately measured (hence the use of the spherometer), as the other factors are both large; the ordinary measurements (to 1 mm. of length or to $\frac{1}{2}$ or $\frac{1}{5}^\circ$ C. of temperature) in their case will be sufficiently accurate. The range of temperature for which the coefficient is determined should be stated.

Effects of Expansion and Contraction.—Many examples can be given of the effects of expansion and contraction. Some of these effects are of great practical use, whilst others are so disadvantageous that methods have been devised to overcome them. Some of the effects are discussed below.

(1) The alternate expansion and contraction of rock surfaces in tropical areas, where it is extremely hot by day and cold by night, has resulted in rock disintegration to such an extent as to form deserts.

(2) Teeth decay owing to the cracking of the enamel covering them consequent upon variation in temperature.

(3) When a hot liquid is poured into a thick glass tumbler it cracks, particularly if standing on a cold or metallic surface. This is due to the sudden expansion inside.

(4) The lead on lead roofs "creeps." When the roof is getting hot the expansion takes place downwards, and on cooling again the lead is pulled from the top downwards.

(5) Metallic structures like railway bridges, girders, railway lines, etc., vary appreciably in length.

For example, Forth Bridge has a 1,700 ft. span (of iron). Assuming the extreme temperatures to be 40° C. in summer and −20° C. in winter, we can calculate the change in length.

1 foot of iron heated through 1° C.
expands by 0·000012 foot.
$\therefore$ 1,700 ft. of iron heated through 1° C.
expands by $0{\cdot}000012 \times 1{,}700$ ft.
$\therefore$ 1,700 ft. of iron heated from -20° C. to $+40^\circ$ C.
expands by $0{\cdot}000012 \times 1{,}700 \times 60$ ft.
$= 0{\cdot}072 \times 17$ ft.
$= 1{\cdot}224$ ft.

Thus in constructions of this type sufficient space must be left for the greatest expansion under the possible conditions. Methods of allowing for this include the freedom of movement of one end on a roller, or a telescopic movement of parts.

(6) The expansion of the moving parts of machinery due to the heat production is an important point, and care has to be taken that "bearings" (axles, shafts, etc.) do not so expand that motion is stopped (*i.e.* that they do not "seize"). This is obviated by the use of lubricating oils.

Some ways in which the effect of expansion is used are given here.

(1) To fix an iron tyre on a wheel, the former, which is made slightly smaller than the wheel, is heated till it slips easily over the wheel. It is then cooled and so contracts, and is fixed firmly to the wheel. By this method crank arms are fastened on engine crank pins.

(2) In the early days of gun making, a hole was bored through a solid piece of metal. Nowadays a gun is built up of a number of cylinders, each one in turn being heated to cause it to expand and slip over the inner one or ones. Thus on cooling there is a large inward pressure, and a gun so built is able to withstand the enormous pressure generated when a shell is fired. It is also less likely to possess flaws than is a gun made from a solid piece of metal.

(3) For cutting glass, two methods are commonly used.

(*a*) A cold steel, or diamond, edge is pressed against the hot glass.

(*b*) A cut or scratch is made on the glass, while cold, by a diamond or carborundum cutter and a hot flame

impinged on it. In both cases the change in size results in the cracking and breaking away of the glass.

(4) Owing to the negligible difference between the coefficients of expansion of platinum and glass (see table on p. 43), a piece of white-hot platinum can be pushed through a piece of glass softened by heating, and on cooling the platinum is firmly held in the glass. If this is tried with a piece of copper, the latter falls out of the glass on cooling—it expands and contracts much more than does the glass.

(5) The difference in expansibilities between iron and brass can be shown with a *compound bar*—two long, narrow, flat iron and brass plates riveted together. If the bar is straight when cold, it bends on heating owing to the greater expansion of the brass. On cooling, the bar straightens out again. The only form of *solid thermometer*, due to Breguet, was devised on this principle. It consists of a brass-iron spiral with the more expansible metal on the inner side of the spiral. The latter thus opens out, on being raised in temperature, and a pointer carried on its outer end, the inner end being fixed, is caused to move over a graduated scale, standardised against a reliable thermometer. This form of thermometer, however, has been little used.

Devices to Overcome Effects of Expansion.—A very important development in this work was the discovery by Guillaume in 1904 of an almost non-expansible alloy of steel with 36 per cent. nickel, and called *invar*. Its coefficient of expansion (linear 0·0000009) is so small that it is negligible over small ranges of temperature. This alloy is being used more and more instead of some of the devices given below.

(1) The expansion of steel railway lines has already been mentioned, and the student should notice the arrangement which permits the movement of the rails without in any way weakening their stability, upon which depend the smooth running and safety of trains. Fig. 28 shows how the rail sections, with a gap between, are joined by fish-plates and bolts. Bearing on the top and bottom of the rails, to which they are firmly bolted, these

fish-plates keep a level surface to the rails. The bolt-holes in the fish-plates are longer horizontally so that a lengthwise movement of the rails is possible.

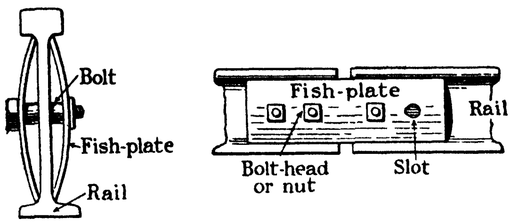


FIG. 28.

(2) The allowance for expansion in steam or hot water pipes takes the form of expansion bends in the pipes or expansion joints, of the telescopic type, packed with asbestos (Fig. 29).

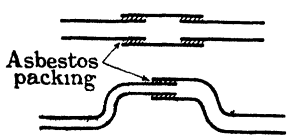


FIG. 29.—Expansion Joints in Steam Pipes.

(3) An interesting example is the "compensating" arrangement in clocks and watches. A clock is controlled by its pendulum, any variation in the swing of which affects its timing. The time of swing of a pendulum varies with its length, increasing when it lengthens and decreasing when it shortens. Thus clocks with pendulums which vary in length with temperature-changes gain in winter and lose in summer. The effective length of the pendulum is the distance from the point of suspension to the centre of gravity, which is approximately the centre of the heavy "bob" fixed at the bottom end of the pendulum. Various devices are used to keep this length constant.

In one type of clock the pendulum is made of wood (or invar) and the bob of a metal which has a much greater coefficient of linear expansion. A nut at the bottom of the wood holds the metal. Any expansion of the wood takes place downwards, and of the metal upwards, the relative lengths being carefully chosen so that the distance of the centre of gravity from the point of suspension is unchanged.

In the case of invar a very short zinc bob can be used to make a pendulum whose length does not vary. (Fig. 30)

Similar in principle is the *mercurial pendulum* invented by George Graham in 1722. This has an iron rod and the bob is an iron vessel holding one or two glass vessels containing mercury, the volume of which is such that its upward expansion compensates for the downward expansion of the rod. (Fig. 31.)

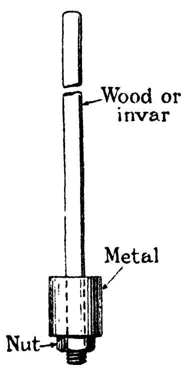


FIG. 30.—Compensated Pendulum.

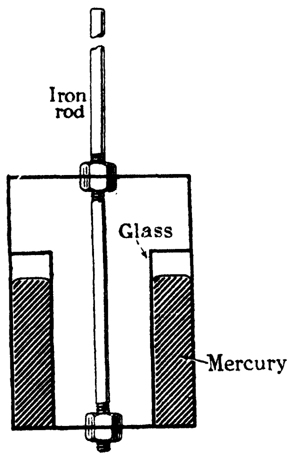


FIG. 31.—Graham's Mercurial Pendulum.

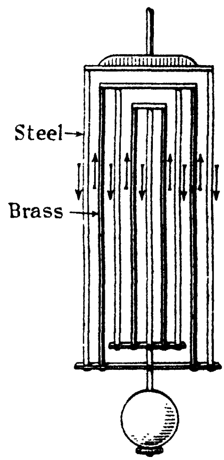


FIG. 32.—Harrison's Grid-Iron Pendulum.

The name of John Harrison, watchmaker, is always associated with the subject of accurate timekeepers. In 1726 he invented *Harrison's grid-iron pendulum*—made up of steel and brass rods so arranged, as shown in Fig. 32, that the steel rods only expand downwards and the brass rods upwards. These expansions are made to neutralise each other by having the total length of steel one and a half times that of brass, since the coefficient of linear expansion of brass is one and a half times that of steel.

John Harrison also invented the modern chronometer—a spring-driven portable clock, so efficient that it

was awarded a valuable prize because of its use for finding longitude at sea. In a watch a balance wheel is pivoted and oscillates about its centre. The time of oscillation depends upon the radius of the wheel—the smaller this is the quicker the oscillation, and so an ordinary wheel oscillates quicker in winter than in summer. A compensated balance-wheel (Fig. 33) is made in two or three sections and the rim is made of two strips of steel and brass, like the compound bar (p. 47), the more expansible brass being on the outside. The spokes expand outwards on a rise in temperature, but the sections curve inwards owing to the unequal expansion, and so a compensation is made. A fine adjustment for local conditions can be made by minute screws on the rim sections. It is interesting to note that wireless time signals have greatly reduced the importance of accurate time-keeping by clocks and watches.

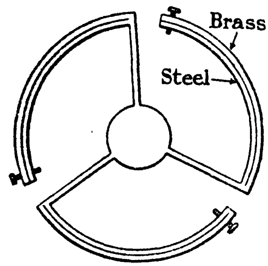


FIG. 33.—Compensated Balance-Wheel.

Area and Volume Expansion of Solids.—We sometimes have occasion to utilise the fact that solids expand superficially (in area), whilst volume expansion is frequently of importance to us. Thus we use coefficients of area and volume expansion in a similar manner to linear coefficients, and

> **the coefficient of area expansion is defined as the increase in area of unit area of a substance when raised in temperature by 1° C.**; whilst **the coefficient of volume expansion is defined as the increase of unit volume when raised in temperature by 1° C.**

This is only so, of course, for substances (called isotropic substances) which expand in all directions proportionately to the length. Metals in general are isotropic, but wood, many crystals, etc., expand differently in different directions.

Area Expansion.—Consider the simple case of a rectangular piece of a substance whose coefficient of linear expansion is α.

Let the lengths of two adjacent sides be l_0 and b_0 at 0° C. (Fig. 34.)

Thus area of rectangle at 0° C. $=A_0=l_0b_0$.

When raised in temperature by t° C. the adjacent sides expand to l_t and b_t, where $l_t=l_0(1+\alpha t)$ and $b_t=b_0(1+\alpha t)$.

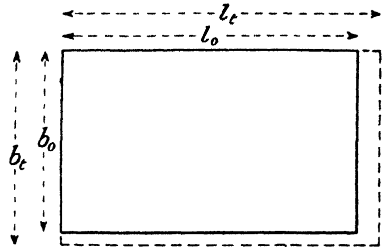


FIG. 34.—Superficial (Area) Expansion.

Hence area of rectangle at t° C.

$$\begin{aligned}&=A_t=l_tb_t\\&=l_0(1+\alpha t)b_0(1+\alpha t)\\&=A_0(1+2\alpha t+\alpha^2t^2).\end{aligned}$$

Thus $A_t=A_0(1+2\alpha t)$ neglecting α^2t^2 since α^2 is an extremely small quantity.

Thus if $A_0=1$ and $t=1$° C.,

$$A_t=1+2\alpha \text{ approximately.}$$

2α is thus the increase in area of unit area when raised in temperature by 1° C., and is thus the coefficient of area expansion. It is usually represented by β;

i.e. **the coefficient of area expansion of a substance $=2\times$ its linear coefficient,**

and we have a general expression, $A_t=A_0(1+\beta t)$, where $\beta=2\alpha$.

The error due to neglecting α^2t^2 can be seen as follows:

In the case of iron, where $\alpha=0{\cdot}000012$ and $\beta=0{\cdot}000024$, part neglected $=(0{\cdot}000012)^2$.

$\therefore$ Percentage error in value for coefficient for 1° C.

$$\begin{aligned}&=\frac{(0{\cdot}000012)^2}{0{\cdot}000024}\times 100\\&=0{\cdot}0006.\end{aligned}$$

Cubical Expansion can be considered in a similar manner. Suppose a rectangular solid of sides l_0, b_0 and w_0 at 0° C., be heated to t° C. (Fig. 35.)

Then volume at 0° C. $= V_0 = l_0 b_0 w_0$.

If l_t b_t w_t are the lengths of the respective sides at

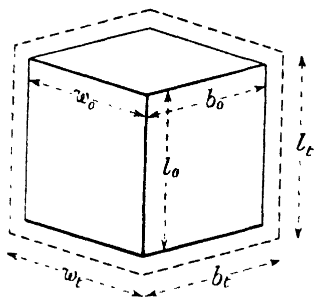


FIG. 35.—Cubical Expansion.

t_0° C., then $l_t = l_0(1+\alpha t)$, $b_t = b_0(1+\alpha t)$ and $w_t = w_0(1+\alpha t)$ where α = the linear coefficient of expansion of the substance.

Then volume at t° C. $= V_t = l_t b_t w_t$

$$= l_0 b_0 w_0 (1+\alpha t)^3$$
$$= V_0(1+3\alpha t + 3\alpha^2 t^2 + \alpha^3 t^3)$$
$$= V_0(1+3\alpha t) \text{ approx.}$$

Hence, if $V_0 = 1$ and $t = 1$° C., then $V_t = 1+3\alpha$ approx., *i.e.* 3α is the increase in volume of unit volume when raised in temperature by 1° C. and is thus the coefficient of volume expansion (γ) ;

i.e. *coefficient of cubical expansion* (γ)

$= 3 \times$ *its linear coefficient* (α),

and the general equation is $V_t = V_0(1+\gamma t)$, where $\gamma = 3\alpha$.

Coefficients of Expansion.—The student should carefully notice the similar nature of the expressions obtained :—

$$L_t = L_0(1+\alpha t)$$
$$A_t = A_0(1+\beta t)$$
$$V_t = V_0(1+\gamma t).$$

We can therefore express these changes in size, with change of temperature, in the general form :—

New size=original size (1+rise in temp.)×the appropriate coefficient of expansion ;

and thus **a coefficient of expansion is the increase in size of unit size when raised in temperature by 1° C.,** where, in each case, size may be length, area or volume.

And finally, the **increase in size=the original size ×the appropriate coefficient of expansion×rise in temperature.**

Exercises on Chapter III

1. Describe two experiments which show that expansion usually occurs when a body is heated. A cheap pendulum clock is found to lose in summer and gain in winter. Explain the cause of this. [*L.G.S.* 1922.]

2. Most substances expand on heating, this expansion sometimes being useful, and sometimes a nuisance. Describe two instances where expansion is useful, and two where it is a nuisance, necessitating a compensating device. Explain the action of the compensation device. [*J.M.B.* 1927.]

3. Describe an experiment to show that the thermal expansion of a solid though small may produce very great pressure. Mention practical examples (two each) of (*a*) the application of this phenomenon, (*b*) precautions taken to avoid its occurrence. [*L.M.* 1925.]

4. Explain the statement that the *coefficient of linear expansion* of iron is 0·000012 in Centigrade units. Describe a method you have witnessed of measuring the expansion of a metal rod and from the results obtained show how to find the value of the coefficient of linear expansion for that metal. If you quote a formula, justify its truth. [*L.G.S.* 1926.]

5. A lead organ pipe is 10 feet long at 0° C. What is its length at 10°, 20° and 30° C. ? (Coefficient of linear expansion of lead=0·000028 per ° C.)

6. A railway line is laid when the temperature is 7° C. If the line is 40 feet long and firmly clamped at one end, how much space must be left between the other end and the next rail to allow for the temperature rising to 27° C. ? (The coefficient of expansion for iron is 0·0000109.) [*J.M.B.* 1923.]

7. Define the coefficient of linear expansion of a solid and describe how it may be measured experimentally. What must be the length of a rod of zinc at 15° C. if its length is to increase by 5 mm. when the temperature is raised to 100° C. ? (Coefficient of linear expansion of zinc=0·000029 per ° C.) [*L.M.* 1928.]

8. The length of a certain copper rod is 30 inches at 0° C. What is the length of steel rod at 0° C. that has the same length as the copper rod at 100° C. ? (Coefficients of linear expansion of copper and steel are 0·000018 and 0·000012.)

9. Define coefficient of linear expansion. How does it depend on the scales of length and temperature used ? The length of a glass tube at 25° C. is 35·82 metres according to the brass scale which is correct at 0° C.

What is the true length of the tube (*a*) at 25° C., (*b*) at 0° C.? (Coefficients of linear expansion of brass =0·000019 and of glass=0·000009.) [*L.G.S.* 1921.]

10. A clock with a brass pendulum gains 10 secs. a day when the average temperature is 40° F.; what will its rate be when the temperature is 60° F.? The coefficient of linear expansion of brass is 0·00002 per degree centigrade. The time of swing of a pendulum varies as the square root of its length. [*L.M.* 1925.]

11. Describe two experiments to illustrate the difference in expansion of solids when heated. The coefficient of linear expansion of brass is 0·000018 and of steel 0·000012 per centigrade degree. Explain how you would construct with bars of these materials, a one-foot standard of length which would be invariable with alterations of temperature. [*L.M.* 1929.]

12. Describe some method of measuring the coefficient of expansion of a metal given a rod made of it. A rectangle of copper measures 50 by 20 cms. at 15° C. At what temperature will its area be 1010 sq. cms.? (Coefficient of linear expansion of copper=0·000018.) [*C.W.B.* 1926.]

13. Define "coefficient of linear expansion." A steel disc radius 5 cms. just fits inside a brass ring at 0° C. What would be the area of the gap between them at 50° C.? The coefficient of linear expansion of steel may be taken as 0·000011 and of brass as 0·000019. [*L.M.* 1920.]

14. A man wishes to fit an aluminium ring on a rod of iron of 1 inch diameter, but it is 0·001 of an inch too small in diameter. How much should its temperature be raised before it will just slip on? Subsequently, he wishes to remove it again, but now he has to heat both metals together. Through how many degrees must this be done? (The coefficient of linear expansion of aluminium is 0·000025 and that of steel 0·000010.) [*J.M.B.* 1926.]

15. A block of iron occupies a volume of 15 c. ft. at 0° C. What will be its volume at 50° and 100° C. if the coefficient of linear expansion of iron is 0·000011 per ° C.?

16. A block of iron occupies a volume of 5 c. ft. at 100° C. If the coefficient of linear expansion be 0·000012 calculate the volume of the block at 0° C. [*L.M.* 1920.]

17. A copper ball has a diameter of 10 cms. at 20° C; find the volume of the ball at 90° C., being given that the coefficient of linear expansion of copper=0·000018. [*L.G.S.* 1919.]

18. A hollow steel ball at 0° C. has an external diameter of 10 cms. and the shell has a thickness of 2 cms. What is the volume of (*a*) the steel, (*b*) the space inside, at 100° C., if the linear coefficient of expansion of the steel is 0·000010 per ° C.

CHAPTER IV

EXPANSION OF LIQUIDS

THE simple experiment described on p. 25, besides illustrating the expansion of a liquid on heating, also clearly shows the most important fact that the expansion of a liquid is complicated by the expansion of the containing vessel. The latter obviously *masks* the expansion of the liquid, and so in dealing with this question we must consider :

(*a*) the *apparent* or *relative expansion* of the liquid in the vessel, neglecting the fact that the vessel expands ; and

(*b*) the *true*, *real* or *absolute expansion* of the liquid, a correction being made for the expansion of the containing vessel.

Thus there are two coefficients of volume expansion (or dilatation) for liquids.

The coefficient of apparent expansion is the apparent or relative increase in volume of unit volume of the liquid in a vessel when raised in temperature by 1° C.

The coefficient of real, true or absolute expansion is the increase in volume of unit volume of the liquid when raised in temperature by 1° C.

Expansion of the Containing Vessel.—Consider a hollow, cylindrical glass vessel of material of coefficient of linear expansion α. (Fig. 36.)

If r=the radius at 0° C., the circumference of cylinder $=2\pi r$.

On heating to t° C. this circumference therefore expands to the initial value $\times(1+\alpha t)$, *i.e.* to $2\pi r(1+\alpha t)$.

But the radius at t° C.

$$=\frac{\text{circumference at } t^\circ \text{ C.}}{2\pi}=\frac{2\pi r(1+\alpha t)}{2\pi}$$
$$=r(1+\alpha t),$$

i.e. *the hollow vessel expands outwards as if it were of solid material.*

This is true of all hollow vessels—they change in size as if they were solid, no matter what shape they are. The problem of liquid expansion is thus simplified, for the following relationship must hold: *the real expansion of a liquid = the apparent expansion + the cubical expansion of the containing vessel* (treated as if the latter were solid and its volume that of the liquid).

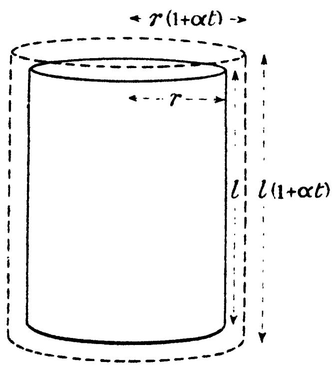


FIG. 36.

On p. 53 we saw that expansion = original volume × cubical coefficient of expansion × rise in temperature.

If a=the coefficient of apparent expansion of a liquid,
r=the coefficient of real expansion of a liquid,
g=the coefficient of cubical expansion of the containing vessel,
and V_0=the original volume of the liquid at 0° C.,

(and thus=volume of vessel at 0° C.).

Then the real expansion of the liquid $=V_0rt$,
the apparent expansion of the liquid $=V_0at$,
and the expansion of the vessel $=V_0gt$.
Hence from the relationship stated above, V_0rt must $=V_0at+V_0gt$,

or $r=a+g$,

i.e. **the coefficient of real expansion = the coefficient of apparent expansion + the cubical coefficient of expansion of the material of the vessel.**

This relationship is extremely important since it is obvious that in practical work we must usually deal with the apparent expansion. Having determined the coefficient of apparent expansion of a liquid, we can calculate the coefficient of real expansion if the coefficient of cubical expansion of the vessel is known. Care should be taken that the cubical coefficient, and *not* the linear coefficient, is used, as the tables generally used give the coefficients of linear expansion. The linear coefficient must, of course, be multiplied by 3 to give the coefficient of cubical expansion of the material.

Change in Density of a Material with Change of Temperature.—When a mass M grms. of a substance (*e.g.* a liquid) is raised in temperature, its volume is increased and so its density becomes less. (Density = mass ÷ volume.)

Suppose the density at 0° C. $=D_0$ and at t° C. $=D_t$
when the volume at 0° C. $=V_0$ and at t° C. $=V_t$

Suppose r = the coefficient of *real* cubical expansion, then $V_t=V_0(1+rt)$ or $\frac{V_t}{V_0}=1+rt$.

But density $=\frac{\text{mass}}{\text{volume}}$, or mass = density × volume, and the mass is constant.

Thus density × volume = a constant quantity,

or $\quad D_0V_0=D_tV_t,$

or $\quad \frac{D_0}{D_t}=\frac{V_t}{V_0}$ (**i.e. the density is inversely proportional to the volume**). This is true for a constant mass of any solid or liquid.

Thus $\frac{D_0}{D_t}=1+rt$ or $D_0=D_t(1+rt)$.

Finding the Coefficient of Apparent Expansion of Liquids.—Strictly speaking, the coefficient of volume

expansion is the increase in volume of unit volume when raised in temperature from 0° C. to 1° C., but in dealing with liquids, as with solids, we do not calculate the coefficient so. We calculate the average increase in volume of unit volume over a range of temperature, and it is therefore important to state the range of temperature for which you have experimented to obtain a value for a volume coefficient.

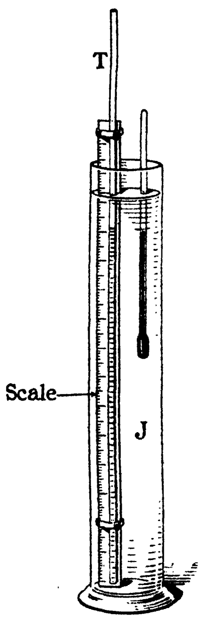


FIG. 37.

(1) *Simple method.*—This can be quickly carried out and involves no volume determinations (Fig. 37). A piece of ordinary glass delivery tubing T (about 4 mm. internal diameter and 60 cms. long) is sealed off and flattened at one end. It is about three-quarters filled with the liquid whose coefficient of apparent expansion in glass is to be determined, and attached to a half-metre scale by rubber bands (these can be cut from bunsen tubing). The whole is lowered into a tall gas jar J (50 cms. high) of cold water, in which a thermometer is suspended, until the level of the liquid is below that of the water. When the level of the liquid in the tube is steady, the length (l_1 cms.) of the column is obtained from the scale and the temperature (t_1° C.) of the water observed. This procedure is repeated with the tube of liquid put into a similar jar of very hot water (the gas jar being carefully warmed before the hot water is put into it) and the length (l_2 cms.) of the column of liquid at the temperature (t_2° C.) of the hot water obtained.

Assuming the tube to be uniform, the volumes of the liquid can be considered to be proportional to the lengths of the columns.

Thus, volume of liquid at t_1° C. is proportional to l_1 and volume of liquid at t_2° C. is proportional to l_2.

Hence for a rise in temperature from t_1° to t_2° C. volume l_1 increased by volume (l_2-l_1), and for a rise in temperature of 1° C. unit volume increased by

$$\frac{l_2-l_1}{l_1(t_2-t_1)}.$$

Thus the coefficient of apparent expansion of the liquid for the range $t_1°$ C. to $t_2°$ C.$=\dfrac{l_2-l_1}{l_1(t_2-t_1)}$

(2) *Weight thermometer method* (or pyknometer).—Weight thermometers are shaped as shown in Fig. 38, but for convenience of filling we use more commonly a specific gravity bottle. This is weighed when clean and dry. (The usual method of drying is to send in a current of hot air, obtained by heating, in a cool bunsen flame, a piece of narrow glass tubing joined to the rubber tubing from a foot-bellows which is worked gently.) The bottle is then filled with the liquid whose coefficient of apparent expansion is to be found, and the ground stopper, with the capillary bore through it, is pushed in, liquid coming

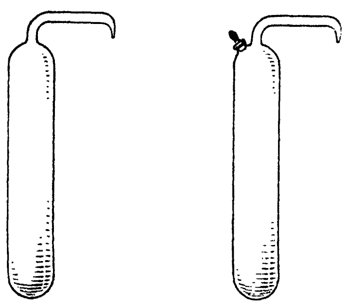


FIG. 38.—Weight Thermometers.

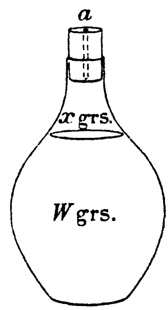


FIG. 39.—Specific Gravity Bottle used as Weight Thermometer.

out at *a* (Fig. 39). This is wiped off, so that the liquid level is just to the top of the stopper, and the bottle is weighed again. The whole is now suspended, immersed to the neck, in a water bath, and is heated up and then kept constant at a suitable temperature which is observed. (If the liquid does not boil at 100° C. the water can be boiling.) The bottle should be left sufficiently long for the whole of the liquid to have attained the temperature of the water, some of the liquid being evacuated at *a* owing to the expansion. Superfluous liquid at *a* is wiped off; the bottle is taken out, allowed to cool, and then weighed. It weighs less, of course, owing to the evacuated liquid.

Suppose the mass of the bottle $= w_1$ grms.,

the mass of bottle full of liquid at $t_1°$ C. (air temperature) $\Big\} = w_2$ grms.,

and the mass of bottle + liquid left in at end of experiment $\Big\} = w_3$ grms.

Then mass of liquid expelled on heating from $t_1°$ to $t_2°$ C. $\Big\} = w_2 - w_3$ grms. $= x$ grms.,

and mass of liquid in bottle at end $= w_3 - w_1$ grms. $=$ W grms.

The calculation is based on (*a*) the fact that this W grms. of liquid exactly filled the bottle at temperature $t_2°$ C., (*b*) the density D grms. per c.c. of a liquid $= \dfrac{\text{its mass in grms.}}{\text{its volume in c.c.}}$

Method 1.—Let D grms. per c.c. = density of the liquid at $t_1°$ C.

Volume occupied by W grms. at $t_1°$ C. $= \dfrac{W}{D}$ c.c.

This volume *expands* to fill the vessel at $t_2°$ C., *i.e. by a volume occupied by x grms. at* $t_1°$ C.

But volume occupied by x grms. at $t_1°$ C. $= \dfrac{x}{D}$ c.c.

Thus $\dfrac{W}{D}$ c.c. of liquid heated from $t_1°$ C. to $t_2°$ C. expands by $\dfrac{x}{D}$ c.c.,

and $\therefore$ 1 c.c. of liquid heated by 1° C. expands by

$$\frac{x}{D} \times \frac{D}{W} \times \frac{1}{(t_2 - t_1)} \text{ c.c.,}$$

or the coefficient of apparent expansion of the liquid

$$= \frac{x}{W(t_2 - t_1)} \text{ (for range } t_1° \text{ C. to } t_2° \text{ C.)}$$

Method 2.—On p. 57 it was shown that

$$\frac{\text{density of liquid at } t_1° \text{ C.}}{\text{density of liquid at } t_2° \text{ C.}} = \left(\begin{array}{c}1 + \text{coefficient of expansion} \\ \times \text{rise in temperature}\end{array}\right).$$

Thus the

$$\frac{\text{apparent density of liquid at } t_1° \text{ C.}}{\text{apparent density of liquid at } t_2° \text{ C.}} = \underline{\underline{1 + a(t_2 - t_1)}}$$

where a is the coefficient of apparent expansion of the liquid.

The mass of liquid filling bottle at $t_1°$ C. $=$ M grms. $=$ W $+ x$ grms.

$\therefore$ Density of liquid at $t_1°$ C.

$$=\frac{W+x}{\text{volume of bottle}} \text{ grms. per c.c.}$$

Since W grms. filled bottle at $t_2°$ C.,

$$\text{density of liquid at } t_2° \text{ C.} = \frac{W}{\text{volume of bottle}} \text{ grms. per c.c.}$$

$$\text{Hence } \frac{\text{Apparent density of liquid at } t_1° \text{ C.}}{\text{Apparent density of liquid at } t_2° \text{ C.}}$$

$$= \frac{W+x}{\text{volume of bottle}} \times \left(\frac{\text{volume of bottle}}{W}\right)$$

$$= \frac{W+x}{W} = \underline{\underline{1+\frac{x}{W}}}.$$

$$\therefore \quad 1+a(t_2-t_1)=1+\frac{x}{W} \quad \text{(underlined quantities),}$$

or $$a=\frac{x}{W(t_2-t_1)},$$ as shown in method 1, for range $t_1°$ C. to $t_2°$ C.

As we have seen, to obtain the coefficient of real expansion of a liquid, the coefficient of cubical expansion of the vessel must be added to the apparent coefficient obtained as above. To avoid this, it is possible to obtain weight thermometers made of silica glass with a negligible coefficient of expansion.

(3) *Dilatometer method.*—A dilatometer consists of a glass bulb, of known volume, with a graduated stem-tube leading from it. In practice, however, a bulb is usually blown on the end of a piece of glass tubing, about 0·8 to 1 cm. diameter, with a bore of about 1 mm. diameter. The mean area of cross-section of the bore is found by weighing the tube empty and then with a measured length of mercury thread in the bore.

Suppose l cms. of mercury thread have a mass m grms. at $t°$ C.

From tables, density of mercury at $t°$ C. $= D_m$ grms. per c.c.

Then $$\text{volume of thread} = \frac{\text{mass}}{\text{density}} = \frac{m}{D_m} \text{ c.c.}$$

$$\text{Thus mean area of cross-section of bore} = \frac{m}{D_m \times l} \text{ sq. cms.}$$

The dilatometer is then filled, to a little way up the stem, with the liquid, whose coefficient of apparent expansion is to be measured, and weighed. From the mass of the liquid, if its density is known, the volume is calculated. The dilatometer is now fastened to a scale by rubber bands, and the bulb and part of the stem heated in a water-bath, levels of the liquid being read at certain temperatures (when, for the time being, the temperature is kept constant).

Thus the volume expansion for various temperatures $=$ expansion in length of column $\times$ area of cross-section of the bore. A graph showing the relation between the volume of the liquid and the temperature can be plotted, and the coefficient of apparent expansion of the liquid for any range of temperature calculated, since it

$$= \frac{\text{volume expansion}}{\text{original volume of liquid}} \times \frac{1}{\text{rise in temperature}}.$$

Dulong and Petit's U-tube Method for the Coefficient of Real Expansion of a Liquid.—This method was introduced in 1817 to study the behaviour of mercury at various temperatures in view of the fact that its suitability as a thermometric fluid was being much discussed. By means of it Dulong and Petit showed the comparative uniformity of expansion of mercury for the range 0° C. to 350° C.

A U-tube, containing mercury, was arranged so that one limb could be maintained at a low temperature, *e.g.* that of melting ice or cold water, and the other limb at a higher temperature, *e.g.* that of steam or oil vapour (Fig. 40). At the start, when both limbs were at the same temperature, the levels were the same (see p. 10). On maintaining one limb at a higher temperature than the other, the liquid in the hotter limb expanded and stood at a higher level than that in the colder limb. On

p. 10 it is shown that (1) the pressure at a point in a liquid is the product of the density of the liquid and the height of the column of liquid above the point, and (2) the pressure at all points at the same level in the same liquid is the same.

Thus, measuring the heights of liquid columns as in Fig. 40,

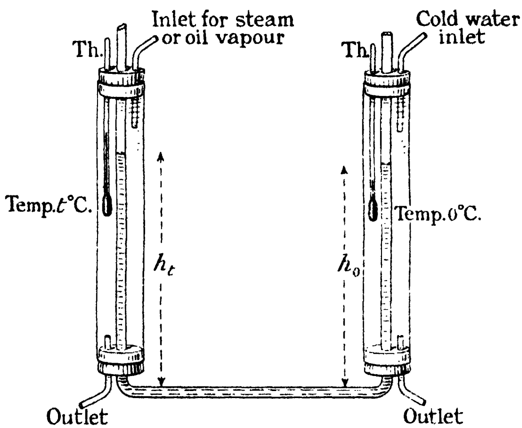


FIG. 40.—Dulong and Petit's Method for the Real Coefficient of Expansion of a Liquid.

if h_0=height of cold column, temperature 0° C., density of liquid D_0

and h_t=height of hot column, temperature t° C., density of liquid D_t,

then pressure at level AB due to hot column$=h_t.D_t$
and pressure at level AB due to cold column$=h_0.D_0$.

Since these pressures must be equal, $\frac{D_0}{D_t}=\frac{h_t}{h_0}$.

But on p. 57 we saw that $\frac{D_0}{D_t}=1+rt$, where r=the coefficient of real expansion of a liquid.

Hence $$1+rt=\frac{h_t}{h_0}$$

or $$rt=\frac{h_t}{h_0}-1=\frac{h_t-h_0}{h_0},$$

and so $$r=\frac{1}{t}\cdot\frac{h_t-h_0}{h_0};$$

i.e. for a range of temperature $t_1°$ to $t_2°$ C., the coefficient of real expansion of a liquid approximately

$$=\frac{1}{\text{difference in temperature between the columns } (t_2-t_1)}\cdot\frac{\text{difference in heights between the columns}}{\text{height of the cold column}}$$

This method is obviously independent of the glass, and so the two limbs can be of different diameters without in any way interfering with the result.

The Expansion and Contraction of Water.—If a series of readings be taken for water with a volume dilatometer (p. 61), from 0° C. upwards to approximately 100° C., a remarkable result will be obtained, as is shown in the graph of Fig. 41, not drawn to an exact scale. As water is cooled down from 100° C. it contracts, but when cooled below 4° C. it expands again, *i.e.* a mass of water occupies its smallest volume at 4° C., or, as we say, **water has its maximum density at 4° C.**

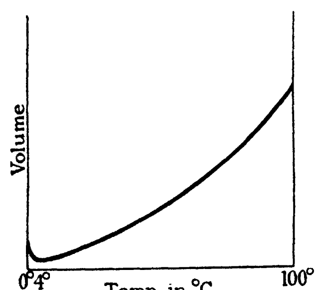


FIG. 41.—Change in Volume of Water between 0° and 100° C. (not to scale).

The expansion of water when cooled or warmed from 4° C. is an important reason why *water is unsuitable as a thermometric fluid.* Another reason can be seen from the graph above and from calculations made from the data—water does not expand uniformly, for its expansion gets greater as the temperature rises. An

experiment often shown to illustrate the behaviour of water is known as *Hope's Experiment.*

A narrow, cylindrical jar, J, of glass, or preferably brass, has an inlet near the top and the bottom (Fig. 42) in which thermometers C and B, in corks, can be inserted, the vessel holding water to a level above the upper thermometer. A is a jacket of brass or copper and is filled with a mixture of small pieces of ice (2 parts) and salt (1 part). This cools down the water in the jar and the readings of the two thermometers are recorded every two minutes and entered in a graph (Fig. 43). With a narrow tube and a good ice-and-salt mixture this experiment can be done in 45 minutes. As is seen from the graph, the cooled water at first contracts, becoming more dense, and so falls to the bottom. The lower thermometer records a lower and lower temperature whilst the upper one remains fairly steady. As the temperature at the bottom approaches 4° C., the temperature at the top is seen to fall and does so gradually till it reaches 4° C., the lower thermometer still

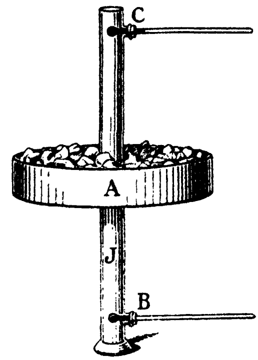


FIG. 42.—Hope's Experiment.

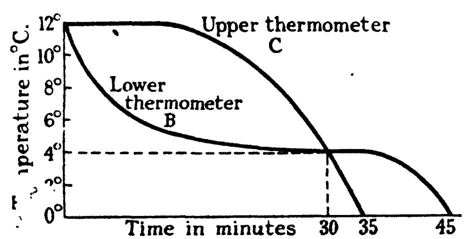


FIG. 43.—Graph showing Results of Hope's Experiment.

remaining at 4° C. The reading of the upper thermometer now falls gradually to 0° C. and ice begins to form *at the top*, whilst there is no change in the reading of the lower thermometer. Thus [illegible] 4° C. the water must have expanded and become less [illegible] It has, therefore, travelled from the centre of the jar

Also since the ice floats at the top of the water it must be less dense than the water at 0° C. Thus **when water turns to ice at 0° C. it expands.**

The peculiar behaviour of water, as illustrated by this experiment, is of great importance; as a result of it, the standard of mass (the gram, in the C.G.S. system of units) is taken as the mass of 1 c.c. of pure water, at 4° C., at which temperature water has its maximum density.

Another important point is that of the formation of ice on the top of ponds. No doubt you have noticed that when a thin layer of ice has formed, intense cold is needed to produce a thicker layer—the ice coating protects the water underneath. If water behaved like most substances and contracted continuously on cooling, and also on solidifying, ice would sink to the bottom of a pond as it formed; in a cold spell the whole of a pond would solidify, and it is doubtful whether the whole of it would melt again if the water were deep. The effect on fish, animal and plant life in the water is obvious.

This, however, is avoided by the remarkable behaviour of water; the layer of ice formed at the top of a pond prevents the water underneath from being readily cooled by the cold air outside. Snow on the ground acts similarly as a protector to the plants, etc., which it has covered over. You can test this by leaving thermometers below and above a snow layer on a cold day, and you will be surprised at the difference in temperatures recorded.

The process of rock and soil disintegration (previously mentioned on p. 45) is also assisted by this behaviour of water on solidifying. During cold spells and winter nights water in the soil and in rock crevices freezes, and so expands, thus forcing the rocks and soil apart. In the daytime the ice usually melts. The continued repetition of the process is very effective in breaking up the rocks and the soil, and the farmer is always pleased when we get much of the warm day, cold night type of weather in winter, after a spell of rain, for h[illegible] his ploughing work will be made much lighter. [illegible] or its.

On the other hand, the householder o[illegible]. An

water-pipes (especially … cold weather, so enor… expansion of water or… filling up a fairly th… tying its cork or stop… leaving it immersed … a crack, like a pist… bottle will be fou… which the water e… as iron, antimony … most substances c… zinc, gold, etc., … be used for cast… to be stamped. … must expand … copper, antim…

A simple ex… be carried ou… (see p. 61) an… about 4° C., s… top pour … Place in … of the p… from liqu… hold the … that all th… before th… of level i… (if stem … volume … be foun… 12 c.c. …

It… or d… met… w… …

…ENTS OF EXPANSION OF LIQUIDS

…< 10^{-5}	Alcohol (ordinary)	. .	122×10^{-5}
…$\times 10^{-5}$	Benzene	. . . .	124×10^{-5}
…5 $\times 10^{-5}$	Chloroform	. . . .	126×10^{-5}
…0 $\times 10^{-5}$	Mercury	. . . . .	18×10^{-5}
…4 $\times 10^{-5}$	Water (mean 20–100° C.)		45×10^{-5}

…ERCISES ON CHAPTER IV

… between the coefficient of absolute expansion of …ient of apparent expansion of the liquid in glass; …fficient of expansion of the glass of a weight thermo…ed. [*L.M.* 1916.]

…at would you make to study the change in the volume of water as the temperature is raised from 0° C. to … the results you would expect to obtain, and explain the …bearing in the freezing of a pond. [*L.G.S.* 1921.]

…onnection between a change in volume caused by heating … corresponding change in density, and describe some …od in which one of these changes is measured by means … [*L.G.S.* 1923.]

… statement that the rate of expansion of mercury is uniform. … compare the rate of expansion of water between 10° C. … its rate between 20° C. and 30° C.? [*L.M.* 1921.]

…lute coefficient of expansion of mercury per °C. is 0·000182 …parent coefficient measured by a glass weight thermometer is … Find the coefficient of linear expansion of glass per ° F., giving …ns for the different steps in the calculation. [*L.M.* 1924.]

…lass bottle at the air temperature of 12° C. contains 56·34 grms. … On raising the temperature by means of a water-bath to 90° C. … were driven out. What is the coefficient of apparent expansion …r in the glass for the range 12° C. to 90° C.?

…ribe how to measure the coefficient of relative expansion of …n glass with a weight thermometer. Supposing this coefficient …00155 and the mass of mercury in the thermometer at 100° C. … grms., find the mass which overflowed when the temperature was …om 0° C. [*L.M.* 1922.]

…xplain the term *coefficient of expansion*. A glass bottle has a … of 50 c.c. What mass of alcohol would it hold at 20° C. if the …ent of linear expansion of glass=0·0000088, the density of alcohol … C.=0·81 grm. per c.c. and its absolute coefficient of expansion …0105? [*L.M.* 1919.]

…. Show how the values of the density of a substance at different …peratures are related to its coefficient of cubical expansion. The …cury filling a density bottle of 15° C. weighs 338 grms. and at 60° C. …ighs 335·8 grms. What is the coefficient of expansion of mercury? Comment on the value obtained. [*L.G.S.* 1929.]

10. What do you understand by the following statements: (*a*) the coefficient of cubical expansion of mercury=0·00018 per degree C., (*b*) the coefficient of linear expansion of glass=0·000010 per degree C.? A glass

vessel just holds a kilogram of mercury at 0° C. What weight of mercury will it hold at 100° C.? [*J.M.B.* 1928.]

11. A glass vessel which holds exactly 1,000 grms. of mercury at 15° C. is put into boiling water at 100° C. What weight of mercury is expelled? Coefficient of dilatation of mercury=0·00018; coefficient of linear expansion of glass=0·00001. [*J.M.B.* 1924.]

12. A glass flask holds when quite full 100 grms. of mercury at 0° C. Find the weight of mercury that would fill the flask at 100° C., (1) assuming that the glass did not expand, (2) assuming that the mercury did not expand, (3) assuming that both mercury and glass expanded. (The coefficient of linear expansion of glass=0·000009 and of volume expansion of mercury =0·00018.)

13. Distinguish real from apparent expansion of a liquid and describe a method of measuring the latter. A uniform glass tube 1 metre long contains a column of mercury at one end. How long must this be in order that the length of the part of the tube unoccupied by mercury may remain unaltered when the whole is heated? Real coefficient of mercury=0·00018 per ° C.; linear coefficient for glass=0·00001. [*L.M.* 1923.]

14. What is meant by the apparent expansion of a liquid? A piece of glass tubing whose internal diameter is 0·9 mm. is sealed to a glass bulb of 10 c.c. capacity. The bulb is then just completely filled at 0° C. with liquid whose apparent coefficient of expansion in glass is 0·00048. How far will the liquid rise in the tube if the temperature of the bulb is raised to 30° C.? [*L.G.S.* 1920.]

15. State what you consider to be the three most important errors for which readings on a mercury thermometer should be corrected. Find the volume of the bulb of a thermometer tube such that a rise of temperature of 1° C. causes the mercury to rise 2 mm. up the stem, the diameter of the latter being 0·2 mm. Coefficient of real expansion of mercury=0·000181 per ° C.; coefficient of linear expansion of glass=0·000009 per ° C. [*C.W.B.* 1929.]

16. Show how the density of a substance at different temperatures is connected with its coefficient of expansion. A U-tube containing a liquid has the two limbs maintained at 15° C. and 100° C. respectively. On reaching a steady state the lengths of the liquid columns are 97 cms. and 102 cms. What is the coefficient of expansion of the liquid? Is this the real or apparent coefficient? Give reasons. [*L.M.* 1928.]

17. A copper cylinder weighs less in water than in air. How would this difference in weight be affected by raising the temperature of the water? Give reasons for your answer. [*L.G.S.* 1924.]

CHAPTER V

EXPANSION OF GASES

ON p. 25 a simple experiment was described to show that air, or any gas, expands very appreciably on heating. In fact, the expansion of a gas, for quite a small change in temperature, is so appreciable that, in comparison, the expansion of the containing vessel can ordinarily be neglected. Thus there is no need to consider, as with liquids, a real *and* an apparent coefficient of expansion.

However, another complication arises in dealing with gases. We have already seen that a gas exerts a pressure —that the particles are considered to be moving at a great speed, and their continuous bombardment of the sides of the containing vessel constitutes a pressure. Thus, when a gas is heated, not only will it tend to expand, but the molecules of gas will move faster and so the pressure will be affected. You probably know how a somewhat deflated ball bounces much better after it has been warmed up in an oven or in front of a fire.

The problem arises—does this pressure change have any effect on the volume of the gas? The answer to this is given by a famous experiment performed in 1660 by *Robert Boyle*, an Irishman of noble descent, who devoted his life and fortune to scientific study. For several years he worked in a laboratory he had erected at Oxford, and was led to study the " spring " of the air—how it behaved when acted on by a compressing (or squeezing) force, and how it recovered on the release of the pressure. For his experiments he had a tube made as shown in Fig. 45. This " Boyle's Tube " (or Boyle's Law Tube) was a U-tube with a very long and a short limb, the latter being sealed, the former open. A little mercury was

poured in to fill the bend and adjusted by tilting, till the levels *aa*, were the same in both limbs. The pressure of the volume of air enclosed in the short limb was therefore atmospheric. Mercury was then poured into the longer limb B, and the level of the mercury became higher in the shorter limb—the volume of the enclosed air getting smaller. This was continued till the column of air in A was reduced to exactly one-half (after a little time had been allowed for the air inside to attain the same temperature as the air outside, supposing a change to have occurred during the experiment). It was then found that the height of the mercury in B above the height of that in A was approximately 29 ins.—*i.e.* the extra pressure on the air enclosed in A was equal to 1 atmosphere. Thus the total pressure was 2 atmospheres when the volume of the air was reduced to one-half its original volume. When more mercury was added to B till the volume of the enclosed air was reduced to one-third, it was found that there were now 2×29 ins. of mercury in B above the level of the mercury in A, *i.e.* the total pressure on the enclosed air was 3 atmospheres, its temperature being the same as the outside air. Thus Boyle found that the air in A decreased in volume proportionately as the pressure increased. This is known as **Boyle's Law,** which states that **at a constant temperature the volume of a gas is inversely proportional to the pressure.**

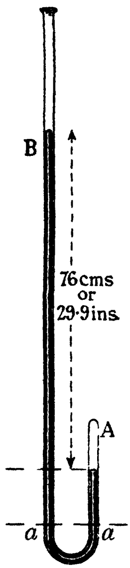


FIG. 45.—Simple Boyle's Law Tube.

Thus, if P=the pressure and V=the volume, for the gas, at a fixed temperature, the product PV=a constant quantity.

Boyle tested this relationship, which is called an *isothermal* one, over a range of pressure $1\frac{1}{4}$ ins.—$117\frac{9}{16}$ ins. of mercury.

Nowadays we use a form of apparatus in which the

pressure is varied more conveniently than by adding mercury. As shown in Fig. 46, it consists of a glass tube, A, sealed at one end, fixed against a vertical metre scale and joined to another open glass tube, B, by flexible rubber "pressure" tubing (*i.e.* tubing of sufficient thickness to withstand pressure). The upper part of B widens out since it is a reservoir for mercury, which encloses air in tube A, whilst B can be moved up and down a vertical rod and clamped in any position near to the vertical scale. The apparatus can be used to verify Boyle's Law as follows:

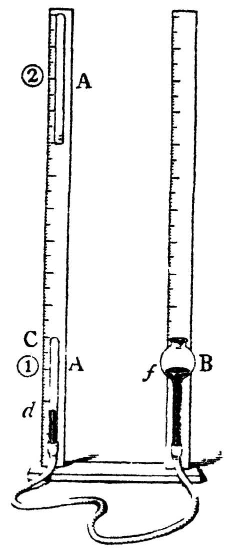


FIG. 46.—Modified Boyle's Law Apparatus.

(*a*) At pressures higher than atmospheric—by fixing the glass tube A in the lower position (1) and raising B to increase the pressure.

(*b*) At pressure lower than atmospheric—by fixing the glass tube A in the upper position (2) and lowering B to decrease the pressure.

In the latter case, less air must be enclosed in A at the start, at atmospheric pressure, otherwise its volume will become greater than the length of the glass tube A and the length of the column will not be measurable. In experimenting, when the pressure is altered a little time should be allowed for the air enclosed to reach the outside air temperature. Results can be entered in a table as given (see Fig. 46):

BAROMETER READING FOR ATMOSPHERIC PRESSURE = CMS. OF MERCURY

Level C cms.	Level *d* cms.	Level *f* cms.	Volume of air enclosed (V) proportional to (C − *d*) cms	Height of mercury column exerting pressure = (*f* − *d*) cms	Total pressure P = at. pr. + (*f* − *d*) cms. of mercury.	PV	$\frac{1}{V}$ (use reciprocal tables).

A graph should be plotted of P against $\frac{1}{V}$ *and this should give a straight line (since* $P \propto \frac{1}{V}$*).* The results can also be used to measure the atmospheric pressure, the latter being unknown. Plot a graph of the excess pressure due to the mercury column $(f-d)$ cms. against the reciprocal volume. It should give a

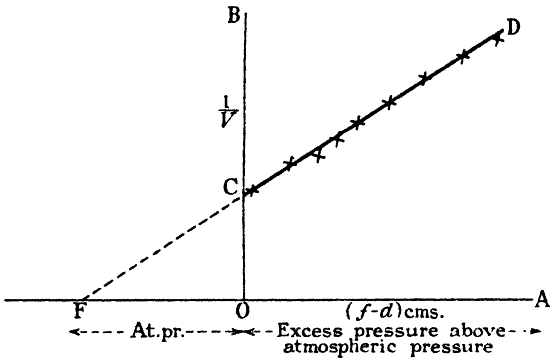


FIG. 47.

straight line CD as shown in Fig. 47. Produce CD backwards to cut the axis OA at F.

Since $P \propto \frac{1}{V}$, when $\frac{1}{V}=0$, then P must $=0$.

Thus F must be the position of zero pressure.

Hence readings from F along FA will give the total pressure and FO must equal the atmospheric pressure.

It is interesting to note that this law is still called Mariotte's Law in France. Mariotte published in 1676, sixteen years later than Boyle, a treatise "On the Nature of the Air," and French scientific historians will not admit the precedence of Boyle.

During the nineteenth century it was pointed out by French scientists, including Regnault, that their experiments showed that this law was not absolutely true—that at high pressures V seemed to decrease more rapidly than according to the law, and Amagat, in 1880, made a full

study of this, using a long U-tube (longer limb 200 ft.) built up of glass sections with steel joints and set up in a coal-mine shaft. He conclusively showed that this law only held sufficiently well over appreciable ranges for gases such as air, hydrogen, nitrogen, etc., and hardly at all for gases, such as carbon dioxide, which were easily converted into liquid. But for all gases the law did not hold at extreme pressures.

Expansion of Gases with Rise of Temperature.—It should now be clear that, when we wish to study the change in volume of a gas with change of temperature, we must keep the pressure constant, since any pressure change will affect the volume.

Charles, a Parisian Professor of Physics, was the first to express the relationship between the volume and temperature of a gas at a constant pressure. He was led to study the subject because of his interest in balloons. It was he who raised the first hydrogen balloon at Paris in 1783. (The first balloons were sent up ten years before; the balloon envelope contained hot air, a fire being carried in the basket underneath.) Charles found the relationship in 1787, but he did not publish his work, and it was left to Gay-Lussac to give it to the world when he accidentally found Charles' manuscripts. Thus the relationship, called **Charles' Law,** is sometimes called Gay-Lussac's Law. **Charles' work showed that—**

(1) **all gases behave similarly when heated under the same conditions**—or equal volumes of all gases expand equally when raised in temperature by the same amount at a constant pressure;

(2) **at a constant pressure, the volume of a gas changes by $\frac{1}{273}$ of its volume at 0° C. for each degree C. rise or fall in temperature.**

The first part of the law can be shown fairly simply by setting up two exactly similar flasks, delivery tubes and collecting jars, one flask containing air and the other coal-gas (*or any other gas conveniently obtained*). Both flasks are heated up in water-baths, till the water is boiling thoroughly in each bath. It is seen that equal

volumes of air and gas are driven out by expansion into the collecting jars.

The volume relationship with temperature can be studied with a modification, due to Jolly, of the apparatus described on p. 73. To it is attached a bulb B (Fig. 48), with connecting tube, instead of the straight tube. The bulb should contain a little strong sulphuric acid to remove water vapour from the air inside. The bulb is surrounded by melting ice and the movable tube M adjusted till a and b are at the same level, a

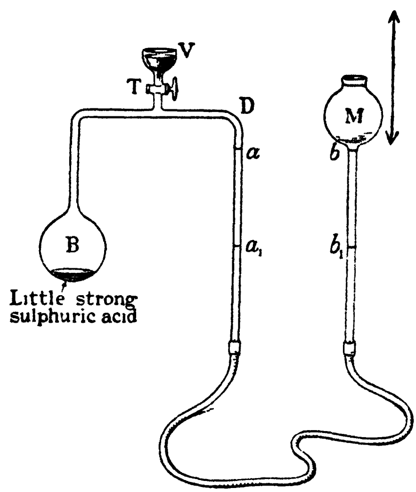


FIG. 48.—Jolly's Apparatus.

being near to the bend D in the tube. The provision of a small tap T in the tube enables this to be done. Often the air enters through a vessel V above the tap, passing through a solid mixture of soda lime (to remove carbon dioxide) and calcium chloride (to dry it). The bulb is then put in a water bath, instead of ice, and kept at 10° C. The rise in temperature causes the air volume to increase, level a falling and level b rising. Thus the pressure of the air in the bulb is now greater [illegible]an atmospheric by a mercury column $(b-a)$ cms., and so [illegible] must be lowered till a and b are again at the same level ([illegible]). The distance aa_1 is observed, and then the process

is repeated, the bulb being maintained as nearly as possible in turn at temperatures of 20° C., 30° C., . . . 100° C. If a graph of volume increase (proportional to length of column aa_1, etc., in the uniform tube) and temperature rise is plotted, it should give an approximately straight line.

The volume of the bulb B and tube to first level a should be found (*e.g.* by weighing empty and then with water in it), and the tube should be calibrated from a to a_1 and downwards. Thus, for a known commencing volume of gas, the volume increase for a known rise of temperature can be found.

Now, Charles realised that the *volume expansion of a gas* (at constant pressure) *must be based on the volume at* 0° *C.* We have already seen the simple volume relationship with temperature change to be (usual symbols) $\text{Vol}_t = \text{Vol}_0\ (1 + t \times \text{coefficient of volume expansion})$.

Charles realised that Vol_0 must be strictly interpreted as the *volume at* 0° *C.* and not, as we did in dealing with solids and liquids, the volume at the original temperature.

Thus the calculation for the coefficient of expansion is sometimes a little more complicated, since it is not always convenient to start the experiment at 0° C.

Hence, suppose volume of gas at 10° C. = 107·6 c.c.
and ,, ,, ,, 100° C. = 141·8 c.c.

Then, if x = the required coefficient of increase of volume at constant pressure,

using $V_t = V_0(1 + xt)$,

$$V_{10^\circ \text{C.}} = 107{\cdot}6 = V_0(1 + x.10)$$

and $$V_{100^\circ \text{C.}} = 141{\cdot}8 = V_0(1 + x.100)$$

or $$\frac{107{\cdot}6}{141{\cdot}8} = \frac{1 + 10x}{1 + 100x}$$

$$\therefore\ 107{\cdot}6 + 10{,}760x = 141{\cdot}8 + 1{,}418x$$

whence $x = \frac{1}{273}$ approx. (or 0·00366).

Charles found that the coefficient of volume expansion at constant pressure $= \frac{1}{273}$ or 0·00366.

Zero Volume of Gases.—A very interesting theoretical result can be deduced on the assumption that Charles' Law remains true for all temperatures. Since a gas changes its volume by $\frac{1}{273}$ of its volume at 0° C. for each 1° C. change in temperature,

if the volume of a gas is 273 c.c. at 0° C.
the volume of the gas is 274 c.c. at 1° C.
" " 275 c.c. at 2° C., etc., etc.

also the volume of the gas is			$273-1=272$ c.c. at	$-1°$ C.
and	"	"	$273-2=271$ c.c. at	$-2°$ C.
"	"	"	$(273-10)=263$ c.c. at	$-10°$ C.
"	"	"	$(273-100)=173$ c.c. at	$-100°$ C.
"	"	"	$(273-273)=0$ c.c. at	$-273°$ C.

This temperature, $-273°$ C., at which, theoretically, the volume of a gas would be zero, is called the **absolute zero.** Only a gas which obeyed the gas law implicitly would occupy a volume of zero (such a gas is called **a** perfect gas)—the idea of a zero volume for a gas is contrary to any of our material ideas of atoms and molecules of matter. It is, however, convenient to formulate another temperature scale, *called* **the absolute scale of temperature,** with its zero position at this special temperature. This scale has degrees of the same value as centigrade degrees, and is thus merely a displaced centigrade scale, *e.g.*

0° on the absolute scale	=	$-273°$	on the centigrade scale		
273°	"	"	=	0°	" "
373°	"	"	=	100°	" "

or **absolute scale value = centigrade scale value + 273.**

We have already seen that

$$\text{Volume of a gas at } t_1° \text{ C.} = Vt_1 = V_0(1+xt_1)$$

and " " " $t_2°$ C. $= Vt_2 = V_0(1+xt_2)$

where

$$x = \text{coefficient of expansion at constant pressure} = \tfrac{1}{273}.$$

$$\therefore \frac{Vt_2}{Vt_1} = \frac{1+xt_2}{1+xt_1} = \frac{1+\frac{1}{273}t_2}{1+\frac{1}{273}t_1} = \frac{273+t_2}{273} \cdot \frac{273}{273+t_1}$$

$$= \frac{273+t_2}{273+t_1} = \frac{\text{absolute temperature } T_2}{\text{absolute temperature } T_1} \text{ (and so } V \propto T).$$

We thus see a very simple way of expressing **Charles' Law—at a constant pressure, the volume of a gas is directly proportional to its absolute temperature.**

The student is strongly recommended to learn this form of the law, for it is very useful in calculations.

Problems.—(1) 75 c.c. of a gas at 10° C. are heated till the temperature is 200° C. What will be the volume of the gas if the pressure is unchanged?

Since volume $\propto$ absolute temperature,

$$\frac{\text{volume at } 200^\circ \text{ C.}}{\text{volume at } 10^\circ \text{ C.}}=\frac{273+200}{273+10}$$

and this $$=\frac{\text{required volume}}{75 \text{ c.c.}}.$$

$\therefore$ Required volume at 200° C. $=\frac{473}{283}\times 75$ c.c. $=125{\cdot}35$ c.c.

(2) The volume of a gas at 12° C. is doubled by heating it at constant pressure. To what temperature is it heated?

Suppose gas is heated to t° C., or $(273+t)^\circ$ absolute.

Since its volume $\propto$ absolute temperature,

$$\frac{\text{volume at } t^\circ \text{ C.}}{\text{volume at } 12^\circ \text{ C.}}=\frac{273+t}{273+12}=\frac{2}{1}$$

$$\therefore 273+t=2\times 285=570$$

$$\therefore \underline{t=297^\circ \text{ C.}}$$

Relation between the Pressure, Volume and Absolute Temperature of a Gas, when all are changed.

Suppose a mass of gas at pressure p_1 and absolute temperature T_1 has a volume v_1
and the same mass of gas at pressure p_2 and absolute temperature T_2 has a volume v_2,
the relationship between these quantities can be found as follows:

Imagine a change from the first set of conditions to the second set to be carried out in two stages.

(1) An *isothermal change* at T_1—the pressure p_1 changing to p_2 and the volume changing from v_1 to some value v (unknown). Then by *Boyle's Law* (pressure $\times$ volume $=$ a constant)

$$p_1v_1=p_2v \quad \text{or} \quad v=\frac{p_1}{p_2}\,.\,v_1.$$

(2) *At a constant pressure* p_2 let the temperature change from T_1° absolute to T_2° absolute—the volume must thus change from v to v_2.

Then by *Charles' Law* (volume $\propto$ absolute temperature)

$$\frac{v}{v_2}=\frac{T_1}{T_2} \quad \text{or} \quad v=\frac{T_1}{T_2}.v_2.$$

Thus equating the values of v,

$$\frac{p_1}{p_2}.v_1=\frac{T_1}{T_2}.v_2$$

or
$$\frac{p_1 v_1}{p_2 v_2}=\frac{T_1}{T_2}, \quad \text{or} \quad \underline{PV \propto T.}$$

PV thus=RT where R is a constant called the *Gas Constant*, and PV=RT is called the *Gas Equation*, but R in the equation is not the same for all gases—equal masses of gases have not the same volume.

The relation deduced can be applied to many problems, e.g. 20 *c.c. of gas at* 12° *C. at atmospheric pressure has its volume halved at a constant temperature and is then brought back to its original volume by being heated at a constant pressure. What is the final temperature?*

In the first case, by Boyle's Law, to halve the volume, the pressure had to be doubled, *i.e.* final pressure is 2 atmospheres.

Thus using symbols as in above work,

$p_1=1$ atmosphere. $v_1=20$ c.c. $T_1=(273+12)°=285°$
$p_2=2$ atmospheres. $v_2=20$ c.c. T_2 is required.

As deduced,

$$PV \propto T, \quad \text{or} \quad \frac{p_1 v_1}{p_2 v_2}=\frac{T_1}{T_2}$$

$$\therefore \frac{1\times 20}{2\times 20}=\frac{285}{T_2} \quad \text{or} \quad T_2=2\times 285=570,$$

or final temperature=$(570-273)°$ C.=$\underline{297° \text{ C.}}$

Change in Pressure of a Gas when its Temperature is changed at Constant Volume.—This is merely a special case of the above general case PV$\propto$ T, where V is kept constant.

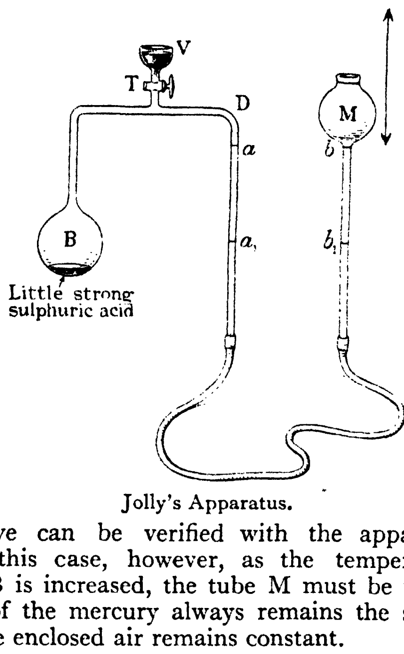
Thus obviously P$\propto$ T,

i.e. **the pressure of a gas is directly proportional to its absolute temperature at a constant volume.**

This is exactly analogous with the volume change with

temperature at constant pressure, and thus we have an equation for pressure similar to the volume equation $V_t=V_0(1+xt)$; *i.e.* $P_t=P_0(1+xt)$, the coefficient of increase of pressure at constant volume being $x=\frac{1}{273}$, the same as the volume coefficient at constant pressure.

This can be applied to pressure changes at constant volume in a similar manner to the volume changes at constant pressure illustrated on p. 77.



Jolly's Apparatus.

The above can be verified with the apparatus shown above. In this case, however, as the temperature of the air in bulb B is increased, the tube M must be raised so that the level a of the mercury always remains the same, *i.e.* the volume of the enclosed air remains constant.

Suppose when bulb is at 10° C., level b is the same as level a,
and ,, ,, 100° C., ,, b is x cms. above level a,
then pressure in first case=atmospheric
and in second case=atmospheric+x cms.

Then $$P_{10}=\text{At. Pr.}=P_0(1+x.10)$$
and $$P_{100}=\text{At. Pr.}+x=P_0(1+x.100).$$

$$\therefore \frac{\text{At. Pr.}}{\text{At. Pr.}+x}=\frac{1+10x}{1+100x}$$ and this is solved for x.

If this experiment be carried out for a series of temperatures and the values of the excess pressure over atmospheric pressure (*i.e.* height of level *b* above level *a* in cms. of mercury) be plotted against the absolute values of the temperature, they should give a straight line, and the value of the atmospheric pressure can

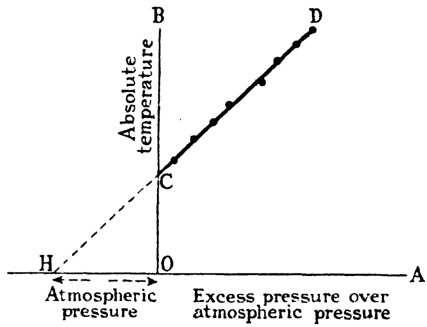


FIG. 49.

be deduced from the graph (see Fig. 49). If P does $\propto$ T, then T=o, P=o.

The straight line DC produced therefore cuts the OA axis at H where H must be the value of the real pressure and equals o when T=o° absolute.

Thus HO must be equal to the atmospheric pressure.

Gas Thermometers.—In the previous work on thermometers we saw that an essential property of a thermometric substance is an appreciable coefficient of expansion. Gases expand much more, bulk for bulk, than do liquids, so much so that with them the amount of, or any lack of uniformity in, the expansion of the containing vessel is negligible. Thus a gas should make an excellent thermometric fluid. This is so, but when used, the thermometer, called a *Gas Thermometer*, is not easily portable, and acts slowly as the gas takes time to heat up. Hence gas thermometers are chiefly used for standardising work in special laboratories. Gases such as hydrogen, oxygen and nitrogen expand very uniformly and so make very exact thermometers, but above 600° C. nitrogen is the most reliable.

The methods of using them are on the principle of the experiments described on pp. 76, 81.

It will be readily understood from those experiments that gas thermometers are of two types.

(1) *Constant Pressure Gas Thermometers*, where the volume of the enclosed gas increases as the temperature rises at constant pressure (see p. 76), the change in volume being used to indicate the temperature—the thermometric scale being marked on the left-hand limb of the U-tube, in which the level *a* moves with the change in temperature (Fig. 48)

(2) *Constant Volume Gas Thermometers*, where the pressure of the enclosed gas increases as the temperature rises at constant volume (see p. 81, and Fig. 48). The scale is then an external one against which the right-hand limb of the U-tube moves as the volume of the gas is adjusted to its constant volume. The common form as shown in Fig. 48 is known as *Jolly's Constant Volume Air Thermometer*.

Of the two kinds, constant volume air or gas thermometers are much preferable for the following reasons:

(1) The level of the mercury in the one limb is much more quickly adjusted to a fixed point than to the same level as the mercury in another tube.

(2) The length of tubing, containing gas, outside the temperature bath is always kept the same in a constant volume gas thermometer. Thus the stem-correction (correction for the fluid outside the temperature bath) only depends on the temperature of the bath. In the constant pressure air thermometer, the length is variable and so it is a difficult matter to correct for it.

Exercises on Chapter V

1. State Boyle's Law. Write a careful description of a method of showing that the law is true. How would you set down the results of experimental observations in your note-book?

2 State and explain Boyle's Law. The pressure of oxygen inside a steel cylinder of 2 cu. ft. capacity is 100 atmospheres. How many cu. ft. would this oxygen occupy at a pressure of 1 atmosphere? [*J.M.B.* 1927.]

3. The capacity of a motor tyre containing air at a pressure of 30 lbs.

per sq. in. is 600 cu. ins. On a day when the barometer pressure was 15 lbs. per sq. in., the valve was opened until the pressure fell to 25 lbs. per sq. in., without appreciably altering the size of the tyre. What proportion of the enclosed air escaped, and what volume did it occupy after its release? [*J.M.B.* 1926.]

4. Describe how you would use a Boyle's Law apparatus to determine the barometric pressure. A thin metal vessel of volume 4 cu. ft. containing air at a pressure of 5 atmospheres is placed in a closed room measuring 4 ft. by 15 ft. by 6 ft. where the pressure is one atmosphere. If the metal vessel bursts, what will be the new air pressure in the room? [*J.M.B.* 1926.]

5. State Charles' Law. Describe with care any experiment you have performed which shows the truth. [*J.M.B.* 1923.]

6. State the law of expansion of gases at constant pressure. Describe how you would measure the coefficient of expansion of air at constant pressure. [*L.G.S.* 1922.]

7. State the law of expansion of gases at constant pressure. A flask containing 300 c.c. of air at 100° C. is closed and then opened under water. It is found that 70 c.c. of water enter the flask, the temperature now being 13° C. From these figures find a value for the coefficient of expansion of air. [*L.G.S.* 1925.]

8. Define *coefficient of expansion* for a gas. A flask provided with a short delivery tube is first weighed empty and then heated in boiling water for some minutes. The delivery tube is closed for a moment while the flask is immersed in water at 25° C. and then opened under water. The flask, together with the water sucked in, is weighed, and is then completely filled with water and weighed a third time. If the three weights are 30·3, 81·0 and 280·88 grms. wt., find the coefficient of expansion of air at constant pressure. [*L.M.* 1923.]

9. State the relation connecting the pressure, volume and temperature of a given mass of gas. A gas container on being raised in temperature from 15° C. to 20° C. shows an increase in pressure of 0·6 atmosphere. What would be the pressure of the contained gas at 35° C., assuming that the volume of the container remains constant? [*L.M.* 1928.]

10. What are the relations between the pressure, volume and temperature of a gas (*a*) when each of the three in turn is maintained constant, and (*b*) when all three may vary? Describe an experiment to verify any one of the relations given under (*a*). If the coefficient of expansion of a gas at 15° C. were 1/280 per degree C., what would be the "absolute zero"? [*C.W.B.* 1927.]

11. A spherical glass bottle whose capacity is 1,000 c.c. at 0° C. is left for a time unstoppered in a room where the temperature is 15° C. and the mercury barometer stands at 755 mms. The stopper is inserted and the bottle is then immersed in a cooling mixture at −10° C. Find the pressure of the air in the bottle, assuming that no air passes the stopper during the cooling. The contraction of the glass may be neglected. [*J.M.B.* 1922.]

12. A mass of air has a volume of 150 c.c. at atmospheric pressure and temperature, namely, 75 cms. of mercury and 12° C. Determine the volume of the air when it is measured at a pressure of 80 cms. of mercury and at 31° C. Give a sketch and a brief description of the apparatus you would use to verify the truth of the result obtained. [*L.G.S.* 1927.]

13. A copper sphere contains air exerting a pressure of 2 atmospheres at 15° C. Assuming the coefficient of linear expansion of copper to be

0·000017, what pressure will the gas exert when the sphere is placed in boiling water?

14. Describe a method of investigating the variation of pressure with temperature for a gas kept at constant volume. Find the percentage increase of pressure in the tyres of a bicycle taken out of the shade (59° F.) into the sun (95° F.) disregarding the expansion of the rubber. [*L.M.* 1922.]

15. A closed glass bulb contains 50 c.c. of mercury and 50 c.c. of air at 15° C. and 740 mm. pressure. *Explain* carefully how the pressure of the enclosed air would be affected by heating the bulb to 100° C., and calculate the new pressure, assuming the volume of air unchanged. [*C.W.B.* 1928.]

16. Explain how the volume of a gas is affected by changes in pressure and temperature respectively. How would you use a constant volume air thermometer to measure the melting point of paraffin wax? [*C.W.B.* 1926.]

17. Describe a constant volume air thermometer. Air at 50° F. is heated at constant volume till the pressure is increased by 10 per cent. Find the new temperature (° F.). [*L.M.* 1923.]

18. Describe a constant volume air thermometer. The air in a closed vessel, the volume of which does not vary with the temperature, is at a pressure of 15 lbs. to the square inch, the temperature being 10° C. What will the pressure become if the temperature is raised to 100° C.? [*L.G.S.* 1925.]

19. Describe a form of constant volume air thermometer. The bulb of such a thermometer having been placed in melting ice, the mercury level in the open tube was 2·6 cms. below the constant volume mark, the barometer reading 77·4 cms. In a later experiment the corresponding reading was 11·6 cms. above the mark, the barometer reading 75·5 cms. What was the temperature of the bulb in the second experiment? [*C.W.B.* 1929.]

20. Describe how to measure the increase in the pressure exerted by a gas when it is heated at constant volume. Draw a graph to illustrate the result which would be obtained from this experiment, and point out any important conclusions which may be drawn from it. [*L.M.* 1921.]

21. Describe a form of air thermometer and discuss the relative advantages of air and mercury for a thermometric fluid.

CHAPTER VI

MEASUREMENT OF QUANTITY OF HEAT—CALORIMETRY

ALTHOUGH the early material or Caloric Theory of Heat did not survive long in the nineteenth century, it played a very important part in the development of heat knowledge, for it led to a distinction between the degree of hotness of a body (temperature) and the heat (caloric) possessed by a body. Probably the first scientist to understand this clearly was *Joseph Black*, of Irish descent, who lived 1728–1799. Whilst a Professor at Glasgow, he was much struck by the slowness with which ice melts and water boils. He observed that during each of these changes the temperature remained constant, yet much heat had to be given to the ice or water. Ordinary matter was said to have an attraction for caloric, and so Black realised that much caloric had to be given to ice to change it into water, or to water to change it into steam, in each case at a constant temperature. He realised, too, that when the reverse change of state took place (*i.e.* steam condensing to water or water freezing to ice) the caloric was set free, and he experimented to show that the same amount was set free as was previously received. He said that the caloric absorbed in ice changing to water, or water to steam, was **latent** or hidden, and he explained its disappearance as due to some form of chemical combination between the matter and the caloric.

One of Black's chief experiments was the placing of dry ice in a metal vessel with a very thin base and standing it on a metal plate fixed over a steady fire. The plate attained a temperature which remained constant and

thus supplied heat (caloric) to the metal vessel at a uniform rate. Black observed the times taken for all the dry ice to melt, for the melted ice to come to the boil, and for all the boiling water to turn into steam. He then compared the quantities of caloric required for these three steps according to the observed times. The idea of an "attraction" between matter and caloric, or the "capacity" of matter for heat, also interested Black.

He believed that different bodies had different attractive powers, or capacities, for caloric, and so he devised a method for testing his belief, thus inventing the *calorimeter* (measure of caloric). He hollowed out a block of ice and covered the top with cardboard or wood. Into the hole in the block (dried inside by blotting-paper, or a sponge) he quickly placed a piece of substance which had been allowed to stand in boiling water for several minutes, put back the top (so that air would not melt some of the ice inside) and left it for a time. The substance cooled down to the temperature of the ice, giving up caloric which caused some ice to melt. The quantity was found by absorbing the water inside the block by a weighed dried sponge or piece of blotting-paper. He repeated this with many substances and so compared the masses of ice melted by equal masses of the different substances. The results thus gave the relative quantities of caloric given out by these substances respectively in cooling from the temperature of boiling water to that of ice: these values are relatively the same quantities received by the substances in being raised in temperature from that of ice to the boiling-point of water, *i.e.* through the same range of temperature, and so gave a comparison of the "attraction" or "capacity" of the substances for caloric or heat.

Heat Units.—Black's work on heat quantities, and their measurement, naturally led to the need for a unit of measurement, and undoubtedly the first was the quantity of heat required to melt 1 grm. of ice. But in experimental work it is not always convenient or advisable (because of the melting during handling) to deal with ice, and early in the nineteenth century the **unit of heat**

in England was the **quantity of heat necessary to raise the temperature of 1 lb. of water by 1° F.** This unit, the pound-degree Fahrenheit, is still used by British engineers and is called the **British Thermal Unit (B.Th.U.).**

It has recently become of more importance owing to the agreement, supported by the Board of Trade, that gas supply companies should sell their gas not by volume, as they used to do, but by its calorific (heating) value. Nearly all the larger companies have accepted this idea, but the supply of gas is still measured by volume (cubic feet). Each company, however, declares the heat value of the gas it supplies (the value depends on the details of the process of manufacture and on the plant used), and the Board of Trade protects the consumer by bringing into force the penalties imposed in the Gas Regulations Act, 1920, if the standard is ever lower than the declared calorific (heat) value. The volume of gas supplied is converted into heat units and charged for accordingly.

The *Therm*, which is equal to 100,000 British Thermal Units, is the practical unit used. At present, the calorific value of the gas supplied by the South Metropolitan Gas Company, London, is 560 British Thermal Units per cubic foot, and the price charged is $8\frac{3}{4}d.$ per therm.

The Calorie or Gram-Calorie.—The unit of heat used by scientists in all countries, and which is based on the **C.G.S. system of units, is the calorie** (from "caloric"), and is **the quantity of heat necessary to raise the temperature of 1 grm. of water by 1° C.** (and thus sometimes called the **gram-degree Centigrade**). The range was at first taken to be from 4° to 5° C., *i.e.* starting at the temperature of the maximum density of water, since it was observed that the quantity of heat necessary to raise the temperature of 1 grm. of water by 1° C. varies with the range (it gradually decreases as the temperature of the water is raised from 0° C. to 40° C. and then increases as the temperature is raised above 40° C.). Thus a more exact unit is the *mean calorie*, $\frac{1}{100}$th of the quantity of heat necessary to raise 1 grm. of water from 0° C. to 100° C. This value is the same as required for the range 15° C.–16° C., a convenient range since it is the normal laboratory temperature. For all ordinary laboratory

work (such as we shall be concerned with in this book) we shall consider that the same quantity of heat is necessary to raise 1 grm. of water by 1° C., whether it be from 0°–1° C., from 21°–22° C., or from 99°–100° C., etc.

Thermal Capacity.—We have seen Black's work on the capacity or attraction, by bodies, for heat, or, as we call it, *thermal capacity*. Later, the thermal capacity of a substance was measured, in C.G.S. units, as the quantity of heat necessary to raise the temperature of the substance by 1° C. (In English units, of course, it was measured per degree F.) Obviously, therefore, the thermal capacity of a substance depends on the mass as well as the nature of the substance.

A very simple series of experiments to illustrate the different thermal capacities of equal masses of different substances is as follows:

(1) Into a 200 c.c. glass beaker pour exactly 50 c.c. of tap water. Heat some water to about 60° C. and pour out 50 c.c. into a measuring cylinder. Take the temperature of the cold water in the beaker by an ordinary 0°–100° C. thermometer, transfer the latter to the hot water, stir well and observe its temperature. Take the thermometer out, put it back into the cold water in the beaker and quickly pour in the 50 c.c. of hot water. Stir well and observe the highest temperature reached by the thermometer, *i.e.* the temperature of the mixture before it begins to cool. You should find your results to be something like the following:—

Temperature of hot water 58° C. > fall 22° C.
Temperature of mixture 36° C. > rise 21·5° C.
Temperature of cold water 14·5° C.

Thus the hot water cools through 22° C. in giving out heat to raise the temperature of an equal mass of cold water by practically the same number of degrees C.

(2) Repeat the experiment having 50 c.c. (*i.e.* 40 grms. approx.) of cold paraffin in the beaker, and pour in 40 c.c. (*i.e.* 40 grms.) of hot water. This time your results will be very different, *e.g.*:

Temperature of hot water 58° C. > fall 15° C.
Temperature of mixture 43° C. > rise 28·5° C.
Temperature of cold paraffin 14·5° C.

Thus the hot water only cools through 15° C. in giving out

heat to raise the temperature of an equal mass of cold paraffin by 28·5° C., or the water cools 1° C. in giving out sufficient heat to raise an equal mass of paraffin by approximately 2° C.

(3) Take a piece of metal (about 60–80 grms.) and find its mass to the nearest grm., say x grms. Place x c.c. (*i.e.* x grms.) of water into the beaker as before. Suspend the piece of metal in boiling water for 10 minutes (*i.e.* till it is heated right through to 100° C.) and then quickly take it out, shake to remove any water on it, and lower into the cold water, observing the rise in temperature as before. Enter your results as in the previous experiments.

Temp. of metal (same as the boiling water) 100°C. >fall 78° C.
Temp. of mixture, 22° C. >rise 7·5° C.
Temp. of cold water, 14·5° C.

Thus the metal is cooled by 78° C. in giving out sufficient heat to raise the temperature of an equal mass of cold water by 7·5° C., or the metal cools through 10° C. approximately in giving out sufficient heat to raise the temperature of an equal mass of water by 1° C. Conversely, therefore, a mass of water in cooling 1° C. would give out enough heat to raise the temperature of an equal mass of the metal by 10° C.

From these experiments it is thus seen that if equal masses of the substances used fall in temperature by the same number of degrees centigrade, the water gives out approximately ten times as much heat as the metal and twice as much as the paraffin. Hence the thermal capacity of water is ten times that of the metal and twice that of paraffin. Water is found to have the highest thermal capacity for all substances, when equal masses are compared, with the exception of gases, such as hydrogen and nitrogen peroxide, and also lithium. This, then, is **another reason why water is unsuitable as a thermometric fluid** (see p. 64).

Specific Heat.—To compare the heat capacities of various substances, it has been found convenient to compare them with an equal mass of water.

Thus the **Relative Thermal Capacity of a substance (or Specific Heat)**

$$= \frac{\textbf{Thermal capacity of a substance}}{\textbf{Thermal capacity of an equal mass of water}}$$

i.e. it is a ratio.

This, then, in C.G.S. units

$$= \frac{\text{heat required to raise the temperature of the substance by } 1^\circ \text{ C.}}{\text{heat required to raise the temperature of an equal mass of water by } 1^\circ \text{ C.}}$$

or, comparing 1 grm. of each,

$$= \frac{\text{heat required to raise the temperature of 1 grm. of substance by } 1^\circ \text{ C.}}{\text{1 calorie.}}.$$

Similarly, it (in British units)

$$= \frac{\text{heat required to raise the temperature of 1 lb. of the substance by } 1^\circ \text{ F.}}{\text{heat required to raise the temperature of 1 lb. of water by } 1^\circ \text{ F.}}$$

or

$$= \frac{\text{heat required to raise the temperature of 1 lb. of substance by } 1^\circ \text{ F.}}{\text{1 pound }^\circ \text{ F. (B.Th.U.)}}.$$

Thus, the **Relative Thermal Capacity**, or **Specific Heat, of a substance is numerically equal to the heat required to raise the temperature of unit mass of the substance by one degree** (the appropriate units being used).

This, then, serves as a basis of most of the calculations in heat measurement, *e.g.*

If s=the specific heat of a substance,
then the heat required to raise the temp. of 1 grm. of the substance by 1° C.=s calories,
or, the heat required to raise the temp. of 1 lb. of the substance by 1° F.=s B.Th.Units.

Since each grm., or lb., of a substance behaves similarly, *and assuming the value of the specific heat holds*, no matter what range the ° C. or ° F. is taken, we have:

Heat required to raise the temp. of m grms. of the substance by t° C.=mst calories.
or heat required to raise the temp. of m lbs. of the substance by t° F.=mst B.Th.U.

Determination of Specific Heats of Substances.—For obvious reasons this work is known as *Calorimetry*, and, in the common methods, is carried out in a calorimeter. The usual method is known as the *Method of Mixtures*, for a known mass of the substance, whose specific heat is to be found, is mixed with a known mass of liquid

with which it does not react chemically (for in chemical reactions heat is usually evolved or absorbed, and so the calculation would lose its value). In most cases water is used, and its specific heat is taken as 1 for the range of temperature involved in the experiment. One of the two substances used is made hot for the purpose of the experiment, and the temperature of each just before mixing, and that of the final mixture, is observed. One substance cools down, giving out heat, whilst the other receives heat and is warmed up. Assuming that the experiment is rapidly made, there is a negligible loss of heat to the air, and we can therefore consider that the *heat gained by cooler substance = heat lost by warmer substance*. We can equate these values and so calculate the required specific heat.

Example.—51·3 grms. of water at 52° C. are poured into 46·5 grms. of paraffin at 14° C., and the final temperature is 40° C. What is the specific heat of the paraffin ?

Calculation. — Heat given out by 51·3 grms. of water (sp. ht. 1) in cooling from 52° C. to 40° C. $= 51{\cdot}3 \times (52-40) \times 1$ calories.

Heat absorbed by 46·5 grms. of paraffin (sp. ht. s) in being raised from 14° C. to 42° C. $= 46{\cdot}5\ (42-14)\ s$ calories.

Equating, $46{\cdot}5 \times 28\ s = 51{\cdot}3 \times 12$

Whence $s = \underline{0{\cdot}48 \text{ approx.}}$

In practice, however, the method is not so simple, for the vessel containing the substances during the mixing must be influenced and so receive heat (or give out heat in a cooling operation). There are two possible methods of getting over this difficulty.

(*a*) Use a vessel which absorbs or loses heat very slowly. It is then assumed that, for the short time that elapses during the experiment, no heat is gained or lost by the vessel. This method is rarely used, as the ideal material for such a vessel, to be cheaply made, has not been found.

(*b*) Use a vessel which absorbs heat very rapidly, and calculate the heat absorbed or lost by it during an experiment. This is the usual method, and so calorimeters

are of polished aluminium or copper (the reason they are polished will be seen in Chapter XII). The calorimeter is mounted in such a method that a gain or loss, of heat from, or to, the air is so small as to be negligible, for the short time the experiment takes. The ordinary method is to support the calorimeter, by wood or cardboard, inside another larger polished metal vessel.

Experimental Determination of the Heat Capacity of a Calorimeter. Water Equivalent.—Weigh a calorimeter (the inner vessel only), fill to about one-third with cold water and weigh again. Heat some water to about 50° C. Carefully take the temperature of the cold water and then that of the warm water. Quickly transfer the thermometer to the cold water in the calorimeter, pour in an approximately equal volume of the warm water, stir well and read the temperature of the mixture. The thermometer is put in the cold water *before* adding the hot water because a mercury thermometer should be read rising if possible (p. 32). During the mixing the thermometer rises and the highest temperature it reaches is the temperature of the mixture. Weigh again at the end of the experiment. Your results should be entered as shown, at least two columns for experimental results being allowed, as it is important that two results, of an approximately similar figure, should be obtained and the mean value taken.

RESULTS

		Expt. 1.	Expt. 2.
Mass of calorimeter	$=m_1$ grms.		
Mass of calorimeter+cold water	$=m_2$ grms.		
Mass of cold water used . .	$=(m_2-m_1)$ grms.		
Mass of calorimeter+cold water and warm water	$=m_3$ grms.		
Mass of warm water used . .	$=(m_3-m_2)$ grms.		
Temperature of cold water . .	$=t_1$° C.		
Temperature of warm water .	$=t_2$° C.		
Temperature of mixture water .	$=t_3$° C.		

Calculation. Heat given out by the Warm Water.—

1 grm. of warm water in cooling through 1° C. gives out 1 calorie.

(m_3-m_2) grms. of warm water in cooling from t_2° to t_3° C. give out $(m_3-m_2)(t_2-t_3)$ calories.

Heat received by the Cold Water.—

1 grm. of cold water in being raised by 1° C. receives 1 calorie.

(m_2-m_1) grms. of cold water in being raised from t_1° to t_3° C. receive $(m_2-m_1)(t_3-t_1)$ calories.

The calorimeter was at first at the temperature of the cold water (t_1° C.) and was then heated up to t_3° C.

Thus the *heat received by the calorimeter* = the total heat given out — the heat received by the cold water.

This $=(m_3-m_2)(t_2-t_3)-(m_2-m_1)(t_3-t_1)$ calories.

But this heat raised the temperature of the calorimeter (t_3-t_1)° C.

∴ **Heat received by calorimeter in being heated by 1° C.**

$$=\frac{(m_3-m_2)(t_2-t_3)-(m_2-m_1)(t_3-t_1)}{(t_3-t_1)} \text{ calories}$$

$$=\frac{(m_3-m_2)(t_2-t_3)}{(t_3-t_1)}-(m_2-m_1) \text{ calories}$$

= **the thermal capacity of the calorimeter.**

Suppose the value is 12 calories. Then the calorimeter requires as much heat to raise it 1° C. as does 12 grms. of water. Now, in calorimetry experiments water is nearly always used in the calorimeter and the other substances warmed up and added. It is thus very convenient to express the heat capacity of the calorimeter in terms of water and so consider the calorimeter and the water in it as only water. Its thermal capacity is thus that of the water in the calorimeter together with that of the mass of water to which the calorimeter is equivalent. Thus we often call the heat capacity of a calorimeter the **Water Equivalent of the Calorimeter,** and instead of writing it in heat units (*i.e.* 12 calories) we express it in terms of the water and say the water equivalent of the calorimeter is 12 grms. Once a value has been experimentally found it can be used in the calculation for any other calorimetry experiment carried out with that calorimeter.

The water equivalent of a calorimeter can also be calculated if the mass (m grms.) and the specific heat (s) of the material are known, for:

Thermal capacity of calorimeter = heat required to raise its temperature 1° C.

= mass × sp. ht.
= *ms* calories.

Hence water equivalent of calorimeter = ms grms.

The experimental method is better since it corrects for the thermometer and any stirrer used, and the same thermometer and stirrer should be used in subsequent experiments. Usually, if care is taken, the thermometer can be the stirrer.

Specific Heat of a Solid—Method of Mixtures.— The method is similar to that described for finding the water equivalent of a calorimeter In dealing with a solid (which is heated up and lowered into cold water

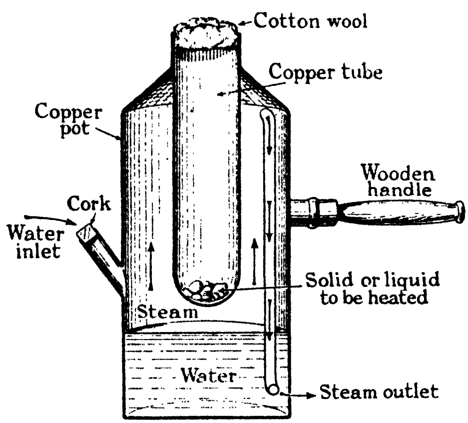


FIG. 50.—Heater for Specific Heat Experiments.

in the calorimeter) steps have to be taken to see that the solid is heated right through and its temperature accurately found. Solids are usually suspended for several minutes in boiling water so that they become heated right through to the temperature of the boiling water. The transference of the solid to the cold water must be rapid, and at the same time it must be well shaken to remove any particles of water. For very exact work, the solid should be

broken up, or powdered, and heated in a special tube from which it can be quickly poured (see Fig. 50). This form of heater is often used in dealing with liquids.

Experimental calorimetry is made much easier by the use of automatic scales reading to $\frac{1}{2}$ grm. provided large calorimeters and large quantities of the substances are used, *e.g.* approximately 200 grms. of water and 100–200 grms. of the solid. If, however, small calorimeters are used, weighings to 0·05 grm. are sufficient. It is very important that temperatures should be read as accurately as possible—using the ordinary thermometers, calibrated to 1° C., estimation to $\frac{1}{2}$° C. should be made. Thermometers calibrated in $\frac{1}{10}$° C. are, of course, better, but are not much used in ordinary work on account of their cost. Enter your results and calculations as shown.

SPECIFIC HEAT (s) OF A SOLID

		Expt. 1.	Expt. 2.
Mass of calorimeter	$=m_1$ grms.		
Mass of calorimeter + cold water	$=m_2$ grms.		
Mass of cold water used . .	$=(m_2-m_1)$ grms.		
Mass of calorimeter + cold water + solid	$=m_3$ grms.		
Mass of solid used	$=(m_3-m_2)$ grms.		
Temperature of cold water . .	$=t_1$° C.		
Temperature of hot solid = temperature of boiling water	$=t_2$° C.		
Temperature of mixture . .	$=t_3$° C.		

Calculation.—

Water equivalent of calorimeter = W grms. (by previous experiment)
or = Mass × sp. ht.
= mass × 0·1 (if copper calorimeter).

Heat received by water and calorimeter [equivalent mass is $(m_2 - m_1)+W$ grms] of specific heat 1 raised in temperature from t_1° to t_3° C.

$$=(m_2-m_1+W)(t_3-t_1)\times 1 \text{ calories.}$$

Heat given out by solid (m_3-m_2) grms., of specific heat s, in cooling from t_2° to t_3° C. $=(m_3-m_2)(t_2-t_3)\, s$ calories.

Assuming there is no loss of heat to the air, the heat lost = the heat gained.

Thus $(m_3-m_2)(t_2-t_3)s=(m_2-m_1+W)(t_3-t_1)$,

or $$s=\frac{(m_2-m_1+W)(t_3-t_1)}{(m_3-m_2)(t_2-t_3)}$$

This calculation assumes that there is no chemical action on mixing and that the solid does not dissolve in water, for in such cases we find that heat is evolved or absorbed (" Heat of Reaction " or " Heat of Solution "). If this does happen with water, then another liquid (of known specific heat) with which the solid does not react, or in which it does not dissolve, must be used, or the solid must be sealed up in a thin tube of metal (*e.g.* copper, as long as there is no action between the two solids) and the water equivalent of the tube found and used in the calculation.

Specific Heat of a Liquid.—This, too, is usually determined by the method of mixtures, and the remarks just made regarding chemical action and solution hold, and so we often have the liquid in a sealed metal tube. Also it is often necessary to pour hot water into the cold liquid in the calorimeter when no metal tube is essential, for the liquid may decompose on heating in the air. The calculation is similar to that for finding the specific heat of a solid, except that when the water is poured into the other liquid in the calorimeter, the calculation for the heat absorbed by the calorimeter must be made separately from the calculation for the heat absorbed by the liquid in it (see calculation below).

A more common way is to use a solid, such as a piece of metal, which does not react with the liquid. After heating in boiling water the solid is lowered into some of the liquid in the calorimeter.

RESULTS for SPECIFIC HEAT OF A LIQUID (s) using a solid of sp. heat 0·084

		Expt. 1.	Expt. 2.
Mass of calorimeter	$=m_1$ grms.		
Mass of calorimeter+cold liquid	$=m_2$ grms.		
Mass of cold liquid	$=(m_2-m_1)$ grms.		
Mass of calorimeter+liquid and hot solid	$=m_3$ grms.		
Mass of hot solid . . .	$=(m_3-m_2)$ grms.		
Temperature of cold liquid . .	$=t_1$° C.		
Temperature of hot solid=(temperature of boiling water)	$=t_2$° C.		
Temperature of mixture . .	$=t_3$° C.		

Calculation.—Heat given out by hot solid (m_3-m_2) grms. sp. ht. 0·084, in cooling from $t_2°$ to $t_3°$ C.$=(m_3-m_2)(t_2-t_3)$ 0·084 calories.

Heat received by cold liquid (m_2-m_1) grms. sp. ht. s, in being raised from $t_1°$ to $t_3°$ C.$=(m_2-m_1)(t_3-t_1)s$ calories.

Heat received by calorimeter of water equivalent W grms. in being raised from $t_1°$ to $t_3°$ C.$=W(t_3-t_1)$ calories.

Since heat received = heat given out,

$$(m_2-m_1)(t_3-t_1)s+W(t_3-t_1)=(m_3-m_2)(t_2-t_3)0{\cdot}084$$

or

$$s=\frac{(m_3-m_2)(t_2-t_3)\ {\cdot}084}{(m_2-m_1)(t_3-t_1)}-\frac{W}{(m_2-m_1)}$$

A variation of this experiment is, knowing the specific heat of a liquid and a solid, to determine the temperature of the solid when heated up to an unknown temperature, *e.g.* when heated in a bunsen burner, fire, or furnace, to the highest temperature attainable. The calculation is as above, except that s is known and t_2 is the unknown to be determined.

TABLE OF SPECIFIC HEATS OF SUBSTANCES

Water	1	Mercury	0·033
Ice (−20° C. to 0° C.)	0·5	Copper	0·095
Alcohol	0·55	Aluminium	0·22
Olive Oil	0·47	Iron	0·109
Linseed Oil	0·44	Zinc	0·092
Paraffin Oil	0·52	Quartz	0·18
Glycerine	0·58	Pyrex Glass	0·20
Ordinary Crown Glass	0·16	Silica	0·20

This table should be referred to in a revision of the work on thermometric fluids (p. 32). It should also be observed that kettles, boilers, etc., in which water is heated, are made of substances which have low specific heats (and, of course, are durable).

Atomic Heats.—*Dulong and Petit's Law.*—The ***atomic heat of an element*** is measured by its atomic weight × its specific heat. It is thus the thermal capacity of a mass, in grms., equivalent to the atomic weight.

Dulong and Petit's Law states that the atomic heats of elements in the same state are approximately equal (the value being approximately 6·4). This is illustrated in the following table:

Solid element.	Atomic weight.	Specific heat.	Atomic heat.
Nickel	58·7	0·109	6·4
Potassium	39·1	0·166	6·5
Solid Mercury	200·6	0·0331	6·6
Platinum	195·2	0·0324	6·3
Tin	118·7	0·0536	6·36

If we have equal numbers of atoms of different elements, the masses will be proportional to the atomic weights. But the above law shows that the thermal capacities of these masses are the same. Thus it follows that the thermal capacities of equal numbers of atoms of different substances are the same, and so **atoms of all elements have the same thermal capacity.**

Latent Heat.—At the beginning of this chapter we saw how Joseph Black realised that heat was necessary to change ice into water, or water into steam, without a change of temperature, and that he called this heat latent (or "hidden") heat, since it was stored up and released on the reverse change of state taking place. But, when heat is given to cold water to warm it up, this heat is also "hidden" and is given out by the water on cooling down. The use of the word "latent," as introduced by Black, is therefore a little misleading, and we often distinguish the heat given to a body as—

(*a*) *sensible heat*, when it raises the temperature,
(*b*) *latent heat*, when it changes the state without raising the temperature.

Latent heat is of two kinds, then,

(1) that concerned in the change of state from a solid to a liquid and called the *latent heat of fusion* (or *liquefaction*) of a substance,
(2) that concerned in the change of state from a liquid to a vapour and called the *latent heat of vaporisation* of a substance.

Latent heats depend on the masses of the substances as well as on their nature, and for comparison purposes

we find the heat values for unit mass, *i.e.* for 1 grm. or 1 lb. according to the units used.

Thus the **latent heat of fusion of a substance is the quantity of heat necessary to change 1 grm. (or 1 lb.) of the substance from the solid to the liquid state without changing its temperature** (or released in the reverse change from liquid to solid).

Also the **latent heat of vaporisation of a substance is the quantity of heat necessary to change 1 grm. (or 1 lb.) of the substance from the liquid to the vapour state without changing its temperature** (or released in the reverse change from vapour to liquid). In both cases the latent heats are measured in either calories per grm. or British Thermal Units per lb.

We are more usually concerned with water, and the respective values are usually called the *latent heat of ice* and the *latent heat of steam*. Their values are obtained by mixture methods in a calorimeter.

Latent heat of ice.—A weighed calorimeter is about one-third filled with water, which has been warmed to about 30° C., and weighed again. Some small pieces of ice (as big as hazel nuts) are well dried by blotting-paper. The temperature of the water in the calorimeter is observed and a few grams of the dried ice are dropped in. The whole is well stirred and the lowest reading of the thermometer observed (preferably about 8°–10° C., so that when all the ice is melted, the whole gets warmed up by the air of the room). The water and calorimeter cool down in giving up heat to (*a*) melt the ice, (*b*) warm up the melted ice. Enter your results and calculation as follows:

RESULTS

		Expt. 1.	Expt. 2.
Mass of calorimeter	$=m_1$ grms.		
Mass of calorimeter+warm water	$=m_2$ grms.		
Mass of warm water used . .	$=(m_2-m_1)$ grms.		
Mass of calorimeter+warm water and ice	$=m_3$ grms.		
Mass of ice added	$=(m_3-m_2)$ grms.		
Temperature of warm water .	$=t_1$° C.		
Temperature of melting ice .	$=0$° C.		
Temperature of mixture . . .	$=t_2$° C.		

Calculation.—Let L calories per gram=the latent heat of ice, and W grms.=the water equivalent of the calorimeter.

Heat given out by (m_2-m_1) grms. of water and by the calorimeter in cooling from $t_1°$ to $t_2°$ C.$=(m_2-m_1+W)(t_1-t_2)\times 1$ calories.

Heat received by the (m_3-m_2) grms. of ice,

(*a*) in melting

$$=(m_3-m_2)\text{ L calories,}$$

(*b*) after melting, in being raised from 0° to $t_2°$ C.

$$=(m_3-m_2)(t_2-0)\times 1 \text{ calories.}$$

But heat given out=heat received.

$$\therefore (m_2-m_1+W)(t_1-t_2)=(m_3-m_2)L+(m_3-m_2)t_2,$$

or
$$L=\frac{(m_2-m_1+W)(t_1-t_2)}{(m_3-m_2)}-t_2 \text{ calories per gram.}$$

For ice, the value is 80 calories per grm. (*i.e.* grm. ° C. units per grm.). But in British units it is measured in pound ° F. units per lb. and so its value must be larger in proportion to the ratio of a degree C. to a degree F., *i.e.* 9 to 5. Thus the latent heat of ice in British Thermal Units per lb.$=\frac{80\times 9}{5}=144$. This is true for all latent heats—the value of the latent heat in calories per grm. must be multiplied by 9/5 to obtain its value in British Thermal Units per lb.

To ensure accuracy the following points must be noticed:

(1) A source of very appreciable error lies in the calculation if the measured mass of added substance is not all ice. If great care is not taken some melted ice, *i.e.* water, is added and not dry ice only. The importance of avoiding such an error is seen from the fact that if only 0·2 grm. of water, not ice, is added, in the calculation it is assumed that the heat required to melt it is $0{\cdot}2\times L$, which is equal to $0{\cdot}2\times 80$ or 16 calories. Such an error quite appreciably affects the accuracy of the result obtained.

(2) Time is taken for the ice to melt—yet we assume that there is a negligible quantity of heat given out to

the air. Thus it is important that the water should not be hot at the commencement of the experiment, for if it is above 30° C. it will give out an appreciable amount of heat whilst the ice is melting.

(3) It must be remembered in making the calculation, that the ice, after melting, is warmed up to the final temperature. It is often found that this source of absorption of heat is ignored.

Latent Heat of Steam.—This value is usually obtained by passing steam (temperature measured) into a calorimeter about two-thirds filled with cold water, so that the temperature rises from air temperature to about 50° C. (exactly measured). The increase in weight gives the weight of steam condensed.

RESULTS

		Expt. 1.	Expt. 2.
Mass of calorimeter	$=m_1$ grms.		
Mass of calorimeter+cold water	$=m_2$ grms.		
Mass of cold water used	$=(m_2-m_1)$ grms.		
Mass of calorimeter+cold water +condensed steam	$=m_3$ grms.		
Mass of condensed steam	$=(m_3-m_2)$ grms.		
Temperature of cold water	$=t_1°$ C.		
Temperature of steam	$=t_2°$ C.		
Temperature of mixture	$=t_3°$ C.		

Calculation.—Let L calories per grm.=latent heat of steam.

Heat given out by (m_3-m_2) grms. of steam,

(*a*) in condensing$=(m_3-m_2)$ L calories,

(*b*) in cooling, after condensing, from $t_2°$ to $t_3°$ C.

$$=(m_3-m_2)(t_2-t_3)\times 1 \text{ calories.}$$

Heat received by (m_2-m_1) grms. of water and the calorimeter (water equivalent W grms.) in being raised from $t_1°$ to $t_3°$ C.$=(m_2-m_1+W)(t_3-t_1)\times 1$ calories.

But heat given out=heat received.

$$\therefore (m_3-m_2)L+(m_3-m_2)(t_2-t_3)=(m_2-m_1+W)(t_3-t_1)$$

$$\text{or}\quad L=\frac{(m_2-m_1+W)(t_3-t_1)}{(m_3-m_2)}-(t_2-t_3) \text{ calories per grm.}$$

Sources of Error.—(*a*) The most important point is that the increase in weight of the calorimeter at the

end gives the mass of water collected. This is considered, in the calculation, to have been steam at the beginning. But in generating steam in a boiler, particles of water tend to be carried along by the steam. If this happens a large error is introduced, for only 0·1 grm. of water is carried over and considered to be steam and to release (0·1 × the latent heat of steam) calories, which is equal to 0·1 × 540 calories or 54 calories. Thus one side of the heat equation is 54 calories out, and so a correct value is not obtained. Care, therefore, has to be taken to pass only *dry* steam into the water in the calorimeter, and is secured by the use of steam traps, a common form of which is shown in Fig. 51. B is the steam supply tube bent so as to make it more difficult for water particles to be carried on, whilst the continuous flow of steam through it warms it up so that there is no condensation on it. One steam supply apparatus is usually sufficient in a laboratory, and the members of the class can take their calorimeters to it and use in turn. (*b*) The water in the calorimeter must not be heated to too high a temperature—50° C. is sufficiently high—otherwise an appreciable loss of heat to the air will result.

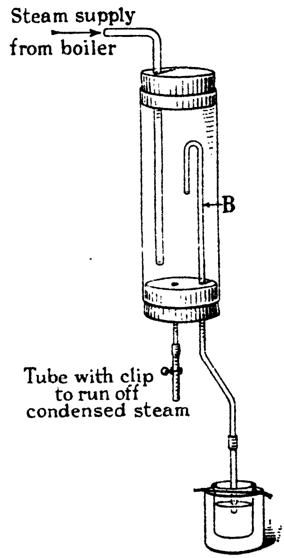


FIG. 51.—Steam Trap.

The determination of the latent heats of vaporisation of other substances is often made by passing vapour into the liquid form of the same substance, which is much below its boiling point. A more convenient form of apparatus, however, is *Bertholet's apparatus* (see Fig. 52) which avoids the error due to partial condensation of the vapour before it enters the calorimeter. The liquid to be used is boiled in an inverted flask, by a small gas ring burner. The vapour passes down into a glass, or preferably metal, spiral tube, immersed in a calori-

meter of water whose rise of temperature is noted, and condenses at the bottom. The gain in weight of the spiral tube gives the mass of the vapour condensed. Knowing the water equivalent of the calorimeter and the spiral, the latent heat of vaporisation of the liquid is readily calculated.

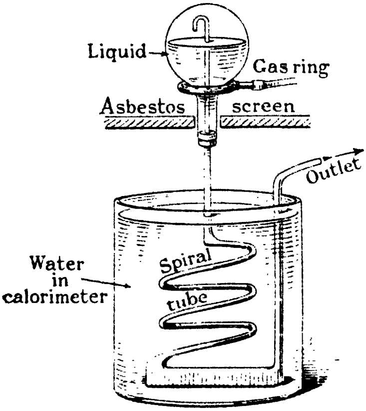


FIG. 52.—Bertholet's Apparatus for Latent Heat of Vaporisation.

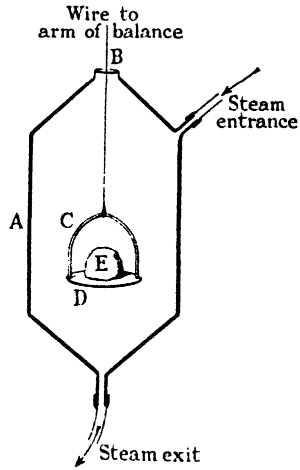


FIG. 53.—Joly's Steam Calorimeter.

Joly's Steam Calorimeter is an excellent apparatus for finding the latent heat of steam (or of another substance), and it has the great advantage of requiring no careful and rapid observations of a thermometer. A simple form is shown in Fig. 53. Through the top of the metal frame A, and without touching it, passes a thin metal wire B carrying a wire frame C with a metal pan D. On this is placed a piece of metallic substance E of known specific heat (s). The wire B is attached to one arm of a balance instead of the usual scale pan, etc. Dry steam is passed through the calorimeter and, in giving up heat to warm up the piece of metal, some condenses. Eventually, when all is heated up to the temperature of the steam, no more condenses and a steady state is reached. The gain in weight of the suspended portion is obtained, this giving the mass of steam condensed (m grms.).

Thus heat given out by the steam in condensing $= m\mathrm{L}$ calories, where $\mathrm{L}=$ the latent heat of steam.

But the mass of metal (x grms.) was raised in temperature from air temperature (t° C.) to the temperature of the steam (suppose 100° C.) and so received $x(100-t)s$ calories.

But heat received = heat given out by the condensed steam.

$$x(100-t)s=m\mathrm{L}$$

or
$$\mathrm{L}=\frac{x}{m}(100-t)s \text{ calories per grm.}$$

A preliminary experiment has to be carried out to find the gain in weight due to the water deposited on the carrier, pan and wire, in warming them up, and this weight is subtracted from the gain in weight obtained in the above experiment. This calorimeter is extremely sensitive and results obtained with it are very accurate. It can, of course, also be used to find the specific heat of a substance (placed on D), given the latent heat of steam.

Some typical numerical problems are worked out below.

EXAMPLE 1.—In attempting to get a rough idea of the temperature of a furnace, a copper cylinder weighing 140 grms. was allowed to reach the temperature of the furnace, and then quickly withdrawn and dropped into 500 c.c. of water at 14° C in a vessel whose water equivalent was 25. The temperature of the water rose to 21° C. Calculate the temperature of the furnace.

(sp. ht. of copper $=0{\cdot}1$ cal. per grm.) (J.M.B Sept. 1928.)

Let t° C. = the temperature of the furnace.

Heat given out by the copper (140 grms. sp. ht. $0{\cdot}1$) on cooling from t° C. to 31° C. $=140(t-31)0{\cdot}1$ calories.

Heat received by the calorimeter (water equivalent 25 grms.) and the water (500 grms.) in being raised from 14° C. to 31° C. $=(500+25)(31-14)\times 1$ calories.

But these must be equal, assuming no heat lost to air, etc., during mixing.

$$\begin{aligned} \therefore\ 14(t-31) &= 525\times 17 \\ 2(t-31) &= 75\times 17 \\ 2t-62 &= 1275 \\ \text{or}\quad 2t &= 1337 \\ \underline{t} &\underline{= 668{\cdot}5^\circ \text{ C.}} \end{aligned}$$

EXAMPLE 2.—A hot water supply is heated by coal gas and delivers 1·5 gallons per minute at a temperature of 54° F. higher than the temperature of the entering water. If one therm (100,000 British Thermal Units) cost tenpence, what is the cost of the gas consumed per hour, assuming that 80 per cent. of the heat goes to the water? A gallon of water weighs 10 lbs. (L.M. 1928.)

In 1 min. 1·5 gallons=15 lbs. of hot water are supplied.

∴ In 1 hour 15×60 lbs. of hot water are supplied.

∴ In 1 hour $15\times 60\times 54$ lbs. ° F. (or B.Th.Units) are used since the water is raised 54° F.

Since only 80 per cent. of the heat of the gas goes in the water, the gas supply must be equivalent to $\frac{100}{80}$ times this amount;

$$i.e.\ \tfrac{100}{80}\times 15\times 60\times 54 \text{ B.Th.Units.}$$

Since 100,000 B.T.U.s=1 therm, the gas supplied in 1 hour

$$=\frac{1\cancel{0}\cancel{0}}{8\cancel{0}}\times\frac{15\times 6\cancel{0}\times 54}{100{,}0\cancel{0}\cancel{0}} \text{ therms.}$$

$$\text{Cost at 10}d.\text{ per therm}=\frac{15\times 6\times 54\times 10}{8\times 1000}\text{ pence}=\tfrac{243}{40}$$
$$=6{\cdot}075 \text{ pence.}$$

EXAMPLE 3.—Steam at 100° C. is blown into a mixture of 40 grms. of melting ice and 60 grms. of water at 0° C. in a copper calorimeter of water equivalent 20 grms. and the final temperature reached was 50° C. What mass of steam was condensed? Latent heat of ice and steam are 80 and 540 cals. per grm. respectively.

In working this example, the student should notice how a pictorial representation of the operation and heat transferences assists in the calculation, and should endeavour always to give such a diagram with calculations he is making. (Fig. 54.)

Let m grms. of steam be condensed.

Heat given out by this m grms. of steam,

(*a*) in condensing $=m\times 540$ calories,

(*b*) in cooling from 100° C. to 50° C.$=m\times(100-50)\times 1$ since sp. ht. of water=1.

Heat received,

(*a*) by 40 grms. of ice in melting $=40\times 80$ calories,

(*b*) by the melted ice, water and calorimeter (total equivalent $=40+60+20$ grms. $=120$ grms.) in being raised from 0° C. to 50° C. $=120\times(50-0)\times 1$ cals.

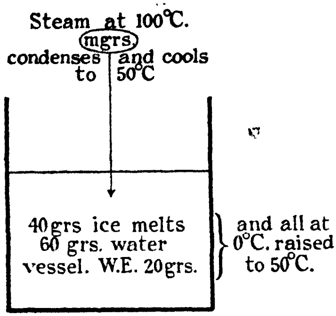


FIG. 54.

But the heat given out=heat received.

$$\therefore\ 540m+50m=3200+120\times 50$$
$$590m=9200$$
$$m=15{\cdot}6 \text{ grms. approx.}$$

EXERCISES ON CHAPTER VI

1 Explain *thermal capacity* and describe the experiments you would perform in order to show that the thermal capacity of a body depends upon the mass and the nature of the body. What is the difference between thermal capacity and specific heat?

2. What is meant by the specific heat of a substance? Describe how you would find the specific heat of copper turnings, or iron filings, and show exactly the calculations you would make.

3. Explain the use of the expressions: temperature, quantity of heat, calorie, specific heat; and show how the specific heat of a liquid such as castor oil might be measured.

4. Define *water equivalent.* 20 grms. of water at 50° C. are poured into a vessel containing 20 grms. of water at 20° C. and the temperature of the mixture is observed to be 31° C. Next 20 grms. of water at 20° C. are poured into 20 grms. of water at 50° C. in the same vessel. The resulting temperature is 36° C. Calculate the water equivalent of the vessel from each experiment and suggest an explanation for any discrepancy in the result. [*L.M.* 1925.]

5. Explain "the specific heat of lead is 0·031." 200 grms. of lead are heated up to 100° C. and dropped into a calorimeter containing 100 grms.

of water at 12° C. Calculate the final temperature of the water if the calorimeter absorbs no heat.

6. *Explain the terms calorie, specific heat, latent heat, water equivalent. 30·5 grms. of water at 40·2° C. are poured into 70·4 grms. of water at 15·4° C.* contained in a copper calorimeter and the resulting temperature is found to be 22·1° C. Calculate the water equivalent of the calorimeter. [*L.G.S.* 1929.]

7. Describe carefully any method you have used for measuring the specific heat of a metal. A brass calorimeter, mass 115 grms., contains 84 grms. of water at 13·6° C. when a piece of brass, mass 94 grms. at 100° C. is dropped in. If the temperature of the water then rises to 20·5° C., find the specific heat of the brass. [*L.G.S.* 1920.]

8. Explain the terms: quantity of heat, calorie, thermal capacity, specific heat, and describe a method for determining the specific heat of a liquid which must not be mixed with water. Copper of mass 150 grms. and specific heat 0·095 is heated, and immersed in 100 grms. of water at 0° C. and the final temperature is 20° C. What is the initial temperature of the copper?

9. Define specific heat. 180 grms. of copper at 97° C. are immersed in 150 grms. of a liquid at 8° C. and the final temperature is found to be 21° C. If the specific heat of copper is 0·1, calculate the specific heat of the liquid. What correction should be made in such an experiment?

10. Explain the term specific heat of a substance. 280 grms. of lead at 98° C. are dropped into 150 grms. of a liquid at 8° C. contained in a copper calorimeter of water equivalent 12 grms. The resultant temperature of the mixture is 15° C. What is the specific heat of the liquid? Specific heat of lead=0·031. [*L.M.* 1928.]

11. 100 grms of copper (specific heat 0·095) are heated to 100° C. and put into 150 grms of petroleum at 10° C. contained in a copper pot weighing 40 grms. The temperature rises to 19·5° C. Find the specific heat of the petroleum. [*L.G.S.* 1923.]

12. The temperature of 500 grms. of water, contained in an iron saucepan weighing 750 grms., is raised from 15° C. to the boiling point over a gas ring in 5 minutes. The gas ring weighs 1,000 grms and the average temperature is 75° C. when the water boils. What is the least number of calories the burner must supply per minute? Why would the actual gas consumption differ from that calculated? Specific heat of iron=0·11. [*L.G.S* 1924.]

13. Describe and explain the reason for the various precautions that should be taken in performing experiments for the measurement (*a*) of the specific heat of a metal, (*b*) of the latent heat of water, and (*c*) of the latent heat of steam. [*L.M.* 1919.]

14. Define the latent heat of fusion of a solid. 12 grms. of ice at its melting point were dropped into 61 grms. of water at 24·5° C., contained in a copper calorimeter of mass 50 grms. and specific heat 0·1. The final resulting temperature was 8·5° C. Calculate the value of the latent heat of fusion of ice. State concisely the precautions advisable in performing this experiment. [*L.G.S.* 1928.]

15. What do you understand by the "latent heat" of ice, and how would you determine it experimentally? 100 grms. of ice are dropped into 100 grms. of oil (specific heat=0·5) at 15° C., contained in a calorimeter (specific heat=0·1) weighing 50 grms. How much ice will be melted? [*C.W.B.* 1928].

16. Explain the statements that the latent heats of vaporisation of water and of fusion of ice are 536 and 80 calories per grm. respectively. Some water at 0° C. is raised to the boiling point (100° C.) by condensing in it 6 grms. of steam. The whole is then cooled down to the freezing point (0° C.) by the addition of ice. How much ice is required ?

17. What experiments would you perform to show that when ice melts there is (*a*) a change in volume, (*b*) an absorption of heat ? How would you determine accurately the quantity of heat necessary to melt 1 grm. of ice ? [*L.G.S.* 1924.]

18. Define the term *latent heat*. How does the numerical value of this quantity in any special case depend on the scale of temperature employed ? A pond 50 sq. metres in area is covered with ice at 0° C. How much ice will melt per hour if it absorbs heat from the sun at the rate of 0·25 calorie per sq. cm. per minute ? [*L.M.* 1921.]

19. Explain what is meant by the terms specific heat and latent heat. 100 grms. of ice at −10° C. are put into water at 0° C. and steam at 100° C. is passed into the mixture at the rate of 1 grm. each minute. Assuming that no heat is lost or gained by radiation during the process, find how long it will take for the ice to be just melted. Specific heat of ice=0·5, latent heat of steam=540, latent heat of water=80. [*J.M.B.* 1922.]

20. A vessel of water equivalent 20 grms. contains 500 grms. of water mixed with 100 grms. of ice. If 50 grms. of steam at 100° C. are passed into it, find the resultant temperature. (Latent heat of steam=540, latent heat of fusion=80). [*L.M.* 1924.]

21. Explain clearly the meaning of the statement that the latent heat of steam at 100° C. is 540. In carrying out an experiment to find the latent heat of steam, 5·5 grms. of steam were passed into a calorimeter weighing 51 grms. containing 96·8 grms. of water, the temperature of which rose from 14·1° to 40·4° C. Calculate how much of the steam was condensed before its passage into the calorimeter (sp. ht. of calorimeter=0·10). [*L.G.S.* 1920.]

22. How is *heat quantity* measured ? What mass of steam at 100° C. must be passed into a swimming bath 50 ft.×40 ft.×6 ft. so that the temperature of the water may rise from 3° C. to 13° C. ? Latent heat of steam=540. 1 cu. ft. of water weighs 62·5 lbs. [*L.M.* 1920.]

23. Define *specific heat*, *latent heat* and *capacity of a body for heat*. If 10 grms. of steam at 100° C. are blown into 200 grms. of water at 10° C., what will be the resulting temperature of the water ? The latent heat of steam is 537 calories per grm. [*J.M.B.* 1928.]

24. When steam at 100° C. is passed into a mixture of ice and water, it is found that 15 grms. are condensed before all the ice is melted. How much ice was there with the water when the steam began to pass ? Latent heat of vaporisation of water=540. Latent heat of fusion of ice=80. [*J.M.B.* 1924.]

25. Define *calorie*, *British Thermal Unit*, *Therm*. A gas ring burning 30 cu. ft. of gas per hour is used to heat water. If the calorific power of the gas is 500 B.Th.U. per cubic foot and 60 per cent. of the heat gets to the water, how long will it take (*a*) to raise the temperature of 5 pints of water from 62° F. to the boiling point, (*b*) for the kettle to boil dry after boiling commences ? (1 pint of water weighs 20 ozs. Latent heat of vaporisation of water=972 B.Th.U. per lb.). [*L.M.* 1925.]

26. The modern gas-bill is made out in terms of *therms* instead of cubic feet. Explain this. With gas at 1*s*. 4*d*. per therm, how much would

it cost to fill a 20-gallon bath by means of a geyser, assuming no waste of heat, if the geyser raised the water temperature from 58° F. to 108° F.? (1 gallon of water weighs 10 lbs; 1 therm = 100,000 B.Th.U.) [*J.M.B.* 1927.]

27. Explain what is meant by the *water equivalent* of a body. Steam at 100° C. is passed into a vessel containing 240 grms. of a liquid at 6° C. When the temperature reaches 27° C. the steam jacket is removed and, on weighing, it is found that 5 grms. of steam has been condensed. If the specific heat of the liquid is 0·56, what is the water equivalent of the vessel? (Latent heat of steam = 540 calories per grm.) [*L.G.S.* 1929.]

28. (*a*) The latent heat of fusion of sulphur which melts at 113° C. is 9 calories per grm. (*b*) The specific heat of solid sulphur is 0·17 calorie per grm. Explain the meaning of the above statements. Find the rise in temperature to the nearest degree when 35 grms. of liquid sulphur at its melting point are poured into a copper calorimeter weighing 40 grms. and containing 100 grms. of water at 14° C. (Specific heat of copper = 0·1 cal. per grm.) [*J.M.B.* 1929.]

29. Heat is supplied to a copper calorimeter weighing 60 grms. and containing 70 grms. of aniline by electrical means at the rate of 500 calories per minute. Assuming that no heat is lost and the initial temperature of the aniline is 12° C., determine (*a*) the time required to reach the boiling point of the aniline, (*b*) the further time required for half the aniline to boil away. (B.Pt. of aniline = 183·9° C.; sp. ht. of aniline = 0·54; L. Ht. of vaporisation of aniline = 104 cals. per grm.; sp. ht. of copper = 0·093.) [*L.M.* 1930.]

30. Explain what is meant by saying that the specific heat of paraffin oil is 0·55. Why is this number unaltered when either the unit of mass or the scale of temperature is changed? 300 grms. of lead shot are raised to a temperature of 100° C. and quickly transferred to a copper calorimeter weighing 75 grms. and containing 120 grms. of paraffin oil at 15° C. What will be the resulting temperature? (Specific heat of copper = 0·095; of lead shot 0·034.) [*L.M.* 1929.]

CHAPTER VII

RELATION BETWEEN HEAT AND WORK— MECHANICAL EQUIVALENT OF HEAT

IT has already been shown that the early material theory of heat, the caloric theory, played an important part in the development of our knowledge of heat, and was responsible for the idea that a quantity of heat could be measured. It thus led, in the eighteenth century, to the realisation of thermal capacity (and later specific heat) and latent heat. But, as is often the case, the new knowledge led to observations and deductions which did not altogether agree with the original theory.

The first man to doubt the truth of the caloric theory was **Count Rumford** (1753–1814), a man who had a very adventurous life. Born Benjamin Thompson at Woburn, U.S.A., he studied medicine at Harvard in his spare time, whilst employed as a clerk in Boston. In 1772 he took up a teaching post at Concord, and later married a wealthy young widow. The acquisition of money gave him more time for his studies, whilst he also commenced a military career. Owing to jealousy regarding his position as Major, he felt compelled to resign, and he came to England where, on account of his work in connection with artillery and marine signals, he was made a Fellow of the Royal Society of England in 1779. He fought on England's side in the American War of Independence (1781), and after this he settled in Bavaria, subsequently holding important civil and military appointments. In 1791 he was made a Count of the Holy Roman Empire and took the name Rumford (another name for Concord).

In 1798 he reported to the Royal Society his observations made whilst superintending the boring of cannon

as Chief of the Munich Naval Arsenal, and which led him to doubt the caloric theory.

When a body is moving over a surface there is an opposition to movement, known as a force of friction. When friction is being overcome—when a body is moving—heat is generated. Upon this depends the striking of the ordinary red-topped, non-safety match, whilst man's earliest method of lighting a fire was by means of heat generated by rubbing two flints together. It is commonly observed that the soles of one's boots become warm when sliding, or playing football or netball on a surface like asphalt.

By the caloric theory this phenomenon was explained as follows : that heat was rubbed or squeezed out, because the filings, or small particles, rubbed off in the rubbing of one body over another, were of lower thermal capacity than the solid material. But Rumford observed in the making of cannon that the blunter the borer used, the fewer the filings, although the amount of heat generated was very much greater, and, in fact, the supply of heat by such a means appeared to be inexhaustible. Rumford, for instance, showed an experiment in which the heat generated by a blunt steel borer made 19 lbs. of water boil in $2\frac{1}{2}$ hours in a gun-metal cylinder. He also experimented and found that iron filings had the same thermal capacity as an equal mass of solid iron, and further that the presence or absence of air made no difference.

In 1799 Sir Humphry Davy rubbed together two blocks of ice in a space from which most of the air had been removed by an air-pump, and showed that they melted. But water has a greater thermal capacity than ice (specific heat of water$=1$, and that of ice$=0{\cdot}504$). Yet, according to the caloric theory, the water should be of lower thermal capacity than the solid blocks of ice. The caloric theory thus failed to explain the origin of the supply of latent heat necessary for the ice to melt.

Davy, himself, did not realise the importance of his work, and for a time his experiments were ignored. About 1812 he concluded that Rumford's and his own experiments showed that friction caused vibration of

particles of matter and that this vibration was heat. However, although the caloric theory was condemned, at the beginning of the nineteenth century nothing had arisen to take its place.

Relation between Work Done and Heat Generated.—Experiments carried out by James Prescott Joule, about 1840, gave the death-blow to the caloric theory, and gradually the modern kinetic theory received general acceptance. Joule (1818–1889) was born at Salford and studied under John Dalton, the chemist of Atomic Theory fame, becoming inspired to take up original research. He carried this on privately—for he was the proprietor of a large brewery. Joule really continued to a logical conclusion the observations of Rumford, thereby realising that when a body moves over a surface, the opposition to motion, the force of friction, is overcome and heat is generated. Joule experimented to find whether there was a relation between the heat generated and the work done in overcoming the frictional resistance. He first gave his results in a paper to the British Association in 1843, and again, with additions, to the meeting at Oxford in 1847, when the matter was taken up by William Thomson (afterwards Lord Kelvin). The paper caused a great sensation, and the kinetic theory of heat quickly developed and gained favour.

Joule worked in 1847 with paddle vanes M moving in water in a copper calorimeter C (Fig. 55). These vanes were set in motion by just sufficient falling weights WW to move them against the frictional resistance of the water. If the water only moved round as a whole the aim of the experiment would be defeated, and so the calorimeter was divided into separate portions by fixed vanes F. Each time, after the weights had fallen and rotated the paddle vanes, the axle A was released at R from the spindle S and the weights were wound up without rotating the paddle vanes. The arrangement of the string on the weights and the spindle was, of course, such that when the two weights were free to pull, the string unwound, rotating the axle. The work done was calculated in ft. lbs., from the total weight in lbs. weight × the distance fallen in

feet × the number of times the weights fell. The heat generated was calculated in British Thermal Units from the total water equivalent of the calorimeter and its contents in lbs. × the rise in temperature in ° F.

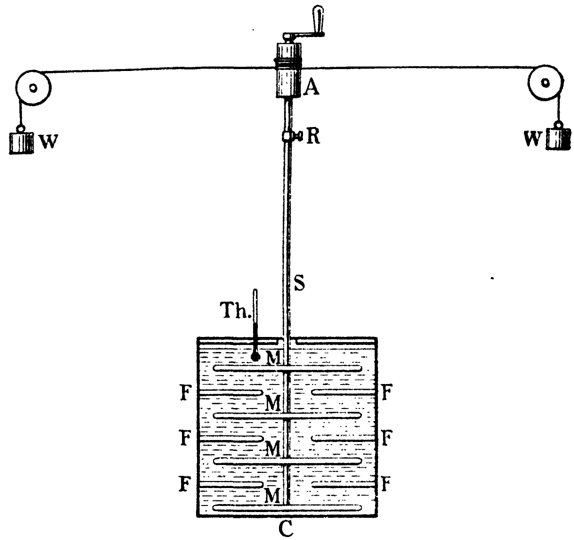


FIG. 55.—Joule's Rotating Paddle Method for the Mechanical Equivalent of Heat.

To get as exact a result as possible, Joule even made corrections for

(1) loss of heat by the calorimeter ;
(2) friction outside the calorimeter ;
(3) the loss due to the speed at which the weights WW hit the ground on falling, even though the speed was extremely low ;

and (4) the loss in the generation of sound in the humming in the calorimeter.

He found, from a series of experiments, that for each British thermal unit of heat generated 772 ft.-lbs. of work had to be done.

Between 1870–1878, Joule repeated his work, at the

request of the British Association, using a larger apparatus and both water and mercury, and his results give a mean value of 773·4 ft.-lbs. of work done for each British thermal unit of heat generated. Thus Joule showed that

$$\frac{\textbf{the work done (W)}}{\textbf{the heat generated (H)}} = \textbf{a constant quantity.}$$

This constant is called **the Mechanical Equivalent of Heat** or **Joule's Equivalent of Heat (J)**. Thus $W = J.H$ in the appropriate units.

Prof. Rowland, of Baltimore, U.S.A., published in 1879 the results of a repetition of Joule's experiments on a much larger scale. The work was done by an engine, and so more was done in a shorter time; this resulted in a smaller error of measurement. He also tested Joule's thermometers against a hydrogen thermometer and found they were inaccurate; the error in them accounted for the difference between Joule's results and these later ones, which give a mean of 778 ft.-lbs. of work per British thermal unit of heat generated. This is now the accepted value for the constant J.

In the C.G.S. system of units, the work W is measured in ergs, and the heat in calories. Thus the mechanical equivalent of heat J is expressed in ergs per calorie (grms. ° C.).

The value can be calculated as shown, since 1 ft. $= 12 \times 2{\cdot}54$ cms.; 1 lb. $= 453{\cdot}6$ grms.; 1 gr. weight $= 981$ dynes.

Thus J

$$= \frac{778 \text{ ft.-lbs.}}{1 \text{ lb. } ^\circ \text{F.}} = \frac{778 \times (12 \times 2{\cdot}54 \text{ cms.}) \times (453{\cdot}6 \text{ grms. wt.})}{(453{\cdot}6 \text{ grms.}) \times (\frac{5}{9}\ ^\circ \text{C.})}$$

$$= \frac{778 \times 12 \times 2{\cdot}54 \times 453{\cdot}6 \times 981 \text{ dyne-cms. or ergs}}{453{\cdot}6 \times \frac{5}{9} \text{ grms. } ^\circ \text{C. or calories}}$$

$$= 4{\cdot}187 \times 10^7 \text{ ergs per calorie, approx.}$$

In 1867 Joule also experimented to obtain the constant J by an electrical method, passing electricity through a coil of very thin insulated wire (of high resistance), immersed in water in a calorimeter. Work was done in overcoming the electrical resistance and heat was generated, the water and calorimeter

being raised in temperature. The heat generated was, of course, measured by the total water equivalent of the calorimeter and contents $\times$ the rise in temperature.

Joule showed that the work done in sending electricity along a wire was measured by $C^2Rt \times 10^7$ ergs,

where C = the current in amperes,
R = the resistance of the coil in ohms,
and t = the time in seconds for which the current is passing (see any electrical text-book).

Thus J is calculated.

The Kinetic Theory.—It was realised that Joule's experiments showed that whenever work was done, *e.g.* in overcoming friction, the energy necessary to do that work was set free in another form—usually as heat. Further, the same amount of heat was always generated when the same amount of work was done, which led to the idea that in transference of energy from one form to another there was no loss or gain. Robert Mayer in 1843 was the first to grasp this idea—which later was expressed as the "Conservation of Energy"—energy can neither be created nor destroyed; it can be changed from one form to another, but there is no change in the amount of energy involved in the transformations. At the same time came the conception, previously stated, that when a body is heated, the energy received is in the form of kinetic energy but is released again as heat energy in cooling down.

A story is told of Mayer, that whilst discussing Joule's experiments, directly after their publication, with another German scientist, Jolly, the latter remarked that if the ideas Mayer had expressed regarding them were true, water could be warmed up by shaking—it seemed to him a ridiculous idea. Mayer did not reply, but a few days later rushed up to Jolly and excitedly said, "It is so! It is so!" and eventually was able to make his friend understand what he meant—that he had tested and found that water was heated by shaking. It is thus possible to measure the mechanical equivalent of heat by a simple, though tedious, experiment. Obtain a long cardboard tube, at least a foot long and 3 to 4 ins. diameter, with tight-fitting end covers. Place a few grams of lead shot, or mercury, in one cover and carefully measure its temperature. Now close the tube over it and invert

carefully, with one distinct action, so that the contents are carried to the top and then fall to the bottom. Continue this, making exactly 1,000 turns. Care should be taken that the shot or mercury does not bounce when it strikes the bottom of the tube after falling. Quickly open the tube and measure the temperature of the lead shot or mercury. It will be found to have been raised.

Suppose $t°$ C. = rise in temperature.

Consider the metal, mass m grms., at the top of the tube after inversion, but just before it falls to the bottom of the tube, which is of length l cms.

The metal, by virtue of its position, possesses potential energy which is transformed into kinetic (motion) energy in falling, and into heat on striking the bottom (if there is no bounce).

$$\text{Potential energy} = m \times l \text{ grms. weight} \times \text{cms.}$$
$$= m \times l \times 981 \text{ dyne-cms. or ergs.}$$

Thus work done in falling 1,000 times

$$= m \times l \times 981 \times 1{,}000 \text{ ergs.}$$

If s = the specific heat of the metal, heat received by the m grms. of metal in being raised in temperature by $t°$ C. = mst calories.

$$\text{Thus} \quad J = \frac{\text{work done in ergs}}{\text{heat generated in calories}}$$
$$= \frac{1{,}000 \times m \times l \times 981 \text{ ergs}}{mst \text{ calories}}$$
$$= \frac{9{\cdot}81 l \times 10^5}{st} \left\{ \begin{array}{l} \text{ergs per calorie } (i.e. \text{ mass need not} \\ \quad \text{be known).} \end{array} \right.$$

In this manner it is possible to show that the water which has fallen to the bottom of a waterfall is warmer than when at the top of the fall:

e.g. the large fall of Niagara is 150 ft. high.

Thus if m lbs. of water fall over, the potential energy of this at the top = 150 m ft.-lbs. This potential energy is changed into kinetic energy in falling and then into heat on impact with the ground (neglecting the kinetic energy of the rebounding water).

Suppose the m lbs. of water are raised in temp. $t°$ F.

Then total heat set free = $m \times t \times 1$ B.Th.Units.

But work done=J×heat generated

or $150m = 778 \times mt$

$\therefore t = \frac{150}{778} = \underline{0 \cdot 2^\circ \text{ F.}}$

Another similar problem is the following: How many metres must a piece of lead fall through for it to melt on striking the ground, its temperature being 15° C., its melting point 327° C., its specific heat 0·03, and its latent heat 5 calories per gram? (Neglect air resistance.)

Let h metres=$100h$ cms. be the required height.

Potential energy of m grms. weight of lead at that height $= m \times 100h \times 981$ dyne-cms. or ergs.

Heat required to raise m grms. of lead from 15° C. to the melting point (327° C.)$= m \times 312 \times 0 \cdot 03$ calories.

Heat required to melt the m grms. of lead$=5\, m$ calories.

But work done=J×heat generated.

$$\therefore m \times 100h \times 981 = 4 \cdot 187 \times 10^7[(m \times 312 \times 0 \cdot 03) + 5m]$$
$$= 4 \cdot 187 \times 10^7 \times m \times 14 \cdot 36$$

$$\therefore h = \frac{4 \cdot 187 \times 10^5 \times 14 \cdot 36}{981} \text{ metres}$$

$= \underline{6{,}130 \text{ metres, approx.}}$, or 3·8 miles approx.

Exercises on Chapter VII

1. Explain units in which heat and mechanical work may be measured. Describe a simple form of experiment to establish a relation between the two units. [*L.M.* 1928.]

2. Define *unit quantity of heat* in British and Metric units respectively and state the value of each in suitable mechanical units. "Heat is a form of energy.' Describe three examples within your own observation in illustration of this statement. [*L.G.S.* 1926.]

3. Explain the statement "One British Thermal Unit is equivalent to 778 foot-pounds," and describe *briefly* some method of verifying it. [*L.G.S.* 1925.]

4. A tube 6 ft. long containing a little mercury and closed at both ends is rapidly inverted 50 times. What is the maximum rise in temperature of the mercury that can be expected? (Sp. ht. of mercury=1/30. One British Thermal Unit is equivalent to 778 ft.-lbs.) [*L.G.S.* 1923.]

5. What is the meaning of the statement that the mechanical equivalent of heat is $4 \cdot 2 \times 10^7$ ergs per calorie? Give two practical examples of the disappearance of kinetic energy owing to friction. If water falls through a height of 10 metres, what will be the rise of temperature if all the energy remains as heat in the water? (The acceleration due to gravity is 981 cms. per sec. per sec.) [*J.M.B.* 1928.]

6. If a lump of lead fell from a height of 100 ft. on a stone pavement,

by how much would its temperature be raised, supposing that all the heat caused by the blow were retained by the metal? A British Thermal Unit is equivalent to 780 ft.-lbs., and the specific heat of lead is 0·03.

[*J.M.B.* 1926.]

7. What is meant by the "mechanical equivalent of heat"? Describe a simple method of measuring it. A mechanical stirrer, working under water, uses 1/33 H.P. If the temperature rises 0·2° C. per minute, find the water equivalent of the vessel, the weight of water being 3 lbs.

[*C.W.B.* 1927.]

8. What is meant by the mechanical equivalent of heat? A steam engine expends 98 horse-power churning up a stream of water flowing through the vessel per minute, and the difference in temperature between the entering and outflowing water is 75° C.; what is the mechanical equivalent of heat? (One horse-power=550 ft.-lbs. per sec.; a gallon of water weighs 10 lbs.) [*L.M.* 1927.]

9. Indicate a method by which the mechanical equivalent of heat may be determined. At a pumping station it was found that 1,000 gallons of water were pumped to a height of 100 ft. for every 1·2 lbs. of fuel used. Express, as a decimal, the ratio between the work done and the energy of the fuel. (One gallon of water=10 lbs., 1 lb. of fuel=15,000 British Thermal Units. Mechanical equivalent=780 ft.-lbs. per B.Th.U.)

[*J.M.B.* 1927.]

10. An engine uses fuel producing 30 British Thermal Units per sec. What horse-power should it develop if the mechanical equivalent of heat is 772 ft.-lbs. per lb. degree F.? If the actual H.P. produced is 10, find the efficiency. [*C.W.B.* 1926.]

11. A sleigh is pulled forward on a level road at 20 ft. per sec. over some melting snow by a constant force of 30 lbs. wt. Assuming that all the resistance is caused by friction with the snow, how much of this will be melted in one minute by the passing of the sleigh? The mechanical equivalent of heat is 780 ft.-lbs. per British Thermal Unit, and the latent heat of fusion of ice is 144 B.Th.U. per lb. [*J.M.B.* 1926.]

12. State briefly the reasons for the acceptance of the theory that heat is a form of energy in preference to the older theory of a heat fluid (caloric). On test a steam plant was found to use 2 lbs. of coal every minute and to generate 1 H.P. in the engines. The calorific power of the coal was 2,000 British Thermal Units per lb. If 1 H.P.=33,000 ft.-lbs. per minute and J=770 ft.-lbs. per B.Th.U., what proportion of the heat obtained from the coal was turned into work? What happens to the rest of the heat?

CHAPTER VIII

FURTHER CONSIDERATION OF CHANGE OF STATE AND VAPOURS

WE have already seen that substances change state, under normal conditions, at fixed temperatures known as the boiling point of the liquid and the melting point of the solid (or freezing point of the liquid). In calorimetric work it is often necessary to know these so-called *physical constants* for a substance, and so we shall study how they are determined.

Lower Change of State.—The melting point (or, regarding the change conversely, the freezing point) of a substance is determined by the following methods.

(1) The "*cooling curve*" *method* is used when an appreciable quantity is available, and especially if the change of state is a gradual one (*e.g.* like the melting of toffee). Some of the solid substance is melted, and the temperature of the liquid, well stirred, is read by a thermometer every minute, until the substance is solidified. The results are plotted graphically as shown (Fig. 56). At the melting point (or freezing point) it is obvious that the temperature remains steady whilst the change of state is occurring. This shows itself by a

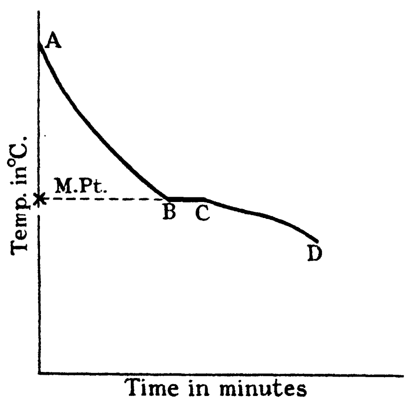


FIG. 56.—Cooling Curve Determination of Melting Point.

straight line in the graph (*e.g.* BC); it is very distinct in some cases, but with waxes (usually mixtures) the change is not so clear.

(2) *Capillary Tube Method.*—This is used when only a small amount of the substance is available. A piece of glass delivery tubing is heated in a flame till it is soft and is then quickly drawn out. About 1 in. of this is taken and sealed off at one end. Into it is introduced some of the substance, ground to a powder, the melting point of which is to be determined. Sometimes it is more convenient to run in the liquid form and allow it to solidify. The tube containing the substance is then attached (by a small rubber band) to the bulb of a mercury thermometer which is then supported in water and carefully heated. The opaque appearance of the solid changes to clearness on melting. The temperature at which this occurs is the point to be noted. The heating is stopped and the liquid allowed to cool, the exact temperature of the return of the opacity being observed. The mean of these two temperatures gives the melting point of the substance. Usually two experiments are necessary, the first to give a rough observation obtained quickly, the second a careful heating from just below the approximate temperature so as to get a very exact value.

(3) A third method, not often employed, is only possible with substances whose change of state is not gradual but abrupt, as in the case of ice formation. It depends on the process known as "super-cooling"—the liquid form, kept cool, can be cooled below its freezing point without solidification occurring. On stirring with the thermometer standing in it, solidification then takes place in bulk and the mercury in the thermometer suddenly rises, indicating the freezing point (or melting point) of the substance. The phenomenon is known as *surfusion*, and ordinary photographic "hypo" (sodium thio, or hypo-sulphate, $Na_2S_2O_3$) shows it very well. It also often exists in the capillary tubes of plants, the water remaining as water despite intense cold.

Freezing Mixtures.—We have already seen that some substances dissolve in other substances with a

lowering of the temperature, e.g. "hypo" dissolved in water, also sodium sulphate, ammonium nitrate, etc. These water solutions are sometimes used to cool down (or even "freeze") other substances, and hence the term "freezing mixture." A more common example, but a more complicated one, is seen in the use of a mixture of ice and salt (usually 2 parts of ice to 1 of salt) in ice-cream freezers. But the use of such a mixture depends on the fact that **the freezing point** (or melting point) **of a pure substance is lowered by dissolving another substance in it.** Pure water freezes at 0° C., but a brine solution may be cooled to −10° C. and not freeze (it depends on the concentration of the brine solution). Thus, when ice and salt are mixed together, some salt dissolves in the ice (with a cooling effect) and the ice melts, since the freezing point of a salt solution is below 0° C. The latent heat of fusion necessary for the ice to melt is absorbed from the mixture, which is thus considerably cooled. It is possible to reach a temperature of −20° C. by this means.

The addition of some calcium chloride to the ice and salt will give an even lower temperature owing to the high solubility of calcium chloride; as low as −50° C. can thus be obtained.

When snow is lying on the ground salt is used by many people to disperse it, the action being the same as with ice and salt. This practice may be effective at the time, but it is apt to be dangerous if cold and frost continue. The dilute snow-salt solution produced runs over the paths, etc., and, if the cold becomes intense enough, freezes in a fine layer of ice which is very slippery, much more so than a semi-frozen layer of snow.

Alloys similarly have melting points lower than either of the constituents. This is why a small amount of substance (called a flux) is added to a substance with a very high melting point to cause it to fuse more easily. In the modern development of the production of metals by electrolysis, the impure ore is rendered fusible on heating (not to a high temperature) by the addition of a small quantity of another compound of the same metal.

The wide use of aluminium, now produced cheaply by electrolysis, is due to the discovery that the use of a little "cryolite" (an aluminium compound) would make the aluminium ore, bauxite, fusible and so suitable for passing electricity, which resulted in the liberation of metallic aluminium. Another example is Wood's metal used for holding wireless crystals. This alloy (parts 1 of tin, 1 of cadmium, 2 of lead, 4 of bismuth) melts at 60·5° C. and is used in automatic fire extinguishers in buildings, so that if a fire spreads, the heat fuses a Wood's metal plug in a water cistern, or in a main water pipe, with a consequent fall or rush of water. It can also be used in boilers for steam engines in case the water-level becomes too low, so resulting in great damage to the copper tubes conducting the steam.

The use of freezing mixtures shows that to cause a liquid to freeze, heat has to be taken from it—heat which was received by the liquid on its previous change from solid form. This explains why just after a fall of snow the air seems warmer. The water in the atmosphere, when chilled, forms snow with the release of its latent heat to the cold air, and so the latter feels warm temporarily.

Effect of Pressure on the Melting Point.—The idea that, since ice or snow contracted on melting, the effect of squeezing or compressing would cause melting was first suggested by Prof. James Thomson. Later, his brother, Lord Kelvin, confirmed it experimentally. Perhaps Prof. Thomson remembered that when a boy wants to harden a snowball he squeezes it. This causes some of the snow to melt to water, which, on the release of the pressure, freezes to ice, and so the snowball is a more effective missile. Lord Kelvin applied pressure to ice, in a cooled glass vessel, by means of a screw tapped into a hole in the cover of the vessel. In it, in a glass case, was a thermometer thus protected from the high pressure. On compression, the ice melted, and it would not freeze, under the pressure, unless cooled below 0° C. The effect is very small, however, and so snow, if cooled appreciably below 0° C. in the air, cannot be squeezed sufficiently by hand to cause any melting, and so snow-

balling is only possible when the snow is very nearly at 0° C.

Tyndall illustrated the phenomenon very effectively. Over a block of ice he placed a thin metallic wire with heavy weights on either end (Fig. 57). The pressure of the wire caused the ice under it to melt and the wire slipped under the water produced, which then froze again as it was at 0° C. and not compressed. In this way the wire cut right through the block of ice, which was left whole. This is known as *regelation* (L. *re*, again; *gelare*, freeze). Other examples of this are seen in

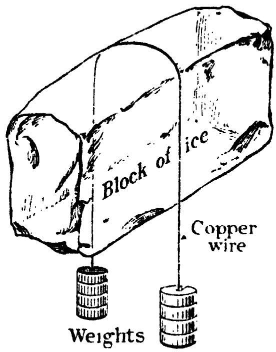


FIG. 57.—Tyndall's Regelation Experiment.

(1) the icy snow of wheel ruts in snow;

(2) the nature and movement of glaciers;

(3) the need for a thin edge on skates, so that the weight of the skater is concentrated on a small area. There is thus an instantaneous melting under the edge and the skater sinks slightly into the ice, receiving a sideways support from the ice around. Freezing, of course, happens as soon as the skater has passed on.

From the special case of ice Prof. Thomson expressed the general case:

Substances which contract on melting (*e.g.* ice) have M.Pt. lowered by pressure increase.
Substances which expand on melting (*e.g.* wax) have M.Pt. raised by pressure increase.

Thus, whereas pressure causes ice to melt, it helps the solidification of wax or tallow.

The Higher Change of State.—Liquids may be changed into vapours by

(*a*) *boiling*—which occurs in the mass of the liquid and, under normal conditions and when the liquid is

pure, at a constant temperature called the boiling point of the liquid;

(*b*) *evaporation*—a slow process which can go on at all temperatures, though not usually at the same rate. Thus pools of water in gutters disappear at ordinary air temperatures.

We have seen that the Kinetic Theory regards all the molecules in substances as moving. In a liquid such as water this movement is very rapid, and, owing to the frequent collisions, at any instant some molecules are moving faster than others. Each molecule is considered to have a "sphere of influence" within which it exerts a gravitational force of attraction on other molecules. Thus, within a liquid, molecules are, on the average, attracted equally in all directions by the surrounding molecules, and so move comparatively freely by virtue of their own velocities. The molecules at or near the surface are only pulled appreciably inwards and so do not easily move out of the liquid. This pull on the surface molecules is called "surface tension," and is often likened to a skin over the surface, which is difficult to penetrate; you can, for instance, put a small piece of wire gauze on water and it will float. A dry sewing needle placed on tissue paper and floated on water can be left floating if the tissue paper be gradually wetted so that it sinks. We thus consider that evaporation is not easy—that only the faster moving molecules manage to shoot out of the surface of the liquid, and so the action is gradual. Further, if the faster moving molecules go out of the liquid, the slower moving ones must remain, *i.e.* the kinetic energy is less and so the temperature of the liquid is lowered by the evaporation, or *evaporation tends to produce cooling*. This will be discussed a little later.

Vapours and Vapour Pressure.—From the Kinetic Theory it is quite simple to understand why air exerts a pressure, and therefore why any vapour or gas should do so. The molecules of any vapour or gas are considered to be in continuous movement, and so frequently collide with the containing envelope or vessel. These collisions, if very frequent, would be like a continuous push, or

pressure. We can show that any vapour exerts a pressure by using a Torricellian tube (see p. 14) set up with dry mercury. (Fig. 58.) Take a bent tube T having a bent portion with a narrow end A and suck some water into it, holding it in by placing a finger over the top B. Place A under the mercury in the trough, so that it is in the entrance to the Torricellian tube D. Release the finger and then gently blow in a little water, from A, so that it rises through the mercury in the Torricellian tube to the vacuum at the top. The water will be seen to vaporise and push down the level L of the mercury. Continue the process carefully until the mercury is no longer depressed (it being at X) and notice that any more water fails to vaporise and remains on the top of the mercury. The space above the liquid is now *saturated* with water vapour.

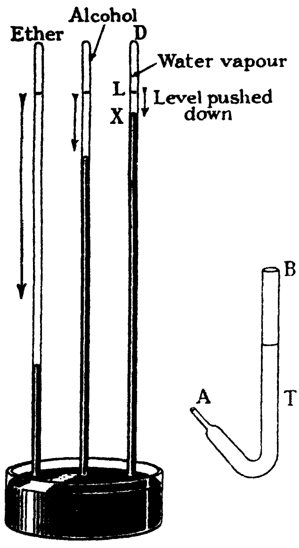


FIG. 58.—Vapours exert Pressures.

The lowering of the mercury level from L to X indicates that the water vapour put in the Torricellian vacuum pushes against atmospheric pressure (which is supporting the mercury in the tube), and it thus exerts a pressure. When the space is saturated with water vapour, the pressure the latter exerts (and equal to LX cms. of mercury) is called the *saturation vapour pressure of water or aqueous vapour.*

If the tube be lowered or tilted, the perpendicular height of the mercury level above that in the trough remains the same, *i.e.* the vapour pressure remains the same. But at the same time the Torricellian space above the mercury becomes smaller, and so some of the molecules of vapour must condense. Conversely, on raising the tube and thus lengthening the Torricellian space, more

liquid must evaporate, the vapour pressure remaining the same.

Now repeat the experiment with other Torricellian tubes, introducing ether, alcohol and petrol. In each case the same effect will be observed as with water, but the final results will vary considerably (as shown in Fig. 58) and you will readily conclude that, *at the same temperature, different vapours exert different saturation* (or maximum) *vapour pressures*. You must carefully notice that it only holds for the same temperature, for if you hold your hands over the Torricellian tube around the space D to X (or warm up with a small, cool, bunsen flame), a little more liquid will be seen to vaporise and the level X will be lowered, thus suggesting that the saturation (or maximum) vapour pressure varies with the temperature—at a higher temperature more vapour can exist in the space and thus a higher vapour pressure is exerted.

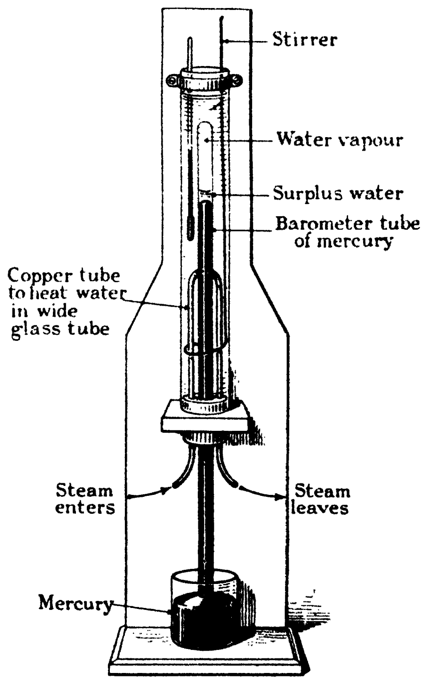


FIG. 59.—Apparatus for Pressure of Water Vapour at Different Temperatures.

That the *pressure of saturated vapour does increase with temperature* can be shown by using two barometers—one as a standard and the other with excess water above the mercury in the Torricellian vacuum—with a water-jacket around the Torricellian vacua. A very suitable form is shown in Fig. 59, the water being heated by steam passed through

it by means of a copper tube. As the temperature rises, the level of the mercury is more and more pushed down, thus showing that the saturation vapour pressure increases with temperature. There is no law relating them, but the curve of vapour pressure plotted against temperature is a smooth one (Fig. 60).

If this apparatus be made so that the water-jacket reaches right down to the trough of mercury, and the barometer tube be surrounded by steam only, it will be found that the level of the mercury in the tube is pushed down to the level of that in the trough, *i.e.* at the temperature of boiling water the saturation vapour pressure of water is equal to the pressure of the atmosphere. This is indicated on the graph in Fig. 60. In the same way, if ether is in the Torricellian vacuum space and the tube surrounded by a water-jacket at 35° C., which is the boiling point of ether, the level of the mercury in the tube descends to that of the mercury in the trough. By such examples it can be shown that **the boiling point of a liquid is the temperature at which its vapour pressure is equal to the atmospheric pressure.**

Another well-known experiment to illustrate this for water is illustrated by Fig. 61. A piece of delivery or dilatometer tubing is sealed at one end and bent so that it can be put in a large round-bottomed flask. Into this tube AB, mercury is put so that air is excluded from the shorter limb B. A little distilled water, freshly boiled to drive off any dissolved air, is introduced into the tube and run to the end B. Some mercury is taken from limb A by a narrow pipette till the level there is below that in limb B. The tube, when in the flask, is heated by steam coming from the water heated in the flask, and the mercury levels are seen to become the same, showing that the pressure exerted by the water vaporised in B is equal to the atmospheric pressure outside the tube. A similar experiment can be carried out with alcohol (heating in a water-bath to 78° C., instead of using steam), or some other enclosed liquid of higher boiling point heated in an oil-bath which is well stirred.

Influence of Pressure on the Boiling Point of a Liquid.—When a liquid vaporises under ordinary

conditions there is a great increase in its volume. Thus, if a pressure were exerted to oppose this volume increase, the effect would surely be to make vaporisation difficult, so making it necessary for a higher temperature to be reached if vaporisation is to occur.

Or, regarding this question from the work just done, if the atmospheric pressure on a liquid becomes greater, then a higher saturation vapour pressure must be reached for the liquid to boil, *i.e.* the temperature of the liquid must be raised. Thus it follows that the boiling point

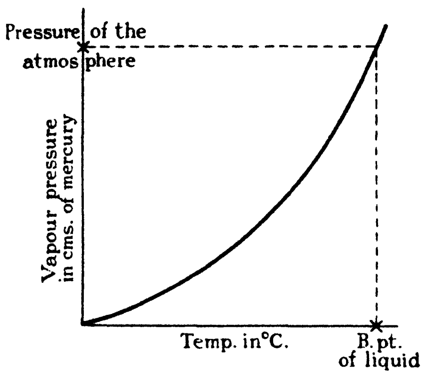


FIG. 60.—Graph showing Variation in Vapour Pressure with Temperature

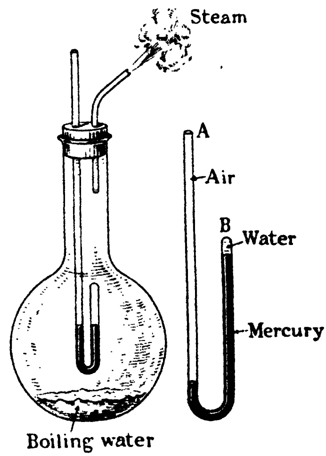


FIG. 61.

of a liquid becomes higher when the atmospheric pressure is increased, and if the pressure is decreased boiling will go on at a lower temperature. You have probably noticed when working in a laboratory that on some days water boils at 99·5° C. and on others at, possibly, 100·5° C. These variations are due to atmospheric pressure changes. Thus, in testing or standardising the higher fixed point of a thermometer, a correction must be made, if necessary, for the atmospheric pressure—the boiling point of water being 100° C. at an atmospheric pressure of 76 cms. of mercury.

Two simple methods of showing this phenomenon should be noted.

(1) Place some hot water, not quite boiling, under the glass receiver of an air pump and reduce the pressure. Bubbles of water vapour are seen to leave the liquid, and the inside of the receiver is seen to become covered with a mist.

(2) Into the mouth of a round-bottomed flask fit a rubber bung. Through this insert a piece of glass tubing, to which a piece of rubber pressure tubing is attached, making it fit tightly. Half fill the flask with water and boil vigorously to cause steam to drive off the air in the flask. Close the latter by screwing up a clip on the rubber tubing. After a little time invert the flask and run cold water on to it, when an appearance of boiling will be seen. Owing to the cooling of the glass and the water vapour (which condenses) in the space inside the flask above the water, the pressure on the water is greatly reduced, and so it commences to boil at that lower pressure, despite the fact that its temperature is below 100° C. The space again becomes filled with water vapour, which exerts a pressure, and boiling eventually ceases. Further cooling (by holding under the tap) results in boiling. This process can be continued for a long time—in fact, you can continue it till boiling is not so vigorous, and you will find, on opening the flask, that the temperature of the water (which was boiling, a moment before, under reduced pressure) is about 30° C.

Application.—(1) The *hypsometer* (Fig. 17) is a vessel in which pure water is boiled and the temperature of the steam observed. It can be used for estimating the height of a place, above sea-level, since the higher it is, the lower is its atmospheric pressure, and hence the lower is the temperature at which water will boil there. Tables are supplied giving the heights above sea-level and the corresponding boiling points of water.

(2) **Papin's Digester.**—At a great height above sea-level, the boiling point of water is so lowered that vegetables, eggs, etc., are not easily cooked by just boiling in the water. A method of increasing the pressure inside the cooker must be used so that the temperature of boiling is higher. Papin's Digester was devised to boil things at higher pressures (and hence at higher

temperatures), and it has now many uses in the industrial arts—it is used, for instance, for extracting gelatine from bones by heating them with water, under great pressure, so that a temperature of about 600° C. is obtained. Another commercial use is for boiling raw wood, under pressure, with caustic soda solution. Almost pure cellulose is left as a pulp, ready for paper making, artificial silk manufacture, etc., etc. In the form of the apparatus shown in Fig. 62, a lever pivoted at F carries a piston A which fits exactly into the gap B at the top of a large boiler. As water is boiled in this boiler, the vapour goes into the space above and exerts a pressure on A. Eventually, sufficient vapour comes off to exert a pressure greater than the downward pressure on A due to the lever effect of W (Pressure on $A \times FA = $ Weight $W \times FD$), and so A is lifted and steam leaves B and the pressure inside is reduced, with the result that A fits into B again, etc., etc. The farther W is away from F, the greater is the pressure on A, and hence the higher the temperature inside before A becomes forced out of the gap. Thus the position of W controls the temperature to which the water must be heated for boiling to commence.

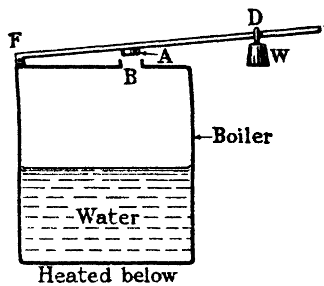


FIG. 62.—Papin's Digester.

(3) **Distillations under Reduced Pressure.**—In the preparation of drugs, etc., purification often entails a process called *distillation*. When a brine solution is boiled in a flask, only steam is driven off. If this is condensed, pure water, called distilled water, is collected, whilst the salt remains in the flask. The apparatus for this process is called a "still." The laboratory form consists of a special distilling flask A with a side-tube B, which is joined to a *Liebig's condenser*—a long tube C, through which the vapour passes, surrounded by a water-jacket D. The vapour is cooled, condenses and runs out drop by drop (L. *di*, one by one; *stillare*, drop; hence

"distillation") at E into a collecting vessel. Such a method is used for obtaining pure water and for the preparation of drugs and food-stuffs. On ships, too, where steam turbines give the motive power, the steam emerging from the turbines is condensed and used again —to avoid the continuous boiling of salt water and the choking of the boilers with salt, and of the pipes with salt particles which are carried over by the steam if the boiling is too vigorous. Sea water, of course, is passed through the water-jackets of ships' condensers.

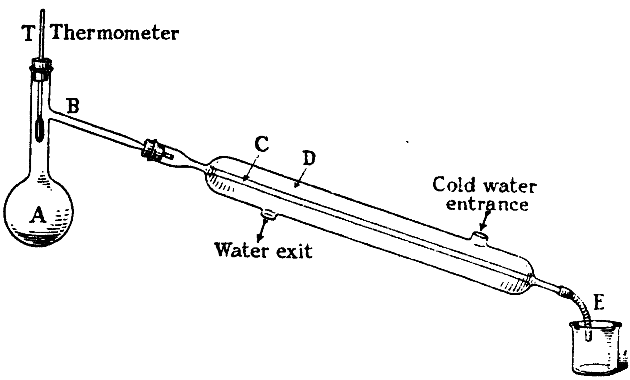


FIG. 63.—Laboratory Distillation Apparatus.

An extension of the distillation process is *fractional distillation*. If a water-alcohol mixture be heated in the apparatus just described, boiling will be seen to go on when the temperature, recorded by thermometer T, is 78° C., which is the boiling point of alcohol. The temperature remains constant for some time and alcohol is collected. After a time boiling almost ceases and the mercury in thermometer T is seen to rise and eventually reaches 100° C., when steady boiling goes on, only water being condensed and collected. A mixture can thus be separated into its parts by this method—hence the name, fractional distillation.

A further use depends on the lowering of the boiling point of a liquid with a lowering of the atmospheric pressure above it. Many substances, particularly drugs, chemically react with air when heated in it and so cannot be purified by

simple distillation in air. They can, however, be distilled *in vacuo*, *i.e.* in the absence of air, and thus under reduced pressure, whence it follows that the temperature of the liquid when boiling is much lower than normally, and so the possibility of chemical decomposition is still less. Such a method can be carried out in laboratories by modifying the apparatus of Fig. 63, using airtight rubber bungs and having another outlet from the distillation flask. This is joined to a filter pump or good suction pump for removing all or most of the air from the closed apparatus. Such a method is used on a commercial scale for the distillation of natural mineral oils to obtain from them the various grades of petrol, petroleum and heavier lubricating oils. (All these vary in density and boiling point, best petrol having the lowest density and lowest boiling point.) It is also used for the distillation of coal *in vacuo* in coal-gas manufacture, the distillation of the coal-tar so produced and from which essential substances for the manufacture of dyes, drugs and explosives are obtained, and in spirit distilleries.

In relation to this, it must be noticed that the boiling point of a liquid is raised by the addition to it of a solid which will dissolve in the liquid, the variation depending on the amount of solid dissolved. This can readily be shown by observing the successive temperatures of boiling of 100 grms. of water with 1, 2, 3, 4, . . . grms. of common salt dissolved in it, the thermometer being in the solution. Notice that if the thermometer is out of the solution it always records the same temperature—that of steam. We thus see one way of testing whether a liquid is pure—if it is, the temperature of the boiling liquid and of the escaping vapour are the same. If it is not pure the temperature of the liquid is higher than that of the vapour.

The "distillation *in vacuo*" apparatus described above can also be used to measure the saturation vapour pressure of a liquid at any temperature, if an accurate manometer is incorporated to indicate the pressure inside when boiling takes place. A common arrangement is shown in Fig. 64, B being a large copper sphere, or a large glass flask in laboratory use, to act as the atmosphere, the pressure of the air in which can be varied by the

air pump, and each time the boiling point of the pure liquid in A and the pressure are observed. Now, we saw on p. 128 that at the boiling point the vapour pressure of a liquid is equal to the atmospheric pressure. Thus in this experiment the manometer reading gives the saturation vapour pressure of the liquid at the temperature at which it was boiling.

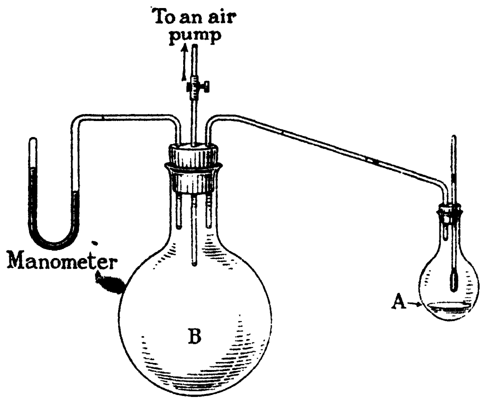


FIG. 64.

Refrigeration.—On p. 125 we saw that evaporation tended to produce cooling, and we should therefore expect that if evaporation was rapid the cooling effect would be appreciable. Rapid evaporation is caused to illustrate this cooling effect in the following experiments :

(*a*) Place a beaker, containing ether, on a wetted piece of wood and through the liquid blow a steady current of air by means of foot bellows. In two or three minutes the beaker will be found to be held to the wood by a thin layer of ice.

(*b*) *Leslie's Experiment.*—Place a little water in a shallow dish, and strong sulphuric acid in another dish, both under the receiver of a good air-pump. Reduce the pressure so that water continuously evaporates, the water vapour produced being just as rapidly absorbed by the acid. The temperature of the water falls (the thermometer standing in it) and, if the pump is a very good one, the water can ultimately be made to

freeze. Carré made an ice machine on the principle of this experiment, but he found later that the evaporation of ammonia was more suitable, and so is often used to-day (see later).

(*c*) *Wollaston's Cryophorus.*—This is a glass vessel containing some water, but the air has been evacuated before sealing. The water is shaken to the end B and then the end A is surrounded by a freezing mixture (as shown in Fig. 65). Water vapour present in the space at A (due to evaporation under reduced pressure) is condensed to water vapour where cooled, and so more water at B evaporates to replace this condensed vapour. These condensation and evaporation processes continue, and the water left in B cools, and eventually a layer of ice is seen to form on the top of it.

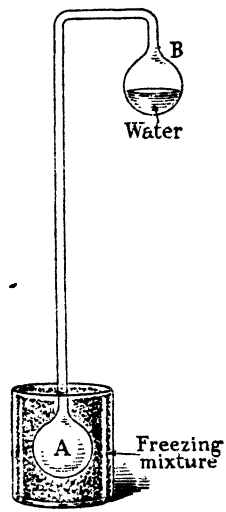


FIG. 65.—Wollaston's Cryophorus.

The cooling effect of evaporation is used to keep vessels cool by wrapping damp cloths around them and leaving them in draughts. In India, water is often kept in porous pots so that there is a slow leakage to the outside resulting in evaporation there, with cooling, and so the water left inside keeps cool. In Australia, porous canvas bags are used, these often being hung in the wind in the shade of trees. In summer our streets are cooled by being sprinkled with water. The evaporation of perspiration from human beings helps to keep them cool—in fact, it is essential to good health in hot weather. This cooling effect makes it unwise for one to sit in a draught when wearing damp clothes. A dog when hot puts out its tongue; it is the only part of its body from which evaporation is possible.

There is another important method of producing cooling—that caused by the expansion of a gas. Gay-Lussac in 1807, and Joule in 1845, carried out experiments to see if a gas cooled when it expanded freely. It seemed reasonable to assume that the small particles of a gas obeyed Newton's Law of Universal Gravitation and

attracted one another. Work would therefore be necessary to overcome this force of attraction if a gas were to expand. If the gas expanded freely—*i.e.* if the confines of the gas were suddenly enlarged or moved out, then in expanding to fill the space, as it naturally would, it must itself supply the energy to do the work of overcoming the attraction. The gas should lose some of its kinetic energy and cool. Joule had two large globes, one containing compressed air and the other evacuated, joined by a tube with a stop-cock, the whole being immersed in water, the temperature of which was observed by thermometers (reading to 1/200° F.). The stop-cock was opened so that gas from one globe expanded into the empty globe, but no cooling of the water was observed. However, *in* 1852, *Joule, working with William Thomson (afterwards Lord Kelvin), carried out the famous Porous Plug Experiment, and showed that nearly all gases cool on free expansion.* They measured the temperature of the gas before and after expansion, as they realised that the fault in Joule's early apparatus was that the water equivalent was so large that no appreciable rise in temperature of the water was possible. The gas was compressed and passed through spirals, in an oil bath, to get it at a measured constant temperature. An engine then forced the gas into a porous plug of silk or wool fibre held in wood. The gas then came out of the large number of orifices in the fibre and expanded owing to the great fall in pressure, and a slight cooling was observed on the delicate thermometers used. Conversely, in the compression of gases heat is generated—you will have noticed this when compressing air into a bicycle tyre.

A modern refrigerating plant incorporates both these methods of cooling. Ammonia (which liquefies under a pressure of only 10 atmospheres at ordinary temperatures) is used in pipes, from which the air is removed, immersed in a brine solution, which can be cooled to −10° C. or so without freezing. A compression pump A (Fig. 66) condenses the ammonia vapour to liquid (a pressure of 155 lbs. per sq. in. will do this at 22° C.), and forces it along pipes over which cold water is sprinkled to cool the ammonia.

The latter then passes out of a very fine hole of an expansion valve E so that there is (*a*) vaporisation, and (*b*) expansion into the pipes beyond where the pressure is low. There is a considerable cooling effect, and the process is carried on so that the brine becomes cooled well below 0° C. Thus rooms R, R, can be kept at the required temperatures

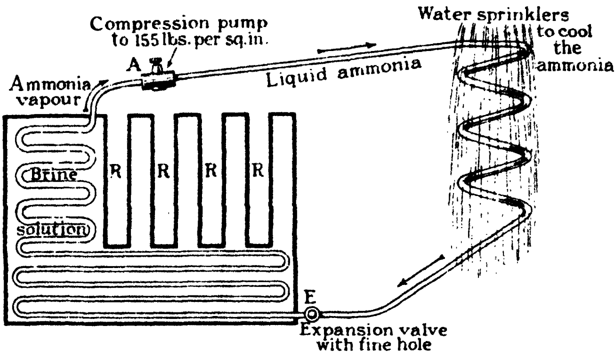


FIG. 66.—Refrigerating Plant.

for different foods in cold storage, or special cases of water can be placed in them, the water frozen, the sides removed and the blocks of ice taken out.

Such a method is used on ordinary ships for cold storage, and on naval ships for cold storage and to keep cordite, since the latter deteriorates if kept above 70° F.

EXERCISES ON CHAPTER VIII

1. Describe how to determine the melting point of a solid by plotting its cooling curve. Explain the theory underlying the experiment. [*L.M.* 1921.]

2. What do you understand by a *freezing mixture?* Explain the use of salt to clear a path after a fall of snow.

3. What do you understand by *regelation?* Describe an experiment to illustrate it.

4. When snow is squeezed it melts, and yet squeezing assists the solidification of ordinary wax or tallow. Explain this statement.

5. What is meant by a saturated vapour? Describe experiments to show on what the pressure of a saturated vapour depends. [*L.M.* 1928.]

6. Explain the full significance of the expression *maximum vapour*

pressure of a liquid at a given temperature. How would you measure this in the case of ether at the ordinary temperature of the laboratory? [*J.M.B.* 1926.]

7. How would you measure the vapour pressure of ether at room temperature? What is its vapour pressure at boiling point, and how would you verify your statement experimentally? [*C.W.B.* 1927.]

8. After setting up a mercury barometer a small quantity of ether or alcohol is introduced into the space above the mercury. Describe and explain what happens. Would the results be different if the experiment were performed at another temperature? What means could you take to test your answer? [*J.M.B.* 1923.]

9. Describe carefully the meaning of the terms *saturated* and *unsaturated* water vapour, and describe simple experiments to illustrate your answer. How would you determine the relation between the pressure of saturated water vapour and the temperature?

10. What is vapour pressure? A long barometer tube can be moved up and down in a deep vessel nearly filled with mercury. The reading is 75 cms. A few drops of water are introduced into the barometer tube, and the height of the mercury column falls to 73·5 cms., a film of moisture being visible on the top of the column. Explain the fall, and state, giving reasons, whether the height of the barometer column or the amount of moisture visible would alter if the tube were gradually (1) raised, (2) lowered, in the vessel.

11. Describe an experiment to measure the maximum pressure exerted by the vapour of petrol at ordinary temperatures. When a liquid evaporates, its vapour exerts the same pressure whether the evaporation occurs in a vacuum or in a space which contains air. How could you modify your experiment to show this? [*J.M.B.* 1927.]

12. How does the temperature at which water boils depend upon the pressure? Describe how you would verify your answer experimentally. [*L.G.S.* 1924.]

13. How does the temperature at which water boils depend upon the pressure? Describe experiments which show how the boiling point changes (*a*) when the pressure is raised above atmospheric pressure; (*b*) when the pressure is reduced below atmospheric? [*J.M.B.* 1928.]

14. Describe experiments to illustrate the effect of change of pressure on the boiling point of water. Point out any practical consequences which follow from this effect. [*L.G.S.* 1920.]

15. Explain any method you have seen used for determining the vapour pressure of a liquid. [*J.M.B.* 1922.]

16. Distinguish between boiling and evaporation. Describe carefully how you would determine (*a*) the boiling point of a solution; (*b*) the boiling point of a solvent. [*L.G.S.* 1922.]

17. Define the *melting point* of a solid. Describe methods of determining the melting points suitable for (*a*) solids that have a well-defined melting point, (*b*) solids that pass through a plastic condition in changing from solid to liquid. [*L.M.* 1930.]

CHAPTER IX

THE WETNESS OF THE AIR—HYGROMETRY

WE know that in the air around us there is water in the form of vapour, which is being continually added to by evaporation from water surfaces; whilst water vapour is cooled, condensed and deposited as rain, mist, dew, etc. This cycle is maintained, and so the amount of moisture in the air varies. On p. 127 we saw how temperature affected the quantity necessary to saturate a space. The rate at which evaporation goes on is not uniform—puddles of water rapidly disappear in summer, whilst in winter they remain much longer—and the deposition of water as rain, etc., is very variable. When the air at any time holds as much water vapour as it can, for there *is* a limit to its capacity, it is said to be *saturated*, but air, as a rule, is not saturated, or very close to saturation point, in England. When it occurs in summer we say it is sultry and we feel languid; if in winter, it is cold and miserable. To indicate the variation in the wetness of the air, devices called *hygroscopes* (Gk. *hygros*, wet+*scopein*, to tell) are used. Their operation depends on the fact that many substances absorb water, according to the amount present in the air, and either swell out, lengthen or twist. A German Cardinal, Nicolaus de Cusa (1401–1464), was probably responsible for the earliest form known. He used a large piece of wool on a balance. The weight of the wool increased when the air was wet and decreased when the air was dry. In the sixteenth century the effect of moisture on gut-strings (twisted cat-gut swells and untwists as it absorbs moisture) was observed and used in mechanical devices. In one form the cowl over the head of a monk was made to

uncover the head in dry weather and cover it in wet weather. Much use has been made of the fact that the human hair absorbs moisture and lengthens. Common salt is, of course, hygroscopic, and seaweed, because of the salt in it, is hard and stiff in dry weather, but when the air is damp it absorbs moisture and becomes soft and limp. Many vegetable and animal fibres show changes according to the amount of moisture in them.

Relative Humidity of the Air.—It is usual to consider the wetness of the air at any time relative to the saturation wetness for the same temperature and pressure of the air. Multiplying this ratio by 100, we obtain the percentage wetness or, as we call it, the *relative humidity*.

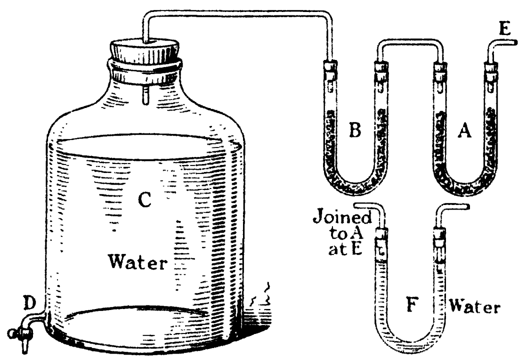


FIG. 67.—Chemical Hygrometer.

Thus the **relative humidity** at any time

$$= \frac{\textbf{Mass of water vapour per c.c. of air at that time}}{\textbf{Mass of water vapour to saturate 1 c.c. of air at that time}} \times \mathbf{100}.$$

A method of obtaining this value is to measure the quantities in the above fraction by means of a *chemical hygrometer* (Gk. *hygros*, wet+*metron*, a measure). The water is absorbed, and weighed, from equal volumes of ordinary air and saturated air. The best absorbent (or hygroscopic substance) to use is phosphorus pentoxide mixed up with broken glass to give a large surface to which the air is exposed and so to ensure that *all* the water vapour is absorbed and weighed. The U tubes

containing this mixture must be kept closed by solid glass rods, fitted in rubber connections, when not in use and when being weighed. In Fig. 67 A is the weighed tube which absorbs the water vapour from the air. B is a similar U-tube, its purpose being to prevent water vapour, from the space above the water in aspirator C, from reaching A and affecting the result. When water is run out at D, the solid tube in the rubber at E being removed, air enters the apparatus at E and its water vapour is absorbed in A. A measured volume of water is run out of D so that the volume of air which enters at E to replace it is known. The gain in weight of A is found. At E is now attached water-tube F, through which an equal volume of air is drawn into the apparatus. The air becomes saturated and its water vapour is absorbed by A, the increase in weight giving the mass of water absorbed from the saturated air. Hence the relative humidity of the air can be calculated. Whilst these slow operations are being performed, the temperature of the air may change, and so it is better to have two sets of apparatus and carry out the two parts of the experiment simultaneously.

Dew-Point.—The experiment described on p. 127 showed that the amount of water vapour existing in a space depended on the temperature. Thus air, under ordinary conditions, might at some time have a relative humidity of 60 per cent., so having 60 per cent. of the water vapour necessary to saturate it under the same conditions. If this air were cooled down, less water vapour would be required for saturation, and so, at the lower temperature, the relative humidity would be higher than 60 per cent. Eventually, on further cooling, the air would reach a temperature at which it was saturated (relative humidity 100 per cent.). The slightest cooling below that temperature would result in the air being over-saturated, with the result that some water vapour would condense. For instance, when a glass of cold water is taken into a warm room a so-called "mist" is seen to form on the outside of the glass. The air adjacent to the glass cools down and reaches the temperature at which the water vapour it contains is saturating it, and a mist or *dew* is deposited on the cold surface, *i.e.* the glass. **Thus we call this temperature, to**

which the air is cooled till the water vapour in it is saturating it, the dew-point.

Importance of Dew-Point.—This depends to a great extent on a very important experimental result. For masses of water of the order of those present in unit volume of the atmosphere, it is found to be approximately true that *the vapour pressure exerted by water vapour in the atmosphere is proportional to the mass of water vapour present.*

The relative humidity of the air at temperature $t°$ C. has been shown to be equal to

$$\frac{\text{Mass of aqueous vapour per c.c. at } t° \text{ C.}}{\text{Mass of aqueous vapour to saturate it at } t° \text{ C.}} \times 100.$$

But the dew-point is the temperature to which the air must be cooled for the water vapour in it to saturate it.

$\therefore$ Relative humidity

$$= \frac{\text{Mass of aqueous vapour per c.c. saturating the air at the dew-point}}{\text{Mass of aqueous vapour per c.c. saturating the air at air temperature}} \times 100.$$

But, as has been stated above, the vapour pressure is proportional to the mass of water present, and the vapour pressure is not appreciably affected by the cooling to the dew-point.

$\therefore$ Relative humidity

$$= \frac{\text{Vapour pressure of saturated air at dew-point}}{\text{Vapour pressure of saturated air at air temp.}} \times 100.$$

Thus, if the dew-point and the air temperature are measured and vapour pressure tables for aqueous vapour are available, the relative humidity of the air can be calculated.

The measurement of dew-point is thus important Being simple and fairly rapid, it gives a better way of finding the relative humidity than the use of the Chemical Hygrometer. Devices for measuring the dew-point are also called hygrometers. In them methods are used to cause a surface to cool rapidly; the air near it thus cools, attains its dew-point temperature, and then begins to deposit a dew on the cooled surface. The temperatures

of appearance and disappearance, on warming, of the film of condensed water vapour are observed and the mean value taken to obtain the dew-point.

The earliest form of dew-point hygrometer was due to Daniell and is shown in Fig. 68. The closed tube, with bulbs A and B, has the air removed from it, but contains some ether. This is shaken into A, which it rather more than half fills. In the rest of the space, of course, is ether vapour. On the outside of bulb B is some flannel, or other absorbent material, on which ether is poured to saturate it. The evaporation of this ether causes a cooling of B and so some ether vapour inside the bulb condenses. Ether from A evaporates to restore the vapour pressure in the space. This process continues, more ether being poured on the outside of B as required. The evaporation of ether inside A causes a cooling there, and so the surrounding air cools. Eventually the dew-point temperature is reached, and moisture begins to deposit on the polished metal band M on the outside of bulb A. The temperature at which this commences is read on the thermometer T_1 and no more ether is poured on B. The apparatus receives heat from the air and the mist on M disappears as its temperature begins to rise above the dew-point, the reading on T_1 again being noted. The mean of the two temperatures gives the dew-point, whilst T_2 indicates the air temperature.

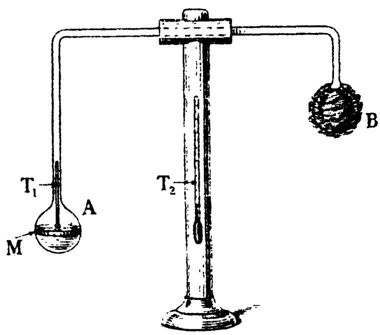


FIG. 68.—Daniell's Hygrometer.

This form of hygrometer is comparatively poor for the following reasons: (1) Ether evaporating outside B contaminates the air and so affects the results. (2) It is not easy to observe the exact moment of commencement of dew deposition and dew disappearance, for there

is no comparison standard and the dew is not easily seen. (3) Evaporation of the ether inside A goes on mostly at the surface of the liquid, which is thus cooled more rapidly than the interior, and so the actual dew-point is not measured.

Another instrument is *Dines' Hygrometer* (Fig. 69), in which water, cooled by ice, flows through a box and against the under side of a black glass window in its

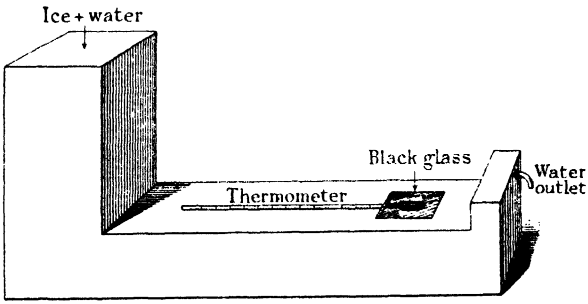


FIG. 69.—Dines' Hygrometer.

top. A thermometer, which can be read from outside, is fixed in the box with the bulb under the black glass. The gradual flow of cold water cools the glass, on which the resulting deposit of dew is much more easily seen than in the case of Daniell's hygrometer. Readings are made as with the latter instrument.

Regnault's Hygrometer (Fig. 70) is a most efficient apparatus consisting of two large test-tubes, A and B, with polished silver thimbles on the bottom. If necessary, before using the hygrometer, the thimbles can be cleaned by rubbing with fine moist sand. Some ether is poured into A and the tubes are joined together as shown. By means of an aspirator air is sucked into the ether, entering at D and leaving at E. Owing to the rapid evaporation, the ether in A cools down to the dew-point and a mist begins to deposit on the silver surface. The silver surface on B, being unaffected, serves as a standard of comparison of brightness, and

the slightest difference between the two indicates that a mist is forming. Immediately the reading of the thermometer T_1 is observed and the aspirator stopped working. The disappearance of the mist on A is carefully watched for and its temperature noted—the mean of the two observed temperatures giving the dew-point. Thermometer T_2 indicates the ordinary air temperature. The bubbling of the air through the ether in A mixes it up, thus obviating the cooling of the surface—a defect of the Daniell hygrometer.

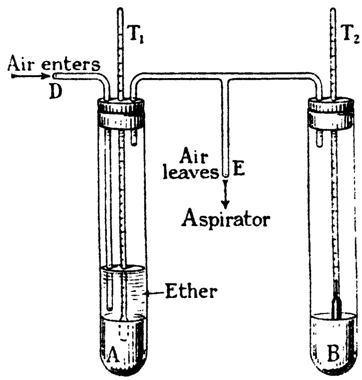


FIG. 70.—Regnault's Hygrometer

In using any hygrometer, observations ought to be made either (*a*) by telescope, or (*b*) with glass between the observer and the apparatus, so that heat from the body, or breath, does not affect the result.

The Wet and Dry Bulb Hygrometer consists of two exactly similar thermometers. The bulb of one is covered with muslin, which is kept continually moist by hanging a free end of it in a tin of water beneath. This thermometer, being cooled by the continuous evaporation of water on the muslin, registers a lower temperature than does the other thermometer which is quite dry. The less saturated is the air, and the warmer it is, the quicker is this evaporation and the more the

cooling, and the greater the difference between the readings of the two thermometers. This hygrometer should be placed in a current of air. It is usually kept in a Stevenson screen—a form of container for protecting meteorological instruments used in the open air. Special tables are used to find the dew-point from the air temperature and the difference between the wet and dry bulb temperatures. These were compiled by Glaisher, who collected many thousands of observations made with a wet and dry bulb hygrometer and a Daniell's hygrometer, at Greenwich Observatory, in India, and at Toronto, for a period of about five years. Glaisher's Tables are used by the Meteorological Office and its stations in this country.

Exercises on Chapter IX

1. What is meant by the *relative humidity* of the air? Describe a method for measuring the amount of water vapour in the air at any given time and explain how you would find the relative humidity of the air at that time.

2. What is meant by the *dew-point?* How may it be determined and what use is it when it has been found? [*J.M.B.* 1923.]

3. Explain the cause of the formation of mist on a cold piece of glass, when this is brought suddenly into a hot room. Describe some apparatus better adapted for the determination of this phenomenon and state what information can be obtained by its aid. [*L.M.* 1920.]

4. Explain what is meant by the term *dew-point.* Describe how the dew-point can be determined for the atmosphere in a room. Explain carefully what effect, if any, there will be on the dew-point (*a*) if a quantity of water is gradually sprinkled in the room, (*b*) if the temperature of the atmosphere in the room is raised. [*J.M.B.* 1929.]

5. Criticise the following statement: "The dew-point is the temperature at which the water vapour contained in the air is just sufficient to saturate it," and suggest another definition of dew-point. If the temperature is 25° C. and the dew-point 5° C., what is the relative humidity?

	5° C.	25° C.
Max. vapour pressure of water .	6·53	23·55 mm. of mercury.
" " density " .	6·9	22·9 grms. per cu. mm.

[*L.M.* 1925.]

6. How is it that a thermometer whose bulb is wrapped round with damp muslin usually indicates a lower temperature than another, by its side, whose bulb is not covered? Point out how the difference of the readings might be expected to vary with external conditions. [*L.M.* 1922.]

CHAPTER X

TRANSFERENCE OF HEAT—CONVECTION—CONDUCTION

IT has probably been realised in the study of heat that the phenomena met with depend on the transference of heat from one body, or point, to another body, or point. Heat is transferred in the following ways: conduction, convection, radiation and evaporation (from a liquid), and they can be summarised briefly as follows:

(1) **Conduction of Heat.**—A silver spoon left standing in a cup of hot tea feels very hot when it is handled to stir the tea. Heat is passing from a hotter to a colder body in contact with it. This method is called conduction and is explained by the Kinetic Theory, already emphasised. The molecules at the end of a piece of substance, in contact with a hotter body, gain energy and so vibrate faster. In so doing they are considered to jog adjoining molecules, and to cause them to vibrate more. This process is carried on along the substance, but there is no appreciable displacement of the particles. This theory supports the practical observation that the substance eventually reaches a steady state. Then the heat conducted along the bar is dissipated from the surface, by methods which will soon be studied, and no point in the substance gains in temperature. Further, as would be expected, the nearer a point in the substance is to the hot body the higher is its temperature.

(2) **Convection of Heat.**—It is common knowledge that in any room the air is much warmer near the ceiling than near the floor. Galleries in theatres and cinemas become very warm when the buildings are crowded with people. The movement of air up a chimney is well

realised and is also indicated by the movement of air into a fireplace. If, in an ordinary living-room with the door closed and a fire lighted, a candle is held near the small gap between the door and its frame, it is seen by the movement of the candle-flame that the inward draught is very appreciable near the floor, but hardly noticeable at the top of the door. The air over the fire becomes heated, expands, thus becoming less dense than the surrounding air, and rises, so causing a draught in towards the fire. The continuous current of heated air moving up is called a *convection current*. The warm air carries its heat to other parts of the room, and this method of heat transference, in which *the particles carrying the heat actually move*, is called *convection*.

(3) **Radiation of Heat.**—A hand held below an electric lamp feels warmer and so must receive heat from the lamp. This cannot be due to convection, for a convection current travels upwards. Conduction is not responsible, for air is a bad conductor of heat (this will be discussed later). The heat must therefore be transferred by another method—we call it radiation of heat. The most common example of this method is the way in which heat is transferred to the earth from the sun.

Heat thus transferred is called "radiant heat." It does not result in any appreciable heating of the earth's atmosphere, and so *heat is considered* to be *transmitted by radiation when it passes from one point to another without raising the temperature of the medium through which it travels*. Further, at a great distance from the earth there is no air, and so radiation can take place through what we term a vacuum.

(4) **Transference of Heat by Evaporation.**—Liquids can lose heat by evaporation, at any temperature, from their surfaces. The faster-moving, or warmer, particles move out of the liquid leaving the slower-moving, or colder, ones behind. The cooling of a vessel of hot liquid is partly due to this, heat being carried away to the surrounding air.

Convection of Heat.—This method of heat transference by which the material particles carrying the heat

move from one point to another can be illustrated by such experiments as the following :

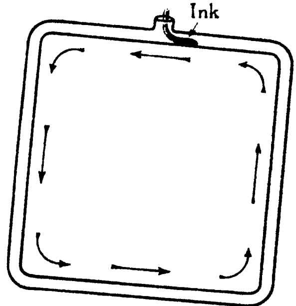


FIG. 71.—Convection in a Liquid.

A. In Liquids.—(1) Almost fill with water a special rectangular shaped piece of glass tubing (bore about ½ in. diameter), and slightly tilt it (see Fig. 71) so that a little red ink, poured in at the top, tends to run down the tube. By means of a small, white, bunsen flame warm the glass tube at the bottom corner towards which the ink is moving. The ink is seen to be driven round the tube in the opposite direction, owing to the convection current set up in the water.

(2) At the bottom of a glass cell, made narrow for use in an optical lantern, use a small cylindrical coil of insulated resistance wire joined outside to suitable cells so that a current can be passed through to heat it. Nearly fill the cell with water and project on to a screen by the lantern. Insert along the centre of the coil a small "lead" from blue copying-ink pencil. Lower into the water and switch on the current at once to warm the water at the bottom which has become coloured, and observe the upward movement of the particles.

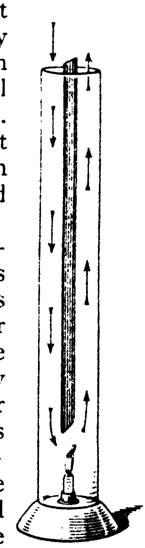


FIG. 72.

B. In Gases.—(1) Take a tall, narrow gas-jar and cut a long piece of cardboard, almost as wide as the internal diameter of the jar. By means of a long piece of stiff copper wire carefully lower a lighted candle to the bottom of the jar and leave it there (Fig. 72). The flame will be seen gradually to become less intense and finally to go out, for the oxygen at the lower part of the jar becomes used up, the heavier carbon dioxide formed remaining at the bottom. If, however, just before the flame is extinguished, the piece of cardboard is quickly lowered in the jar almost to the flame (dividing the jar into two compartments), it will be seen to revive and to continue burning vigorously. The heated air rises up one side of the cardboard

and air from outside descends on the other. The candle thus gets a continuous supply of air from the down draught produced.

(2) Take a box fitted with two glass chimneys and a glass side as shown in Fig 73, and place a lighted candle below one chimney. The heated air rises up the chimney, whilst air from outside descends the other chimney, so ventilating the box.

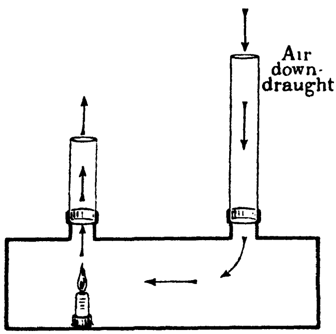


FIG. 73.

If a smouldering piece of paper is held at the top of the jar in the first experiment, or the cold glass chimney in the second, the movement of the smoke clearly shows the direction of the draught.

The latter experiment illustrates the original method of ventilating mines. Two shafts were always dug and at the bottom of one a large fire was kept alight, and so there was a down-draught in the second shaft, which kept a current of air moving through the mine.

Here follow some examples and applications of convection currents in liquids and gases.

(1) **Ocean Currents.**—The possible cause of these currents of water is that the water in the equatorial regions of the earth becomes much more heated than the water near the poles, and so expands. The level of the oceans thus tends to become slightly higher at the equator than at the poles, and so the warm water tends to flow along the surface from the Tropics to the Polar regions. The movements of the currents are modified by the motion of the earth, the configuration of the ocean bed and, in the case of purely surface currents, by the prevalent winds. The Gulf Stream is a well-known convection current which influences our climate. The meeting of the Gulf Stream and the cold Arctic current off the coast of Newfoundland results in the fogs for which that coast is notorious; this is an interesting example of the result of air being suddenly cooled to dew-point.

Further details concerning ocean currents can be read in books on physical geography.

(2) **Hot Water Circulation.**—Convection currents are the means whereby hot water is circulated to different parts of a house for domestic purposes and for heating buildings. Fig. 74A shows the principle commonly used for the former purpose ; it is well illustrated by an experiment using the apparatus shown in Fig. 74B. The flask

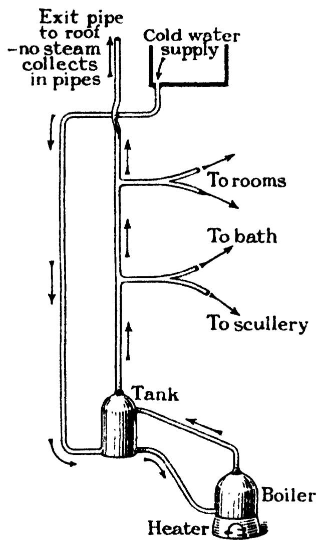


FIG. 74A.—Domestic Hot Water Supply by Convection.

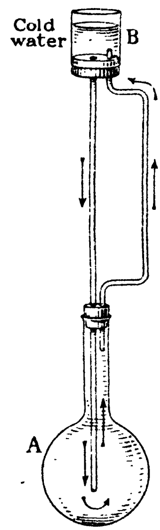


FIG. 74B.

A is filled with coloured water, whilst the tank B and tubes are filled with clear water. On heating flask A a convection current is clearly seen to move in the direction shown by the arrows.

The ordinary low-pressure hot water system (Fig. 75) used for heating buildings is similar in principle to this, except that the pipe is continuous and radiators are used to maintain an equable temperature over all rooms. The heat is carried round by the moving particles of water.

The pipes and the air adjacent to them become heated, convection currents thus being set up in the air. Although there is a little conduction of heat and some radiation from the pipes and radiators, most of the heat is carried around the room by means of the convection currents in the air. In some low pressure heating systems the return of the cool liquid due to gravitation is aided by *accelerators*

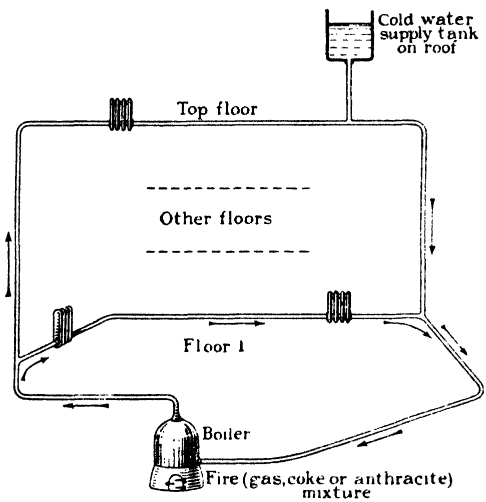


FIG. 75.—Heating Rooms by Hot Water.

or *impellers*. These, driven by electric motors, help to force the water circulation in both pipes simultaneously.

(3) **Steam Heating** is also used for large buildings, such as factories and mills. Steam is generated in a boiler, and mains and radiators carry the steam round the buildings, and other mains convey the condensed steam back to the boiler. Pressures vary from 1/10th to about $2\frac{1}{2}$ atmospheres, the terms low-pressure system and high-pressure system being used accordingly.

(4) The **heating** of buildings **by hot air** is becoming more common. It is economical and, since few pipes are necessary, unaffected by frosts. Usually air is

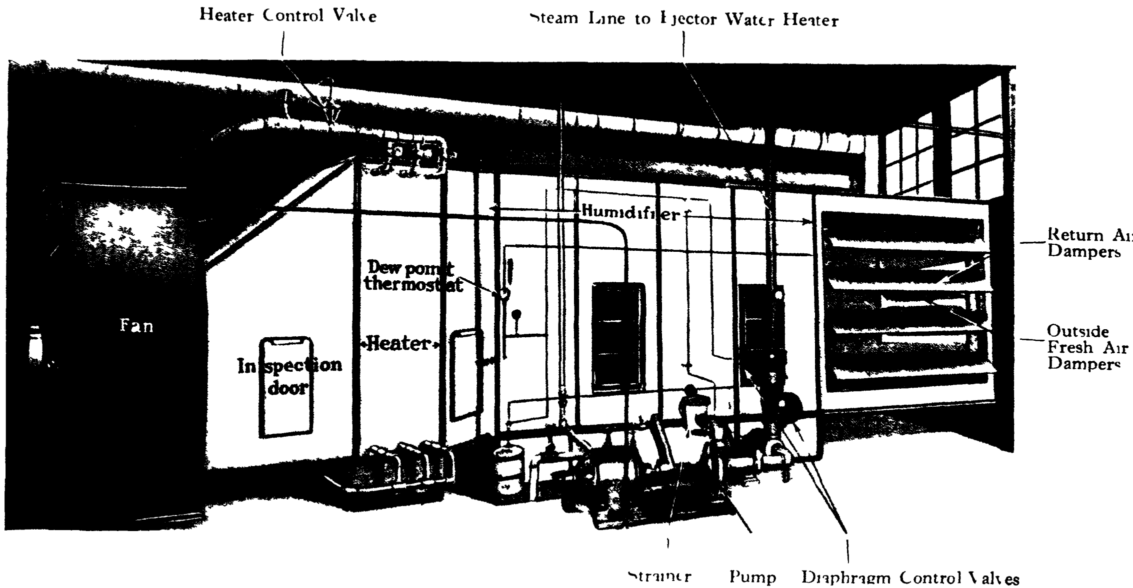


FIG 76 —A typical Unit of Carrier Humidifying and Conditioning Equipment The Air enters at the right, through the Return Dampers or from the Outside through the Fresh Air Dampers, the Proportion of Fresh and Return Air being Automatically Regulated—passes through the Humidifier where it is Cleansed and Conditioned—is drawn through the Heater, where it is brought to an automatically controlled Temperature, and enters the Fan Clean Humidified Heated (or Cooled in Summer) and ready for Distribution to the Building —(*By permission of the Carrier Engineering Co Ltd*)

passed over steam-heated plates in chambers below ground-level, and so rises in convection currents carrying the heat. The hot air passes through gratings in the floors or at the base of the walls of the rooms. In the most recent installations air is drawn in from outside by pumps and washed by a spray of fine cold water and a disinfectant. The liquid condenses and the air passes on to the heating chamber, steam also being added to it to make it sufficiently moist. (See Fig. 76.)

(5) Ventilation of rooms usually depends on convection currents set up by fires. We can see that it is necessary to have an outlet near the ceiling of a room, for the air is heated and deprived of much of its oxygen by the occupants. In large rooms such outlets are provided, but in ordinary houses they are rare, and when a room is occupied the window should be left open a little at the top. In large rooms in public buildings, in addition to the outlet provision, Tobin's tubes are commonly used to supply fresh air. These lead from the open air, near ground-level, to a point in a room about 10 or 12 feet above the floor. Through them a steady flow of air enters the room, and, owing to its greater density, falls to the level of the heads of the people in it and lower.

Winds (*e.g.* trade winds and anti-trade winds) and land and sea breezes are also convection currents. The latter examples will be discussed later.

Conduction of Heat.—Below are given some experiments illustrating the conduction of heat, in which it is transmitted along a substance without any appreciable movement of the particles, and some effects caused by it are noted.

Solids differ greatly in behaviour. Generally speaking, metals are good conductors of heat, whilst substances such as asbestos, rubber, flannel, etc., which are bad conductors of electricity and sound, are bad conductors of heat. When heat is to be transferred quickly, good conductors are used, *i.e.* metals are used for cooking utensils, heating pipes and radiators, etc. Copper is one of the best thermal conductors, and so the end of a soldering iron is made of copper. As its point gives

heat to the material being soldered, more heat is quickly conducted to the point from the rest of the metal. Examples of the use of bad conductors can be seen in household appliances, *e.g.* the hay-box to keep things hot, flannel to keep our bodies warm or to keep ice from melting, tea-cosies and eiderdowns. Many warm-blooded animals have covering of fur or hair, but man has to clothe himself. Snow also is a bad conductor, and snow on the ground protects plants during intense frosts. Steam-pipes and boiler installations need to be insulated, otherwise they lose much heat, and so they are covered with asbestos, or a cement of asbestos and magnesium carbonate. The walls of cooking ovens are packed with a thermal insulator, called slag-wool, so that heat is not wasted and the kitchen does not become over-heated when cooking is going on. One of the very best of modern heat insulators is granulated cork pressed into sheets and baked.

The difference in behaviour between glass and a metal such as copper can be shown by holding equal lengths of thick copper wire and thin glass rod in a bunsen flame with naked fingers. The glass can be held without discomfort for a long time, but the copper is quickly released.

Another well-known experiment is performed with a piece of wood into which some large-headed nails have been driven, the heads being made level with the surface of the wood. The latter is well rubbed with glass paper and a piece of white paper pasted on the surface, which is now passed to and fro through a bunsen flame. A scorched pattern is eventually seen on the paper, the latter being discoloured over the wood but white over the metal heads of the nails, for these conduct heat away from the paper, whilst the wood, a poor conductor, does not.

It is common knowledge in the laboratory that a piece of gauze wire held a little above a lighted bunsen burner controls the flame. Sir Humphry Davy realised this—that the metal gauze, a good conductor, rapidly leads away the heat produced by the burning gas, so that any unburned gas which rises above the gauze is not ignited,

for the temperature there is not high enough. Again, if the gauze is held above a bunsen burner, and the issuing gas lighted *above* the gauze, the gas below the gauze is not ignited for a similar reason.

Davy made use of these observations in his **safety lamp** for miners. In coal mines methane gas (called fire-damp by the miners) often issues from fissures in the coal. This gas, mixed with air, will readily ignite explosively, and the presence of fine particles of coal-dust (carbon), which ignite as well, adds to the devastating nature of the explosion. For this reason naked lights are dangerous in mines. However, methane requires considerable heat to fire it, but gives out little heat in burning. Therefore Davy first used a long metal tube, and then, later, a gauze tube, over the flame (Fig. 77) so that the heat produced by burning was spread by the metal, and distributed, with the result that the temperature around did not become high. The lamp also serves to indicate, by the variations in the nature of its flame, the presence in the air of any fire-damp, for as the latter burns inside the lamp a blue cap appears at the top of the flame of the burning paraffin. The effectiveness of this safety lamp can be shown by directing on to it, when lighted, a stream of coal-gas (which possibly contains 40 per cent. of methane). The gas is seen to ignite inside, but not outside the lamp.

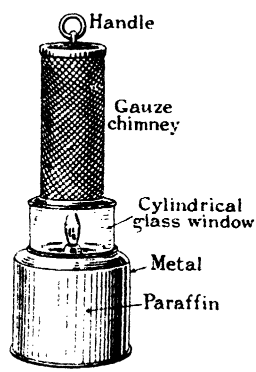


FIG. 77.—Davy Safety Lamps.

Sensations with regard to Heat not Reliable.—Our sensations of the hotness and coldness of the substances that we touch depend upon their conducting powers. In warm air (or rooms) metals always give a sensation of being *hotter*, and in cold air (or rooms) of being *colder*, than other substances which are at the same temperature. This is because the metal either gives heat to, or receives heat from, the person touching it.

In the former case there is a rapid transference of heat from other parts of the metal to replace the loss at the part touched. Thus heat is continuously received by the person, who has a sensation that the substance touched is hot. In the latter case he continuously gives heat to the metal, and so considers it to be cold. Thus it should be understood why in sunshine the sands feel hot to bare feet; on a cold morning the linoleum in a bedroom feels cold, but a carpet or rug much less so; and why woollen undergarments feel much less cold than cotton ones when first put on.

Liquids are bad conductors, with the exception of the metallic liquid mercury. *In testing liquids it is, of course, essential that convection currents are not set up, and so the liquids must be heated from the top.* The heated liquid expands and becomes less dense, so remaining at the top, and most of the heat passed down the liquid is due to conduction. Take a test-tube nearly filled with water and put in the bottom a piece of paraffin wax, round which a piece of thick copper wire has been wound, so that the wax remains at the bottom. Heat the water at the top, where it will be seen to boil vigorously without enough heat being conducted down through the water to melt the wax (temperature required about 52° C.). Thus water is a bad conductor of heat. The above experiment can also be carried out by first heating some wax, at the bottom of the test-tube, and so causing it to melt and adhere to the glass on cooling. If mercury is used in the tube instead of water, the wax is seen to melt and rise to the top of the mercury. Heat has readily passed down through the mercury which is shown to be a good conductor. *Care must be taken, however, that the mercury is only gently heated*, so that *little mercury vapour is given off*, *as it is extremely poisonous.*

Gases are even worse conductors of heat, but the difficulty of preventing convection currents is great. Probably the most successful method of studying conduction in gases is that in which the difference in rate of cooling of a large, heated thermometer is observed, (*a*) in a vacuum, (*b*) in a gas at such a low pressure that

convection currents are almost eliminated. To illustrate the poor conductivity of steam, the following experiment can be performed. Let fall a drop of water on a hot, metallic surface, so that it collects itself into a spheroidal globule of liquid. For quite a time it runs to and fro, as around it is formed a cushion of vapour which separates it from the heated plate and very slowly conducts heat through to the water.

Conductivity along a Rod of a Substance.—We have seen that when one end of a piece of substance is raised in temperature, heat passes along to the other end. In the case of a metal rod the amount of heat can be

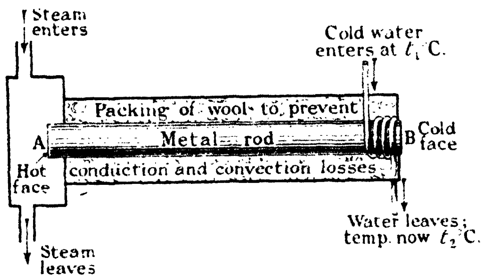


FIG. 78.—Searle's Method to measure Conduction of Heat along a Bar.

measured by a method (for which we are indebted to Dr. Searle of Cambridge) shown in Fig. 78. If the flow of water through the narrow, copper, spiral tube is kept constant, the more the heat conducted across from face A to face B the greater the difference in temperature between the ingoing and outgoing water. Using this method, it is possible to study in turn the effect of the various factors which control the amount of heat passing from A to B. The results obtained show that the *quantity* of heat passing across from face A to face B (Q calories)—

(1) is proportional to the area of cross-section ($\propto$ A sq. cms.);

(2) is proportional to the difference of temperature

[$\propto (\theta_1 - \theta_2)°$ C.], where $\theta_1°$ C.=temperature of hot face A and $\theta_2°$ C.=temperature of face B;

(3) is inversely proportional to the distance between the faces ($\propto 1/l$ cms.);

(4) is proportional to the time for which the conditions are maintained ($\propto t$ secs.).

Thus $Q \propto A \frac{\theta_1 - \theta_2}{l} t$ or Q in calories$= k A \frac{\theta_1 - \theta_2}{l} t$,

where **k is the constant,** depending on the nature of the substance, **called the coefficient of thermal conductivity of the substance.**

If $A=1$ sq. cm., $(\theta_1 - \theta_2)°$ C.$=1°$ C., $t=1$ sec. and $l=1$ cm., then $Q=k$,

i.e. **the coefficient of thermal conductivity of a material is the quantity of heat which flows, in unit time, through unit area of a plate, of unit thickness, having its faces maintained at temperatures differing by 1° C.**

Searle's method, as described above, is used for finding k for a good conductor. The quantity of heat passing along the bar in a known time is measured, by the rate of flow of water through the spiral tube and its rise of temperature, and equated against the value for Q in the equation deduced above, and for which all the factors, except k, are measured.

A suitable method for measuring the coefficient of thermal conductivity is due to Péchet, who did the earliest work in measuring the thermal conductivity of a substance. A calorimeter is made with its base thickness l cms., of the solid under consideration. Into it is placed a known mass of water (m grms.). This is then plunged into a vessel of hot water so that the base is just below the surface. The rate of rise of the temperature of the water in the calorimeter is observed.

Suppose the water rises from temperature $t_1°$ to $t_2°$ C. in x secs.

Mean temperature of the water in the calorimeter during the experiment$=\frac{t_1+t_2}{2}°$ C. and of the hot water$=$T° C.

Thus temperature gradient (fall of temp. per unit thickness) $= \frac{T-(t_1+t_2)/2}{l}°$ C. per cm.

If area of base (of substance whose coefficient of thermal conductivity$=k$)$=$A sq. cms.,

the quantity of heat passing through per sec.

$$=kA\frac{T-(t_1+t_2)/2}{l} \text{ cals.}$$

But heat received by the water in the calorimeter per sec.$=$mass of water$\times$rise in temperature per sec.$\times$specific heat of water

$$=m\frac{(t_2-t_1)}{x}\times 1 \text{ calories.}$$

Equating these, k is calculated.

Comparison of Thermal Conductivities by Ingen-Housz's Method.—In this method uniform rods of different substances, of similar dimensions, are fixed horizontally in corks in holes in the side of a water-bath. The rods are coated with a thin layer of paraffin wax. Hot water is poured into the bath to cover the ends of the rods, and heat is thus passed along them by conduction. The paraffin wax is observed to melt gradually along the rods, at different rates along the different rods. Edser modified this form of apparatus by fixing in the base of a water-bath vertical rods, putting on the top of each a light metal index held by the wax on it (Fig. 79). As the wax melted along a rod the index slipped down, thus more clearly and accurately indicating where the wax melted.

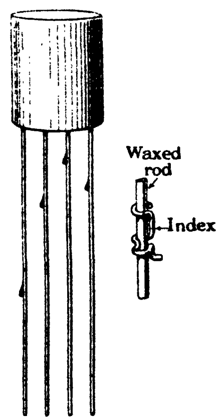


FIG. 79.

The order of quickness of melting of the wax along the different rods is observed. The order obtained by arranging the rods according to the final length of wax melted is not the same. In the first case, heat passing along the rod raises the temperature of successive layers of the substance. Obviously the greater the thermal capacity of a layer, the more heat is required to raise its temperature to the melting point of paraffin wax. Thus

the less is the rate of movement of the index held by the wax. After a time a permanent state is reached. All the temperatures of the various parts of a rod remain constant, for heat is merely conducted along, without any temperature changes, replacing heat lost at the surface of the rod. Thus the process reaches a state of pure conduction between faces maintained at a constant temperature difference; the wax ceasing to melt further along the rods. The water-bath is usually filled with boiling water, and so there is enough heat available for such a state to be reached.

For either of the rods, the quantity of heat passing across per sec. $(Q)=kA\frac{(\theta_1-\theta_2)}{l}$, or $k=\frac{Ql}{A(\theta_1-\theta_2)}$ (usual symbols).

But A is the same for both rods.

θ_1=the temperature of the water-bath } are the same
θ_2=the temperature melting paraffin wax } for both rods.

Thus $k \propto Ql$.

But the greater the length of bar along which the wax is melted, the more the heat that has been transmitted along it. As the indexes fall small distances, the temperature difference between these distances are equal in the two cases, and so the greater the distance the greater the amount of heat which must have passed across. Thus the quantity of heat is proportional to the distance, and so, in the final state, $Q \propto$ length (l) of rod along which the wax is melted.

But $k \propto Ql$.

And so $k \propto l^2$, *i.e.* the ratio of the conductivities of the two rods is the same as that of the squares of the distances along which the wax has melted.

The following are the appropriate values for iron and bismuth :--

	Specific Gravity	Specific Heat	Thermal Conductivity
Iron . . .	7·86	0·11	0·14
Bismuth . .	9·8	0·03	0·019

Thus the thermal capacity of unit volume of bismuth $(9{\cdot}8\times0{\cdot}03=0{\cdot}294)$ is much less than that for iron $(7{\cdot}86\times0{\cdot}11$

$=0{\cdot}8646$), and so the rate of melting of the wax at the beginning is much higher for the bismuth. But since the thermal conductivity of iron is 7 times as great as that of bismuth, a longer length of wax is melted along the iron.

Exercises on Chapter X

1. Explain what occurs when the end of a steel poker is placed in the fire and left there until the whole poker reaches a steady temperature. What differences would you expect to observe if the poker were made of copper instead of steel? [*L.M.* 1923.]

2. Write a short account of the convection of heat and its applications.

3. What are convection currents? How do they aid the transfer of heat from one part of a body to another? Explain how convection currents may be made use of *either* in ventilating *or* in heating a building. [*L.G.S.* 1920.]

4. Describe with the aid of a diagram how a building may be heated by hot water conveyed in pipes from a boiler in the basement. Explain the various modes of heat transference occurring in the passage of the heat from the boiler furnace to objects in a room. [*L.G.S.* 1921.]

5. Give some practical examples of the occurrence and use of convection currents in liquids and gases.

6. Describe the construction of a Davy lamp. What is its purpose and what physical and chemical properties of the substances concerned enable it to fulfil its purpose? [*J.M.B.* 1922.]

7. How would you show that water is a bad conductor of heat? Describe an experiment to show how the heat supplied to the bottom of a tin vessel containing water is carried to all parts of the liquid. Explain the experiment and mention some uses that are made of this method of transferring heat. [*J.M.B.* 1923.]

8. A piece of paraffin wax is placed in a test-tube of water and it melts quickly when a flame is applied at the bottom of the tube. If, however, the wax is fixed at the bottom of the tube and the flame applied near the top a considerable time elapses before the wax begins to melt. Account for this difference in time, and describe how the heat reaches the wax from the flame in each case. [*L.G.S.* 1924.]

9. Contrast the physical processes involved in the conduction and the convection of heat. Describe experiments to show that water can conduct heat, but that it is a poorer conductor of heat than mercury.

10. Two glass flasks, similar in every respect, being provided, how would you determine which kind of glass was the better conductor of heat?

11. Spheres of copper and iron of the same diameter and of masses 8 : 7 are both heated to 100° C. and placed on a slab of paraffin wax. It is found that the copper sinks in more quickly than the iron, but that in the end the iron is level with the copper, having melted the same amount of wax. Give an explanation of this. [*L.G.S.* 1925.]

12. Two long rods of the same diameter, one of iron and the other of lead, are coated with paraffin wax and have small metal spheres attached to their lower surfaces by the wax. Two ends, one of each rod, are then heated together and the spheres drop off as the wax melts. Describe and explain the order in which the spheres drop off.

Thermal conductivity . . .	of iron=0·14,	of lead=0·08.
Specific heat	of iron=0·11,	of lead=0·03.
Specific gravity	of iron=7·5,	of lead=11·4.

Sketch the arrangement of the apparatus you would fit up to carry out this experiment. [*L G.S.* 1928.]

CHAPTER XI

RADIATION OF ENERGY—WAVE-MOTION

IN the previous chapter it was stated that heat, or, as it is better to say, heat energy, is transferred, or *radiated*, from the sun to the earth. In the process, the medium between the source of the heat energy and the point of its reception is unaffected. Heat energy is not the only form transferred from one point to another in this manner. The sun itself is radiating light energy at the same time as it is radiating heat. Perhaps you have not realised that light is a form of energy. Think of the amount of electrical energy which is used up in generating light (and heat at the same time) to give an illumination which is much less than that of daylight, light due to the sun. Consider also the modern development of television (Gk. *tele*, at a distance). For this, light energy is converted into electrical energy and back again into light energy, the former involving the use of a so-called photo-electric cell (Gk. *photos*, light). Light meeting a layer of potassium or cæsium (selenium was originally used) has been found to cause an emission of electrically charged particles, *i.e.* a current of electricity, in proportion to the intensity of the light falling on it. The current changes are transmitted to a distance where they are made to pass through a Neon lamp already faintly illuminated. This changes in brightness very appreciably with the small current changes, and so brightness at the transmitting end gives rise to brightness at the receiving end. The process is undergoing development.

Another example of energy radiation is that of wireless telegraphy and telephony—energy is radiated via the

aerial of a transmitting station and is intercepted at distant receiving stations.

Before dealing with the case of heat energy radiation, we shall study some of the facts and ideas regarding radiation in general Prehistoric man must have realised that the sun supplied him with heat and light—for when night followed day there was no supply of light or heat (except, at times, the light of the moon). So we find that he regarded the heavenly bodies as having power over the phenomena of the world and over human fortune in particular. Thus he rendered special homage to these bodies, arranged special festivals in their honour ; in fact, he worshipped them as gods, the greatest being the sun. In the song of Homer, dated about 750 B.C., the sun and the other planetary bodies were regarded as gods. This solar worship led to the construction of calendars to mark the festivals, whilst the earliest large stone pillars were probably erected to fix the direction of the sun and so to observe when the sun rose at the same point on the horizon, *i.e.* the recurrence of years. Solar temples were built so that on midsummer day the sun, on rising, shone through an aperture and illuminated the altar, not only marking the day but impressing the people assembled. Stonehenge, in Wiltshire, was possibly built for that purpose.

Further knowledge of the heavenly bodies, gained by careful observation, resulted in a realisation of an order in this universe of which the earth is but one part. With this came a gradual realisation of the existence of "animus," or the mind of man, and with it the idea of a supreme being, the Creator of the universe. So man ceased to worship the heavenly bodies, though still awed by their splendour.

Thus thought was directed to the phenomena of heat and light, and eventually attempts were made to explain them. The first theory was a very obvious one. Suppose one is hit from behind by a snowball, whilst walking along a street, and on turning round sees an impish-looking boy a little distance away. The obvious conclusion is that he threw the snowball. So the early Greeks,

Pythagoras (582–500 B.C.), and Democritus (460–370 B.C.) said that vision was caused by the projection of particles from the object seen, *e.g.* the sun, into the pupil of the eye. A similar theory held in the period of the renaissance of science in Europe, A.D. 1600 up to as late as A.D. 1800. Just as heat was considered to be due to a fluid, called caloric, so it was held that the sun was continually sending out in all directions, or *radiating*, a fluid which produced in us the sensation of light. As the little particles of fluid responsible for the sensation of light were called corpuscles, the theory is called the *Corpuscular Theory of Light*.

But another theory to explain light phenomena was first put forward by Hooke and much developed by *Christian Huyghens about* 1678 (published 1690). *This theory considered light to travel in the form of waves.* Huyghens' arguments did not convince Sir Isaac Newton, who, after much study, supported the Corpuscular Theory. Owing to the belief at the time in Newton's ability, this theory became general and probably lasted a hundred years more than it would have done without his support.

In the nineteenth century the fluid theories were set aside and wave theories became general, whilst about the same time it was established that heat was due to molecular motion. The conception of its transformations received general acceptance, and so the radiation of energy became the subject of much interest. As we shall see, there has been great development of knowledge and ideas, and man has now many methods by which he is able to radiate energy. In fact, the time may not be far distant when large amounts of electrical energy may be transferred from place to place through space without in any way affecting intervening matter.

Wave-motion.—Most of us have, at some time, amused ourselves by throwing stones into a pond or lake and by watching the rings, or ripples, which spread out in concentric circles from the place where the stone entered the water. Many of us have stood on the end of a pier watching the waves rolling by. In both cases we have watched what is called wave-motion.

When water is displaced, whether it be by stones, the wind, the paddles of a steamer or the oars of a rowing-boat, waves are invariably produced and are seen to travel with quite an appreciable speed. There is something, however, about this wave-motion which is not so obvious. Perhaps you have noticed when fishing in the sea how the float rises up and down with a slight to-and-fro motion. Or maybe you have been in a small boat on a river when a large steamer has ploughed its way by, and have noticed how your boat rose and fell as the waves, made by the steamer, passed under you. If you have not noticed it, float a piece of wood on a pond, throw a stone into the water a couple of yards away from it, and notice its movement. Floating bodies are not carried away by waves (though they are by water-currents), they merely rise and fall rhythmically, with a slight to-and-fro motion. [Many of you, no doubt, have noticed that movement when bathing in the sea.] When at the top of their motion they are said to be at the *crest* of a wave, when at the bottom, at the *trough* of a wave. It is seen, then, that when we watch wave-motion, although the crests, for example, appear to move forward, yet really the water does not move forward appreciably—it chiefly moves up and down perpendicularly in a regular manner. When a stone is thrown into water, *i.e.* a hole is made in it, water rushes in from the sides to fill up the hole just as it does when you push some of the water aside in your bath by means of a sponge or by your hand. The inrush of water more than fills up the hole—it causes the water to be lifted up in a heap above. This water, under the gravitational force of the earth, falls back on the surface of the water around. Thus the water is first pushed down and then lifted up again, and this action goes on a little while, getting weaker and weaker, the up-and-down motion being accompanied by a slight to-and-fro one. Consequently the displaced particles collide with adjacent particles and set them in motion. These particles then set particles adjacent to them in motion and the effect of the original displacement travels outwards, *i.e.* a wave travels outwards. Thus **wave-**

motion in water is really a particular state of motion handed on from one portion of water to another. The waves are thus called *progressive waves.* The wave-motion in water, where the movement of the water particles is perpendicular to the movement of the wave itself, is called *transverse progressive wave-motion.* Some very wonderful photographs of the production of such waves are to be seen in an excellent book called *The Splash of a Drop*, by Prof. A. M. Worthington (S.P.C.K. publication).

In order that a *medium* may be *set into wave-motion,* (1) *the medium must be disturbed,* (2) *the medium must oppose the disturbance and so tend to recover from it.* It must thus be elastic in some sense. Probably you have seen the toy which consists of a light ball, or a monkey, fastened to the end of a piece of thin rubber. When the object is pulled away from its mean position by the stretching of the rubber, the latter opposes the stretching. The more it is stretched, the greater is its opposition, and eventually it is sufficient to prevent further movement of the object. Its resisting force then operates to pull the latter back. The inertia, or tendency of a body to continue its state, whether of rest or of motion, causes the object to return past its mean position and so the rubber becomes compressed. Opposition is offered to this by the elastic rubber, and so the motion is gradually stopped and then reversed again. This goes on—the object jumping up and down, or vibrating, or oscillating.

Similarly water, when displaced, resists—it offers resistance to any alteration of level. Water surfaces, unless acted on by external forces, are horizontal and level. Hence water rushes in from the sides, and then heaps up, as we have seen. It cannot remain so, owing to the pull of the earth, *i.e.* gravitation. Thus these influences cause the water particles to move up and down, and not to come to rest at once. Further, the adjacent particles are set in motion, i.e. *energy is transferred.* This, the essential point in regard to wave-motion is very obvious when you consider that waves produced in the centre of a pond travel out and can lift up a small

boat at the edge of the pond. Thus the moving particles hand on energy in turn, and so transfer it from one place to a distant place. Such, then, is our conception of the propagation (transmission) of radiant energy, *i.e.* the radiation of energy. An elastic medium, one which will oppose a disturbance and so tend to recover from it, is necessary for wave-motion to be possible. In the case of waves in water, the particles are raised above their mean positions, losing speed and thus kinetic energy, but gaining energy of position, *i.e.* potential energy. When they are at the level of the rest of the water they have no energy of position but have an appreciable speed, and hence kinetic energy. And so a change from one to the other continues in the particles during the wave-motion.

A succession of waves is termed a *wave-train*, and some important facts are to be observed in regard to wave-trains. If you throw a stone into water and watch two or three separated floating objects as the waves pass through the water, you will see that the floating objects start moving at different times, the object which is nearest the displaced water commencing to move first. Again, the floating objects, although they undergo the same up-and-down movement, at any given moment are at different stages of that movement. Thus in wave-motion in water not only do the particles move in exactly the same way, describing equal and similar paths in equal intervals of time, but, as we pass from particle to particle in the direction of movement of the wave, the simultaneous positions occupied by the particles in their respective paths are different.

As an example, let us consider a series of spheres equally spaced along a line AB. Suppose that the first one is caused to move up and down along a line perpendicular to AB and that in so doing it jogs the next ball and sets that one moving similarly—but a little later—and that this process is continued along the row of spheres. Then at any particular instant the positions of the respective spheres will be as shown in Fig. 80, the maximum displacements being AX and AY, the complete

vibration, or oscillation, being from X to Y and back again, or its equivalent.

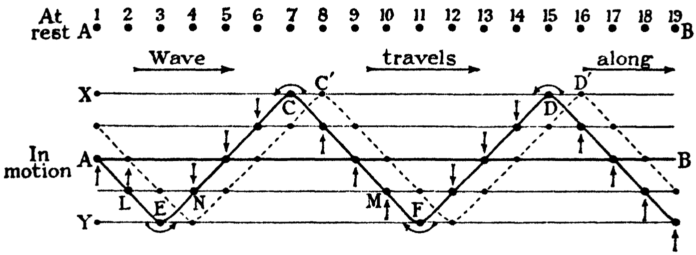


FIG. 80.—Illustrating Transverse Wave-Motion.

If No. 1 sphere is in the mean position moving upwards to X, and each succeeding sphere is considered to be $\frac{1}{8}$ of a complete vibration, *i.e* $\frac{1}{8}$ of twice XY,

which $= \frac{1}{2}$ of AX or AY, behind,

Then No. 2 sphere is displaced this distance, or $\frac{1}{2}$ of AY, and is moving towards X ;

No. 3 sphere is displaced distance AY and is reversing its motion ;

No. 4 sphere is displaced distance $\frac{1}{2}$ AY and is moving towards Y ;

No. 5 sphere is in its mean position, and is moving towards Y ;

and so on.

The appearance of the displaced spheres is thus as indicated by the curve drawn through their positions, and is in the form of a wave similar to that seen on water.

The positions of maximum displacement in one direction, *e.g* positive towards X, are called crests, and in the other direction, *i.e.* negative towards Y, are called troughs.

Thus C and D are crests and E and F are troughs.

If we consider the positions of the spheres a little later, after the time for $\frac{1}{8}$ of a complete vibration, the

positions of the spheres are indicated on the dotted line curve. No. 2 will have moved to its mean position, No. 4 to its maximum displacement AY, etc., etc.

Crests C and D will have moved to C′ and D′ respectively, and so it is seen that each sphere, from No. 1 onwards, attains its position of maximum positive displacement a little later than the sphere immediately on its left. We say the spheres are in *different phase.*

Two spheres, *e.g.* Nos. 7 and 15, in positions C and D respectively, are said to be in the *same phase*, and (1) their displacements are equal, (2) their speeds are equal and are in the same direction.

Spheres Nos. 2 and 10, in positions L and M, are in the same phase, but not 2 and 4 (in positions L and N), for although equally displaced they are moving in opposite directions.

Regarding the above, there are four fundamental definitions which apply to all kinds of wave-motion :

(1) **The distance between any two nearest particles in the same phase is the wave-length** (usually represented by λ). Thus CD, or EF, or C′D′, or LM, in Fig. 80, equals the wave-length.

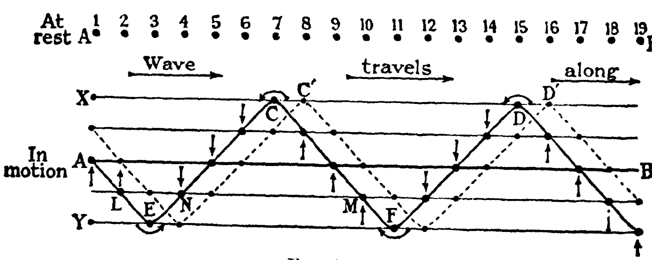


FIG. 80.

(2) **The time taken for each particle to describe completely its path, or vibration, is called the periodic time,** or **period** (usually represented by T and measured as a fraction of a second). It should be noticed that the path is completely to-and-fro, *i.e.* from X to Y and back again, or its equivalent.

(3) **The number of complete vibrations of a particle in one second is called the frequency** (usually represented by η).

Now the period of vibration, T in seconds,

$$= \frac{1 \text{ second}}{\text{No. of vibrations per sec. } (\eta)}$$

i.e. $$T = \frac{1}{\eta} \quad \text{or} \quad \eta = \frac{1}{T}$$

(4) **The maximum displacement of each particle from its mean position is called the amplitude.**

Velocity of Wave-Motion. — The velocity of a wave is equal to the distance the wave, or any part of it (*e.g.* a crest), travels in one second, or the distance travelled divided by the time taken. Now regarding Fig. 80, and considering No. 7 sphere which is at a crest, we have seen that $\frac{1}{8}$ of a period after, No. 8 sphere is at a crest, and in succession particles 9, 10, etc., attain the position of a crest. When the crest has apparently moved from position C to D, No. 7 particle will have gone through its movement of displacement equal to AY and back to C again, and so the time taken will be the period T. Thus the crest moves from C to D, *i.e.* a distance λ in time T.

$$\therefore \text{Velocity of the wave } (V) = \frac{\lambda}{T}.$$

$$\text{But } \eta = \frac{1}{T}, \text{ and so } \underline{V = \eta\lambda},$$

i.e. **velocity of wave-motion = frequency × wave-length.**

Wave-front is a very important conception in the subject of wave-transmission. A very familiar example is the ridge of an ocean swell. It runs at right angles to the direction of the wave-motion. The wave-front through any particle, which is participating in wave-motion, can be obtained thus. Imagine a continuous surface extending out from the particle and containing other particles which are simultaneously in the same

phase. This surface, a wave front, is considered to advance with the wave and so its velocity is the same as that of the wave. There are, of course, any number of wave-fronts, even in a wave-length, but in observing water waves we involuntarily observe the wave-front of crests—progressing forward, and often as a line of so-called " white-horses."

The following are types of waves and wave-fronts, when waves are travelling out in all directions in the same manner and with the same speed :

(1) *Radiation* from a point gives rise to *spherical waves*, the wave-front being the surface of a sphere

(2) *Radiation from very distant sources* (*e.g.* the sun) is, of course, of type (1), but the surface received by us is that of a sphere of extremely large radius, *i.e.* the surface is approximately plane. We thus call the wave a *plane-wave* and the wave-front is, of course, a plane.

The Æther of Space.—Heat and light travel from the sun to us through space, which we know must be devoid of air or other matter. When Huyghens put forward his wave-theory he realised the necessity of a medium for the transmission of the waves. He therefore considered there was a fluid, diffused throughout all space, penetrating everything and yet not affecting our senses in any way. He called this all-pervading substance the *æther of space*, and its particles were said to be exceedingly small, hard and elastic, thus being ideal for transmitting energy in wave form.

It is probable that Newton found himself unable to accept the wave-theory because there was little evidence of the existence of such a medium. But in the last century the wave-theory became established, and light and heat energy are considered to be transmitted as transverse ethereal waves, whilst knowledge is continuously being gained of these and other ethereal waves such as X-rays, wireless waves, etc.

The Inverse Square Law.—When a source is radiating energy in all directions, it is obvious that the amount of energy received at some fixed area depends

upon its distance from the source. As the spherical waves progress outwards their radii get larger and so the surface area of a sphere enclosing the waves becomes larger. The energy is enclosed within this sphere, the energy received per unit area of surface becoming smaller the farther the area is from the source. It is only to be expected that the effect produced over a small area will be in proportion to the amount of energy received (this, however, is not true for large quantities), and if we assume this we can deduce a very important law which is found to hold in many branches of Physics.

Consider a source S (section shown in Fig. 81) radiating E units of energy per second in spherical waves, and let A and B be concentric spheres, with S as centre, of radius a and b respectively.

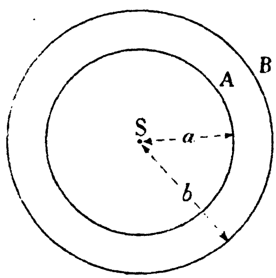


FIG. 81.

Then the energy received per sec. over unit area of surface $A = \frac{E}{\text{surface area}}$ units $= \frac{E}{4\pi a^2}$ units. Also the energy received per sec. over unit area of surface B (when surface A is not in the path)

$$= \frac{E}{\text{surface area}} \text{ units} = \frac{E}{4\pi b^2} \text{ units.}$$

Assuming

$$\frac{\text{intensity of effect at surface A}}{\text{intensity of effect at surface B}} = \frac{\text{amount of energy per sec. received at A}}{\text{amount of energy per sec. received at B}}$$

this $$=\frac{E/4\pi a^2}{E/4\pi b^2}=\frac{4\pi b^2}{4\pi a^2}=\frac{b^2}{a^2},$$

i.e. **the intensity of the effect of the energy at a distant point varies inversely as the square of its distance from the source of energy.** This is the **Inverse Square Law,** which we shall have occasion to use later.

EXERCISES ON CHAPTER XI

1. What do you understand by a *transverse wave?* Illustrate your answer with the aid of a diagram showing the passage of such a wave along a line of equally spaced particles and explain the terms *amplitude, wave-length* and *frequency.* [*L.G.S.* 1923.]

2. What is meant by frequency, wave-length and wave velocity? Obtain a relation between these three quantities for a given wave-motion. [*L.M.* 1925.]

3. What are the main characteristics of wave-motion? Illustrate your answer by a diagram. [*L.G.S.* 1926.]

4. A stone is dropped perpendicularly into still water and produces a series of concentric waves. At the end of 5 secs. there are 30 concentric circular troughs and crests and the boundary of the outermost is a circle of 1·5 metres radius. Find the wave-length and velocity of propagation of the disturbance. Find also the period of oscillation of a water particle.

5. Explain the Inverse Square Law for radiating energy. Deduce it by theoretical methods.

CHAPTER XII

RADIATION OF HEAT

IT has already been stated that heat energy is transmitted by radiation when it passes from one point to another without raising the temperature of the medium through which it travels. Sir Humphry Davy showed that heat passed through a vacuum by electrically heating a wire inside the best vacuum he could produce and detecting the heat radiated outside by a sensitive thermometer (this was covered with lamp-black since it had been found that a black surface was more sensitive to small quantities of heat, a point which will be dealt with later).

Instruments for Detecting Heat Radiations.—The earliest instrument was a simple thermometer tube as used by Davy. But the first scientist to make a study of thermal radiations in detail was *John Leslie* (1766–1832), and for their detection he invented *Leslie's differential air thermoscope* (see Fig. 86). It consists of two air bulbs joined by a long tube, part of which is a U-tube containing a coloured liquid, arranged so that the liquids are level in the limbs of the U-tube when the air bulbs are under the same conditions. Both bulbs are blackened outside to increase their sensitivity. When one bulb only is subjected to heat radiations, air inside it expands appreciably and pushes the liquid level down on that side. Thus small differences in temperature, or heat radiations, are indicated.

A variation of this is the ether thermoscope (Fig. 82) which contains a little ether, CD, but no air. When used the thermoscope is arranged as in the figure, and in the bulbs is ether vapour. Bulb A is blackened and thermal

radiations falling on it cause the bulb and vapour inside it to be heated. The vapour thus exerts a greater pressure, so pushing level C down and level D up.

The most celebrated of radiation instruments is the thermopile (Fig. 83), invented by Nobili in 1830 and based on Seebeck's discovery of a thermo-electric current obtained

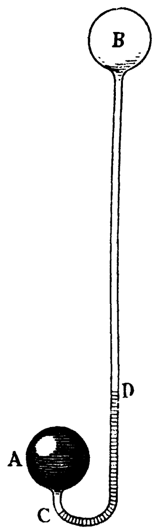


FIG. 82.—Ether Thermoscope.

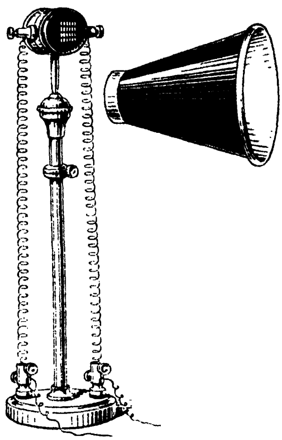


FIG. 83.—Thermopile.

by using a thermo-couple, which is a pair of dissimilar metals joined together at one end of each (see p. 26). The pile consists of a series of thermo-couples of antimony and bismuth insulated from one another, the antimony of one being joined to the bismuth of the next (Fig. 84). The two ends of the pile, or series, so formed are connected to terminals, and these need to be joined to an electrical instrument, a sensitive galvanometer, to indicate the electric current produced. The junctions of the thermo-couples are arranged, in the thermopile, in one plane and are blackened, and the heat radiations are allowed to

fall on them. With this instrument the thermo-electric current set up is approximately proportional to the heat energy received per second, but the comparatively slow

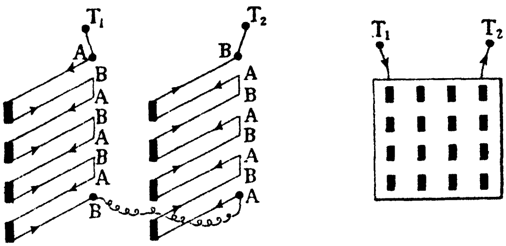


FIG. 84.—Showing Construction of a Thermopile.

rate of indication and the length of time taken for the pile to return to its original state are disadvantages. To this instrument, however, we owe the researches of Melloni and Tyndall, which will shortly be discussed.

In 1875, Sir William Crookes invented the *Radiometer*, which consists of four vanes, delicately pivoted, or suspended, in a bulb from which most, but not all, of the air has been removed (Fig. 85). The vanes are blackened on one side and polished on the other, and arranged so that the black side of one vane faces the polished side of an adjacent vane. When heat radiations fall on the radiometer the black vanes become more heated than the polished ones. Thus the air particles adjacent to the black surfaces become heated to a slightly higher temperature than those adjacent to the polished surfaces. There is thus a greater push on the black surfaces of the vanes owing to the greater velocity of the molecules near them, and the vanes revolve with polished faces leading.

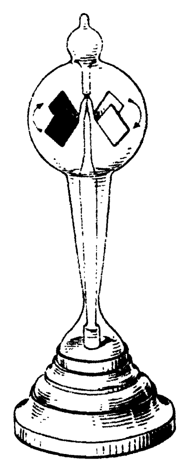


FIG. 85.—Crookes' Radiometer.

A later instrument is the *Bolometer*, introduced in 1881 by Prof. Langley. It makes use of the fact that

the electrical resistance of platinum increases with rise of temperature. Thus a thin strip of platinum (or steel and palladium are now found to be very suitable) covered with lamp-black is used to absorb thermal radiations, and the variation in resistance measured by electrical methods.

An instrument, Prof. Boys' Radio-Micrometer, very much more sensitive than any of the above, has been devised, but its action is outside the scope of this book.

In his work on thermal radiations, Leslie introduced a useful piece of apparatus, known as *Leslie's Cube*, for use in conjunction with his differential air thermoscope. It is a hollow tin cube of about 6-in. side, with a small neck fitted in the top for filling with water. Usually the four side surfaces are of a special nature, one is made dull black by the use of lamp-black or some suitable paint, one is brightly polished, one is coated with white enamel, and the other is roughened, or rusted after the tin coating has been scratched off.

Radiation from Surfaces.—About 1804, Leslie, using his cubes and differential air thermoscopes, showed that the radiation from a body depended upon the nature of its surface. For different surfaces, maintained at the same temperature, he found that a black surface was the best, a roughened surface not so good, a white surface poor and a brightly-polished surface the worst radiator. This can be verified with apparatus similar to Leslie's. The cube is filled with boiling water and then placed with each of two faces near to, and equidistant from, a bulb of the differential air thermoscope (Fig. 86), convection not being permitted to influence the arrangement. If face B radiates better than face P, then the level D of the liquid falls and level E rises. The results obtained are as stated above.

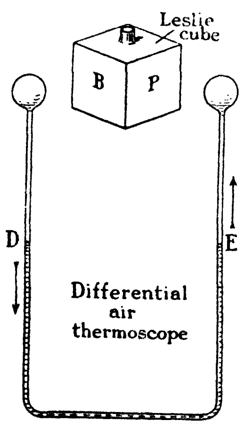


FIG. 86.

Melloni and Tyndall, using a thermopile instead of the differential air thermoscope, confirmed Leslie's results.

A simple way of showing that a black body radiates much better than a polished one is to use two flasks, one clear and the other blackened over, *e.g.* by soot. Fill both with hot water from the same supply, place a thermometer in the liquid in each and allow to cool whilst standing on similar surfaces, a little distance apart so as not to influence one another. Readings on the thermometer in the liquid in the blackened flask are found to fall the more rapidly.

Absorption of Thermal Radiations.—It has already been realised that bodies on which heat radiations fall absorb them, so becoming heated. It is said that in the middle of the eighteenth century Benjamin Franklin, an American famous for his work concerning electricity, put differently coloured cloths on snow when the sun was shining, and found that the snow melted most under black cloths and least under white cloths. He thus gained an idea of the variation in the absorption according to the colour of the absorbing surface, but, of course, no account of the different thermal capacities of the cloths was considered.

Leslie investigated the problem, using a cube having a pair of exactly similar faces. The bulbs of his differential air thermoscopes were coated differently and he experimented with two bulbs held at equal distances from similar surfaces of the cube, which was filled with boiling water, *i.e.* from similarly radiating surfaces at the same temperature. The bulb surface which was the better absorber thus became more heated as did the air inside it, and so pushed down the liquid on its side of the U-tube. In this way it was found that a dull-black surface was the best absorber, a roughened surface not so good, a white surface poor, and a polished surface the worst absorber. Obviously, a surface which does not absorb thermal radiations must either transmit them or throw them back, *i.e.* reflect them. As we shall shortly see, few material substances transmit them, and polished surfaces are very good reflectors of thermal radiations; that, of course,

is why electrical radiators are usually highly-polished behind the heating elements.

The general rule concerning radiation, etc., by surfaces is stated thus:

A good radiator is a good absorber and a poor reflector (black).

A bad radiator is a bad absorber and a good reflector (polished).

A perfect absorber, or radiator, is called a perfect black body. No surface is, of course, perfect—but a surface of lamp-black is very efficient and is considered for practical purposes to be a perfect radiator or absorber.

An interesting example of the difference in the absorbing powers of surfaces is seen in the ordinary utensils in front of a fire. A dull-black fender will be found to feel much hotter than polished copper fireirons near it.

If a piece of white porcelain, with a blue pattern on it, is heated strongly in a blowpipe flame for some time and then observed in the dark, the blue pattern appears brighter than the white surface. This is because the blue pattern surface is a better radiator, and absorber, than the white surface.

Some applications of the effect of the nature of surface on radiation and absorption by a body are seen in the following:

(1) Bulbs of thermometers, and the exposed junctions of thermo-couples, are blackened when used to absorb, and so indicate thermal radiations.

(2) Hot-water pipes, radiators, etc., are painted black, or dark-green, to assist radiation from them; this, however, is not vital as most of the heat given off from them is in the form of convection currents in the air. (This is why they are shaped to have a large surface-area.)

(3) Vessels which are required to retain their heat are made with polished exteriors, *e.g.* teapots, calorimeters, steam-pipes in engines, etc. A *vacuum thermos-flask* is one of the best examples; it also involves the use of precautions to avoid loss of heat by the other causes discussed in Chapter X.

The principle was used in 1892 *by Sir James Dewar to keep condensed air in the liquid state.* It is now used for keeping liquids hot for a period of 24 hours or so. Fig. 87 shows the following points which should be noted :—

(*a*) Convection is prevented by the use of a vacuum space all round the glass container, and by closing in the top.

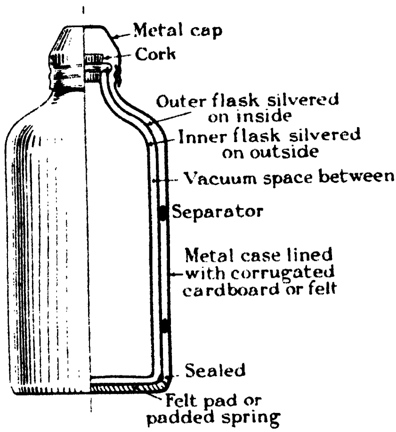


FIG. 87.—Section of a Thermos Flask.

(*b*) Conduction is reduced to a minimum by having the liquid container of a poor conductor (glass), by closing in the top by another poor conductor (cork), and by supporting the outer glass wall by pads of felt.

(*c*) Radiation is reduced as far as possible by the use of polished surfaces. The outer surface of the glass vessel holding the liquid is polished on the outside and so reflects heat back into the interior if the liquid inside is hot. The inside of the outer glass wall is also polished and so reflects back any heat that may be radiated from the inside glass wall, or reflects back heat radiated to the flask from outside sources.

(*d*) Evaporation from a hot liquid inside is prevented by enclosing it in the limited space.

(4) Light suits and flannels, usually worn in the summer, have poor absorbing properties and so are advantageous. It should be noticed, however, that in our endeavours to become cool, great use is made of the power of losing heat generated in the body. Convection is the best aid to this—the air near to one's body becoming heated and passing out. Clothes, particularly under-garments, should therefore be loose and porous, preferably of the cellular type with spaces for air circulation.

An interesting fact, too, is that polar animals are white and thus do not so easily radiate the heat they generate.

(5) The student should carefully think out the methods by which a modern motor-car, or aeroplane, engine is cooled, and particularly for the case of one which is air-cooled. The invention of devices for cooling high-powered engines by air, obviating the carriage of water for cooling purposes, has been an important factor in the fight for supremacy, as regards speed, in air and on land.

(6) *Land and Sea Breezes* have already been mentioned. Water is found to be a bad absorber and bad radiator, whereas rocks, earth, etc., are good absorbers and good radiators. You have no doubt realised, when at the seaside, how quickly the sand and rocks in particular become hot to the touch.

In the daytime the earth, rocks, etc., become heated more quickly than the water, with the result that the air above them gets heated more quickly than the air above water adjacent to the land. Convection currents are therefore produced, air moving in to the land from the sea in what is known as a *sea-breeze*. After sunset the earth radiates its heat much better than the sea near by and so cools down more quickly. Thus it is possible for the air just above the sea to be warmer than the air just above the land, with the consequence that a convection current is set up, air moving from above the land out to sea in what is termed a *land breeze*.

Transmission of Thermal Radiations.—We have

already seen that many surfaces, including those of our own bodies, absorb thermal radiations and become heated. But some substances allow heat radiations to pass through them, *e.g.* rock-salt and fluorspar, and are said to be *diathermanous.* Substances which allow none to pass through are said to be *adiathermanous* or *athermanous.* Melloni studied the diathermancy of solids and liquids, the liquids being contained in tubes with rock-salt ends. A source of heat was placed at one end and a thermopile at the other to detect any transmitted heat radiations. He found that rock-salt and fluorspar are very diathermanous, but most solids transmit very little. Also clear carbon bisulphide transmits about 63 per cent. of the incident heat, but most liquids, especially pure water, are very athermanous.

Tyndall, in 1859, studied gases as Melloni did liquids, and his results and the confirmation of them since are very interesting from the point of view of the effect of the earth's atmosphere on heat radiation from the sun. It has now been established that—

(*a*) pure air is highly diathermanous;

(*b*) aqueous vapour and carbon dioxide present in ordinary air have a well-marked absorbing power. (The behaviour of water vapour was expected from Melloni's results for water.)

The absorbing action by clouds above tends to prevent loss of heat from the earth by radiation and so to maintain an equable temperature. We all know how oppressive it is in summer when there is a low belt of clouds above us. Sandy deserts, mountain tops, and elevated tablelands always cool quickly owing to the absence of absorbing substances, particularly water vapour, above them.

Dew-formation.—The radiation of heat from the surface of the earth is an important factor in dew-formation. Dew is the condensation of moisture of the air on cold surfaces and usually takes place at night or early morning. The formation of a heavy dew is assisted by—

(1) a clear sky which allows rapid radiation from the surface of the earth (heated in the daytime);

(2) a calm state of the atmosphere, so that the air containing much moisture remains stationary. Further, the dew deposition takes place on substances which are (*a*) good radiators, (*b*) bad conductors—so that after they have radiated their heat they are not warmed up by the earth—(*c*) placed near the earth so that the air, warmed by radiation from the soil, does not rise up to the substances. Thus a heavy dew is often formed on grass fields and not on hard roads close by, and also in valleys and not on adjacent hillsides. If an old sack is left lying on a gravel path all night, it is often found to be saturated with dew, whereas the gravel path shows little, if any, signs of a dew.

Law of Cooling.—The radiation from a body has already been shown to depend upon the nature of its surface, whilst the nature of the material is also a factor in its cooling. But the temperature of a body has much to do with the rate at which it loses heat by convection and radiation. You have probably noticed that boiling water poured into a bowl becomes lukewarm very quickly, but remains at that stage of cooling for a very considerable time.

The first detailed study of the cooling of bodies seems to have been made by Sir Isaac Newton, and the result is expressed in **Newton's Law of Cooling,** which states that, **for small ranges of temperature, the rate of loss of heat by a body is proportional to the mean difference of temperature between the body and its surroundings.** This law can be verified quite simply as follows :

Half-fill a calorimeter with water warmed to about 40°-45° C. and stand on a badly conducting surface. By means of a thermometer, read the temperature of the water every two minutes for a period of about 20 minutes, carefully stirring with the thermometer all the time (or hold the thermometer in a wooden clamp and stir with a piece of copper wire made into a loop). Enter the results in a table as follows :

Time in mins.	Temp. of water in °C.	Fall of temp. during 2-minute intervals. = L° C.	Mean temp. of water in 2-minute intervals.	Mean difference of temp. between water and air in the interval (D°C.).	$\frac{\text{Fall of temp.}}{\text{Mean diff. of temp.}} = \frac{L}{D}$ for the interval.
0					
2					
4					
↓					
20					

(Temperature of surrounding air= ° C.)

The results in the last column should be approximately the same, for if Newton's Law is true, $\frac{\text{loss of heat}}{\text{mean diff. of temp. etc.}}$ for equal intervals of time should be the same; for the same mass of liquid, the loss of heat is obviously proportional to the fall in temperature. It must be noted that *this Law only holds for small differences of temperature.*

Specific Heat of a Liquid obtained by Method of Cooling.—By carrying out the above experiment with water, and then with a liquid of unknown specific heat, the latter can be calculated. Equal volumes of the two liquids must be used in the same vessel, so that the conditions are the same—equal surfaces are exposed to the air and to the calorimeter. When this is so, the rates of loss of heat by the two liquids are the same, *i.e.*,

$\frac{\text{heat lost by liquid}}{\text{time taken}}$ = a constant for the same range of temperature fall.

Suppose w grms. of water (sp. ht. taken as 1) and m grms. of liquid of specific heat s cool from 40° C. to 35° C. in t_1 and t_2 seconds respectively. Then heat lost by water $=w(40-35)\times 1$ calories in t_1 seconds; and heat lost by liquid$=m(40-35)\times s$ calories in t_2 seconds. The rates of cooling being equal,

$$\frac{w(40-35)}{t_1}=\frac{m(40-35)s}{t_2}$$

whence $s=\frac{wt_2}{mt_1}$.

Heat and Light Radiations. (***To be read after Light has been studied.***)

The following points regarding thermal and light waves are of interest:

(1) *Both heat and light waves travel through a vacuum.*—Objects can still be seen when in a glass vessel from which as much of the air as possible has been removed. It is possible to detect outside a vacuum, by means of a thermopile, or bolometer, heat generated electrically inside it.

(2) *Heat and light waves travel in straight lines* (see pp. 198, 199).—By using a source of heat, a screen and a thermopile, it can be shown that the screen intercepts all the radiations if placed directly between the source and the thermopile. If, however, a hole is made in the screen in the direct line between them, heat reaches the thermopile. There are also sensitive papers for use in showing heat shadows. The surfaces of these are so prepared that they change in colour when heat falls on them. Fig. 88 indicates their use to show the straight

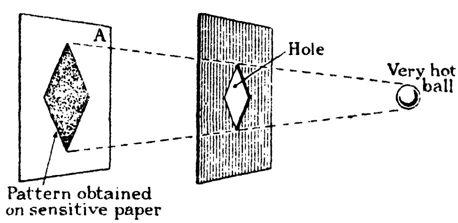


FIG. 88.

line propagation of thermal waves. A may also be tinfoil with a lamp-black coating facing the radiations. On its other side is a wax coating which melts in a pattern as shown, if the source is sufficiently hot.

(3) *The velocity of thermal waves is the same as that of light.*—When a solar eclipse occurs and the light of the sun is obscured, at the same time there is observed a distinct lowering of the temperature.

(4) *Heat and light waves obey similar laws of reflection.*—This can be shown for heat radiations as indicated in Fig. 89, using two long cardboard tubes and a thermopile, or Crooke's radiometer or even an ether thermoscope to receive the radiations. (For light waves, see Chapter XIV.)

(5) *Heat and light waves obey similar laws of refraction.*—When a bright image of the sun is focussed on a piece of paper by means of a lens, the paper is scorched by the heat concentrated at the same time. The shortage of suitable diathermanous

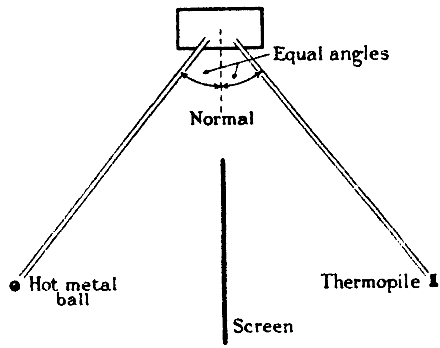


FIG. 89.—Regular Reflection of Thermal Radiations.

materials such as rock-salt makes it difficult to carry out many experiments regarding this.

(6) *The Inverse Square Law holds for heat radiations just as it does for light radiation* (see pp. 207–212).—Tyndall showed this rule for heat radiation in an ingenious manner. The

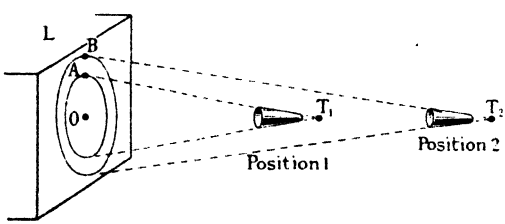


FIG. 90.—Tyndall's Experiment to verify Inverse Square Law for Thermal Radiations.

lamp-black surface of the side of a large Leslie's cube, L, was used to radiate heat, and some was intercepted by a thermopile as shown in Fig. 90. When in position 1, the conical funnel (or receiver) of the pile received heat from the circular surface of radius OA, and, when moved back to position 2, at twice the distance, from the circular surface of radius OB.

Thus $OT_2=2OT_1$ (T_1 and T_2 being the apex of the conical *thermopile (funnel)*).

But triangles AOT_1 and BOT_2 are similar, for AT_1 is parallel to BT_2, and so $\frac{OB}{OA}=\frac{OT_2}{OT_1}=\frac{2}{1}$.

$$\text{Thus} \quad \frac{\text{area of circle of radius OB}}{\text{area of circle of radius OA}}=\frac{\pi.OB^2}{\pi.OA^2}=\frac{4}{1},$$

and so

$$\frac{\text{heat energy radiated from circular area of radius OB, per sec.}}{\text{heat energy radiated from circular area of radius OA, per sec.}}=\frac{4}{1}.$$

But if the inverse square law is true, since $OT_2=2OT_1$, the amount of heat radiation received, from the same source, is inversely proportional to the square of the distance from the source.

∴ Heat radiations from the same source at O would be received at T_1 and T_2 in the ratio of 1 : 4.

But in the experiment the heat sources send out radiations in the two cases in the ratio of 4 : 1, and so the effects at T_1 and T_2 should be the same.

Tyndall verified this result experimentally, thus supporting the correctness of the law assumed, by showing that as the thermopile was moved from position T_1 to T_2 there was no change in the deflection of the sensitive galvanometer connected to the thermopile.

(7) *The transmission of heat radiations by different substances is not necessarily similar to the transmission of light*, i.e. substances transparent to light are not necessarily diathermanous or *vice versâ*. Carbon bisulphide, with iodine suspended in it, is opaque, yet it is appreciably diathermanous.

Glass Hot-houses.—An interesting example of radiation and absorption is that of the hot-house, used for special plants and for forcing growth. In Chapter XVIII it is shown that solar radiations cover a large range of wave-lengths, the longer ones being concerned in the heat energy transmission. Glass is found to transmit only short wave radiations, such as those which are visible and the shorter of the heat radiations. These therefore pass through glass roofs and are absorbed by the plants, earth, etc., inside the structures. The energy is then re-radiated as long heat waves which cannot pass out

through the glass, and so the glass-house becomes warmed and does not quickly lose the heat obtained; hence the name "hot-house."

Panel Warming of Rooms.—A modern method of heating rooms is to distribute heat from surfaces, chiefly ceilings, which are kept heated at about 100° F. by pipes, embedded in plaster, through which water circulates by the usual low-pressure hot-water system.

At 100° F. for a ceiling panel nearly 100 per cent. of heat given off is by radiation
At 100° F. for a wall panel nearly 60 per cent. of heat given off is by radiation
At 100° F. for a floor panel nearly 40 per cent. of heat given off is by radiation.
At 160° F. for radiators about 16 per cent. of heat given off is by radiation.

The panel warming system, particularly for ceilings where the whole of the heat is distributed by radiation, has great hygienic advantages, whilst it removes the necessity for dirt-collecting exposed radiators. With the latter, too, the people nearer them are more heated than those farther away; in a room heated by ceiling panels, the radiant heat travels down continuously and equally over the whole area. A great advantage, too, is that window glass does not conduct the radiant heat to the outside air; neither does it absorb the radiant heat to any degree. Results show that in the case of ceiling panel radiation the floor is about 1° F. warmer than the air just above the floor, the air at the top of the room being about 3° F. higher still.

Exercises on Chapter XII

1. Discuss the three ways in which heat may be transmitted from one region to another, making clear with the help of particular illustrations the essential characteristics of each mode. [*L.G.S.* 1927.]

2. Describe the process of *radiation* and *convection* of heat. What devices would you employ in the case of a vessel containing a hot liquid to reduce the losses by these processes as much as possible. [*L.G.S.* 1929.]

3. Describe the different modes in which heat may be transferred from one place to another. Show how, in the case of a "vacuum flask," heat transference is reduced to a minimum. [*L.G.S.* 1919.]

4. Describe the processes by which a vessel containing a liquid loses heat when brought into a room. What arrangements would you make to diminish the rate of loss of heat? [*L.M.* 1928.]

5. One way of keeping a liquid cool on a hot day is to wrap the containing vessel in a damp cloth; another is to put it in a vacuum flask with silvered sides. Explain the effectiveness of each method. [*J.M.B.* 1926.]

6. Describe and explain two natural phenomena in one of which convection, and in the other radiation, plays a prominent part. [*L.M.* 1920.]

7. Explain why the radiators in a building heated by hot water are usually dark in colour, unpolished and shaped as they are. Describe any experiments you have seen performed which confirm the reasons you have given. [*J.M.B.* 1923.]

8. Radiant heat does not pass through glass. How would you demonstrate this? In summer a greenhouse is at a much higher temperature than the outside air, even if there is no internal heating. How would you explain this statement knowing the first statement to be true? [*J.M.B.* 1922.]

9. Under what conditions are dew, mist and hoar-frost formed? On spreading a woollen rug over a part of a gravel path at night dew will be formed upon the rug far more readily than upon the path. Explain this.

10. Distinguish briefly between conduction, convection and radiation of heat; give an example of each. Explain the part played by each in heating a building by an ordinary hot-water system. [*J.M.B.* 1928.]

11. Steam at 100° C. is blown into a vessel of water at a uniform rate. Explain carefully why you would expect the water ultimately to reach a steady temperature. If 3 grms. of steam are blown in per minute, and the water attains a steady temperature of 80° C., what is the rate of loss of heat? [*C.W.B.* 1929.]

12. Describe experiments to study the power of different substances of transmitting heat radiations. What general results would you expect to find?

13. What is Newton's Law of Cooling and under what conditions does it hold? How would you verify it?

14. Compare the rates of loss of heat of a vessel of water at 50° C. and at 40° C., the temperature of the air being 13° C. Describe how to test any relationship you assume.

15. Explain the meaning of the statement that the specific heat of paraffin oil is 0·52, and describe how you would determine this quantity experimentally, indicating the precautions necessary to obtain an accurate result. [*L.G.S.* 1921.]

16. Describe experiments or observations to show that radiant heat obeys the same laws of propagation as light. [*L.M.* 1925.]

Miscellaneous Exercises.—I. Heat

1. Explain how the fixed points of a thermometer are determined. How could a thermometer be used to find (*a*) whether the atmospheric pressure were above or below the normal, (*b*) whether a given sample of water were free from impurities or not.

2. State the *chief* advantages and disadvantages of mercury and alcohol for use as thermometric liquid. Absolute zero on the Centigrade scale is $-273°$. Express this on the Fahrenheit scale, justifying any formula you use. [*L.G.S.* 1928]

3. What is meant by a *degree of temperature?* How are the three scales of temperature (Fahrenheit, Centigrade, "Absolute") obtained? Express 59° Fahrenheit in the "absolute" scale. [*L.G.S.* 1923.]

4. State and explain three *important* reasons why water is not a suitable liquid to use in the construction of a thermometer. The cold water supply to a geyser is at a temperature of 50° C. Express the rise of temperature of the water (*a*) on the Centigrade scale, (*b*) on the Fahrenheit scale. (A graphical solution will be accepted.) [*L.G.S.* 1927.]

5. What properties should a liquid possess to be suitable for use in a thermometer? Compare the relative advantages and disadvantages of alcohol and mercury for this purpose. [*L.G.S.* 1922.]

6. Compare the thermal properties of mercury with those of water, briefly indicating the evidence on which your statements are based. [*J.M.B.* 1927.]

7. Write a short account of the changes of dimensions which take place in solids, liquids and gases when their temperatures are altered.

8. Define the coefficient of linear expansion of a solid. A barometer with a brass scale reads 29·72 ins. on a day when the temperature is 59° F. If the scale graduations are correct at 0° C., what is the true height of the barometer at this time? (Coefficient of linear expansion of brass=0·000019 per ° C.) [*L.G.S.* 1924.]

9. Define *coefficient of expansion*, and explain the relation between the linear and cubical coefficients. How would you find the coefficient of expansion of turpentine? [*C.W.B.* 1927.]

10. How may the coefficients of expansion of (*a*) air, and (*b*) water be determined experimentally? [*L.G.S.* 1922.]

11. How is the volume of a *perfect gas* affected by changes of (*a*) pressure, (*b*) temperature? 20 c.c. of air at 15° C. are enclosed in a tube connected with a pressure gauge. The pressure of the enclosed air is first increased by 50 per cent. and then its temperature is raised to 100° C. What will be the volume of the air under the new conditions? [*J.M.B.* 1922.]

12. What is meant by a perfect gas? Describe the apparatus used for studying the behaviour of air when heated at C.V. Explain how you would get a range of observations from 0° C. to 50° C. and state what results you would expect to find.

13. What is meant by the temperature coefficient of increase of pressure in the case of a gas? What evidence does the behaviour of gases give for the existence of an absolute zero of temperature at $-273°$ C. Find the rise in temperature required to double the pressure of a quantity of gas kept at constant volume if the original temperature is 20° C. [*L.M.* 1926.]

14. How would you construct a simple mercury barometer, and employ it to measure the pressure of the air? On a day when the barometric height was 76 cms. of mercury, the difference in levels of some water held in a U-tube connected to an open gas pipe was 10 cms. Calculate the pressure of the gas in grms. per sq. cm., the specific gravity of mercury being 13·6. [*J.M.B.* 1926.]

15. Heat is applied at a uniform rate to a block of ice until it is converted into steam. Give a *brief* statement of the changes in volume, condition and temperature which take place, and of the relative times occupied in the various stages of the process. [*L.G.S.* 1923.]

16. Explain the meaning of capacity for heat, specific heat and water equivalent. Describe fully how you would measure the specific heat of turpentine. A calorimeter contains 300 grms. of water and is heated by a steady flame, the temperature rising 10° in 4 minutes. Under the same conditions, but with 400 grms. of water, it required 5 minutes for the same rise of temperature. Find the water equivalent of the calorimeter. [*C.W.B.* 1926.]

17. Show by a curve (not necessarily to scale) the relation between volume and temperature of water between limits −10° C. and 10° C. Taking the specific heat of ice as 0·5 and the latent heat of water as 80, calculate the quantity of heat required to produce the above changes in temperature in 10 grms. of water substance.

18. A book of tables gives the following information about aluminium: coefficient of expansion=0·000023 per degree C., specific heat=0·23, melting point=658° C., latent heat=107 calories per gram. Explain the meaning of these data. Calculate the amount of heat required to melt 100 grms. of aluminium, starting from 18° C. [*J.M.B.* 1927.]

19. Some dry ice at 0° C. is placed in a copper can and 10 grms. of water at 100° C. are poured upon it. Assuming that all the heat given out by the water is taken up by the ice, how much ice must be taken for it just to be melted? What would have been the result if, the quantity of ice being the same as before, twice as much water had been poured upon it?

20. A mixture of 250 grms. of water and 50 grms. of ice is heated in an open vessel till it is all converted into steam at atmospheric pressure. State what you know concerning the changes in volume and temperature which occur. Determine the total amount of heat necessary, given that the latent heat of fusion of ice is 80 and the latent heat of steam 539 in centigrade units. [*L.G.S.* 1926.]

21. Define the latent heat of steam at any temperature. Determine its numerical value from the following results: 10 grms. of dry steam at 100° C. were passed into 300 grms. of water at 10° C., contained in a calorimeter of mass 50 grms. and specific heat 0·1. The final temperature was 30° C. Sketch the apparatus used in this experiment and point out what steps must be taken to obtain a satisfactory result. [*L.G.S.* 1927.]

22. What do you understand by the term *latent heat of water?* It is often noticed that the temperature of the air rises after a fall of snow. Can you account for this? Steam at 100° C. is passed into a vessel, whose water equivalent is 100 grms., containing 2 kilograms of water at 15° C. until the temperature rises to 60° C. What weight of steam would you expect to have been condensed? Latent heat of steam=540. Assume no heat to be lost or gained by radiation, and pressure is normal. [*J.M.B.* 1922.]

23. Taking the latent heat of fusion of lead as 5·0, its specific heat as 0·030 and its melting point as 330° C., find the temperature to which 5,000 grms. of water at 12° C. will be raised by pouring into it 200 grms. of molten lead at its melting point. [*L.M.* 1921.]

24. When a piece of substance at ordinary air temperature is lowered into liquid air, the latter boils. Explain this. A piece of aluminium weighing 5 grms. wt. and at 18° C. is dropped into liquid air and 2·32 litres

of air, at 18° C. and 75 cms. of mercury pressure, boil off. Find the mean specific heat of aluminium. The boiling point of air is −192° C., its latent heat of vaporisation is 50, and its density at N.T.P. is 1·30 grms. per litre.

25. Define the terms *calorie* and *erg*. Describe some simple method for obtaining an approximate numerical relation between these two units. [*L.M.* 1922.]

26. Heat is said to be a form of energy. What evidence can you give in support of this statement? What do you understand by the mechanical equivalent of heat? [*J.M.B.* 1927.]

27. Show the method of calculation you would adopt and the data you would require to solve the following problem: a 56-lb. weight is dropped from a height of 100 ft. into a drift of snow at the melting point. What weight will be melted? A numerical answer is not required. [*J.M.B.* 1927.]

28. Two short cylinders of the same size, one of copper and the other of lead, are fixed at one end to a hot plate, kept at a uniform temperature, while their other ends are coated with wax. It is noticed that the wax on the lead cylinder melts before that on the copper. When, however, long cylinders of the same materials are substituted for the short ones, only the wax on the copper cylinder is melted. How do you account for this? [*L.G.S.* 1924.]

29. Describe some simple experiment which shows that water is a bad conductor of heat. Point out the bearing of this fact upon the temperature of the bottom of a deep pond at different seasons of the year. Indicate any other factor which may have a considerable influence on this temperature. [*L.G.S.* 1920.]

30. Describe an experiment to show that water is a bad conductor of heat, and explain how it is that water is nevertheless used in central heating systems for conveying heat from the furnace to the rooms to be warmed. [*L.M.* 1926.]

31. A vessel of hot water can lose heat in four different ways. Explain the methods by which these losses may be diminished. [*L.G.S.* 1923.]

32. Describe arrangements (*a*) for supplying heat from one part of a building to another, (*b*) for preventing the escape of heat, pointing out in each case the methods by which conduction, convection and radiation are helped or hindered. [*L.M.* 1921.]

33. Give a brief account of the characteristic physical properties of saturated and unsaturated vapours. What is the saturation pressure of water vapour at 100° C.? How could you verify your statement? [*L.M.* 1920.]

34. Give a short account of the chief phenomena occurring in the change of state from solid to liquid. [*L.M.* 1920.]

35. What is the cause of dew? Why are a previous hot day, a still night and a cloudless sky favourable to the formation of dew?

LIGHT

CHAPTER XIII

LIGHT PROPAGATION—SHADOWS AND ECLIPSES—ILLUMINATION AND PHOTOMETRY

IF you refer to a dictionary for a meaning of "light" you will read that it is "that by which we see." The study of light, then, should be of interest since it would tell us something of a vital sense which most people possess—a sense which many consider the most important; though a good subject for debate is whether sight is more valuable than hearing. By the former we learn much of the material world around us; by speech and hearing we communicate with other human beings and learn much that sight alone cannot teach.

As we have already learned, light is a form of energy which comes to us through space. It does so by transverse waves in a non-material substance, the æther of space being the medium. The word "light" is often used to mean

(*a*) the sensation experienced by a part of our body,
(*b*) a group of phenomena—the action and behaviour of light energy. We shall study these.

As we all know, our source of energy in general and of light in particular is the sun. This huge mass of burning material is continuously sending out light and heat energy, some of which is received by other bodies—certain of them being cold—in the sun's region of influence, which is called the Solar System. Certain other bodies—the stars, for example—are also very hot and are themselves sources of light, such bodies being said to be *self-luminous*. (L. *lumen*, light.)

The planets, which we see in the heavens, are not self-

luminous though they are luminous, *i.e.* they can be seen. We see them by the light of the sun, which they send to us by a process of reflection—just as cliffs, etc., reflect sound energy. We shall study the process of reflection of light in the next chapter.

To a certain extent we imitate the sun in many of our terrestrial forms of light—substances being made so hot that they become incandescent (L. *incandescere*—to grow white), *i.e.* they radiate light energy as well as heat energy. Bodies receiving light from luminous objects are said to be illuminated.

The Path of Light Energy.—It is found that when light is travelling in a continuous medium, *i.e.* a medium of the same density throughout, it does so, under ordinary circumstances, along straight lines. This is known as the **rectilinear propagation of light.** But it is not always so, for waves can bend round corners to a certain extent, and there are other influences which may affect the path. However, for terrestrial work, which concerns us, it is quite true to say that light travels in straight lines. In connection with this theory, the following phrases have come into use :

(*a*) *Ray of light*—the straight line path along which light travels from a point in a luminous object.

(*b*) *Pencil of rays* (or a *beam*)—a collection of rays which are proceeding from or towards a point. Thus a pencil may be *divergent* when the rays are proceeding from a point, *convergent* when they are proceeding to a point, or *parallel* when they are proceeding from or to infinity or, for practical purposes, a very distant point. Thus the sun's beam at the surface of the earth is considered to be a parallel beam. This can be seen by shaking a duster in a room when the sun is shining through a window. The fine particles of dust are seen in the rays of light, and the paths of the rays appear to be all parallel.

It is quite simple to show that light does travel in straight lines in a limited space. Take three pieces of cardboard, A, B, C, each with a fine hole through the centre, and fix these boards in clamps so that the fine holes are in the same

straight line (Fig. 91), stretching a piece of fine thread through the holes. Carefully remove the cotton without moving any board and place a source of light S behind A, then look through from the outside of C, at E, when light is seen passing through the holes. Slightly move either of the cardboards and no light is seen. It can be shown that light from S only reaches E directly when the three holes are exactly in the same straight line.

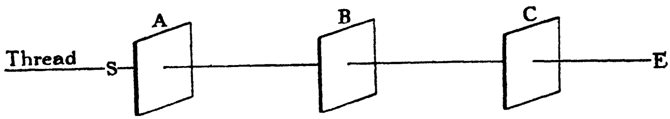


FIG. 91.

Shadows.—*Further evidence in support of our acceptance of the "straight line propagation of light" theory is seen in the formation of shadows.*—Any opaque object (*i.e.* one that does not allow light to pass through it) placed in a beam of light intercepts some of the rays. Thus a sheet of paper, or a screen, which was illuminated

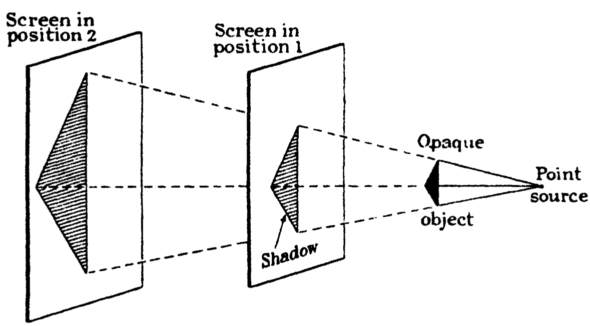


FIG. 92.

by the beam is deprived of some of the light when the opaque object is put between the screen and source of light—the portion not illuminated is known as a shadow. The nature of the shadow varies with the size of the source of light. When the source is very small, so that the rays of light can be considered to radiate from

a point (hence the phrase "point source"), the shadow is distinct and of the same shape as the object intercepting the rays, but its size depends on the relative positions of the source, object, and screen (Fig. 92).

When the source is not a point or is appreciable in size compared with the size of the object intercepting the rays, the following happens. In Fig. 93 S is a source from which light energy is travelling out in all directions. A and B, and C and D are the top and bottom points of the source and intercepting object, O,

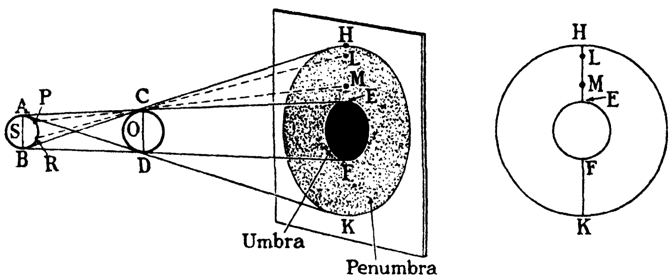


FIG. 93.

respectively. Rays travelling between ACE and BDF are completely stopped by the object, not only in the plane of the paper but in other planes, and thus there is a zone of complete darkness, known as the umbral zone, and a screen intercepting light from S will show a dark space of the same shape as object O. This full shadow, or *umbra* as it is called, is indicated in the figure by blackness. In the figure, BC and AD are produced to meet the screen at H and K respectively. Points outside H and K receive light not affected by the presence of O and so are fully illuminated. Between H and E, and also between K and F, there is an interesting effect, M, which is near the dark point E, and L, which is near the bright point H, are joined to C (in Fig. 93) and produced back to meet the source of light at P and R respectively. Thus point M only receives light from the portion of the source between A and P and thus is weakly

illuminated. L receives light from the portion of the source between A and R and so is nearly fully illuminated. But since the portion between E and M is nct fully illuminated, and is thus in partial shadow, we can say that the shadow is deeper at M than at L. Thus there is a gradual change from full darkness at E to full illumination at H. This, of course, is happening in other planes, and so there is a partial shadow round the umbra, of the same shape as the object. This partial shadow, called the *penumbra* (L. *pæne*, almost+*umbra*, shadow), gradually changes from full density at E to absence of shadow at H. According to the relative sizes of S and O and their position with regard to each other and the screen, so the relative sizes of the umbra and penumbra vary. It is advisable to study this carefully at home, using a gas or electric lamp as a source, a ball or other solid as an intercepting object, and a piece of paper, or the wall, as a screen. To render the phenomena more clear, it is advisable to surround a great part of your source so as to reduce the general illumination of the room.

Eclipses.—The above explains the phenomenon of an eclipse, either of the sun or of the moon. The moon goes round the earth in an orbit once in 28 days approximately, whilst at the same time the earth goes in its orbit round the sun in 365 days approximately. It is thus possible for the moon to pass between the sun and the earth, so intercepting the sun's rays, which otherwise would reach the earth. Thus some part of the earth at that time is in shadow due to the moon's interposition, and the sun is said to be eclipsed. In Fig. 94 S represents the sun, M the moon, and E the earth. Thus a person on the earth between A and B would see nothing of the sun, *i.e.* there would be a *total eclipse;* a person between A and C or between B and D would only see part of the sun, *i.e.* there would be a partial eclipse. The earth is also revolving on its own axis once in 24 hours and so an eclipse does not last long at any place, the line AB, for example, on the earth moves into the shadow and then out of it. (At

the same time, of course, the moon is moving, and the earth is moving in its orbit round the sun, so that the moon is soon out of position to shut off the sun's rays from the earth.)

It would appear that the moon should pass between the sun and the earth once every 28 days. This would be so if the moon's orbit round the earth and the earth's orbit round the sun were in the same plane. But the orbital planes are slightly inclined and so there are only two points in the moon's orbit which are in the

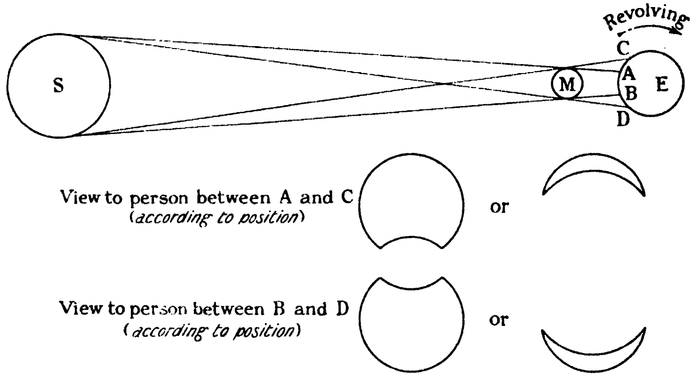


FIG. 94.—Eclipse of the Sun.

same plane as the earth and the sun. It is thus very rare for one of these points to be exactly between the sun and earth, and so eclipses of the sun are uncommon. Another factor that makes eclipses rare (and total eclipses particularly so) is that, owing to the positions and sizes of the sun and moon, the shadow caused by the moon is approximately equal to the average distance of the moon from the earth. But the moon's orbit, like that of the earth, is elliptical, so that at one period the moon is nearer the earth and at another it is farther away. When the moon is nearer the earth, the shadow reaches the earth and a total (and a partial) eclipse is possible as shown in Fig. 94, but, even then, the total eclipse rarely lasts more, and often

less, than one minute at a place—the shadow quickly moving over. But when the moon is farther away from the earth, then the shadow does not reach the earth, and an observer at X in Fig. 95 sees the central portion of

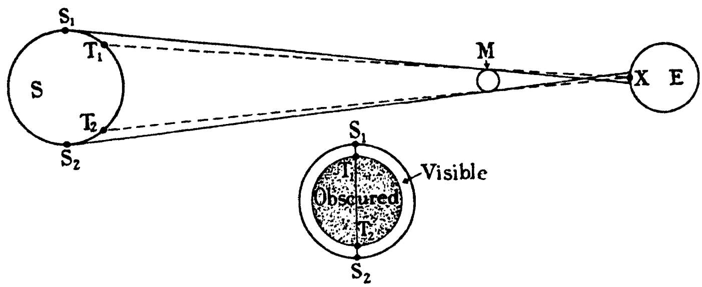


FIG. 95.—Annular Eclipse of the Sun.

the sun between T_1 and T_2 obscured, but he sees the outer ring of the sun. This is known as an *annular eclipse* (L. *annulus*, a ring, and must not be confused with "annual"). Such an eclipse is rare.

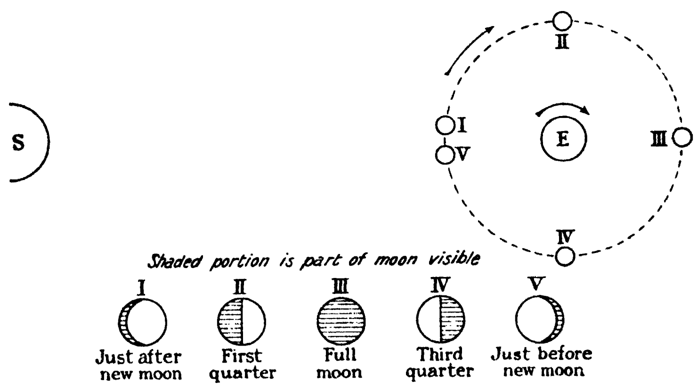


FIG. 96. –Phases of the Moon. (Note that an observer on the earth E is rotated once in 24 hours and so can see these effects.)

It should also be noted that the moon is not self-luminous; it merely reflects light of the sun to us. It therefore follows that the moon, when it comes between

the sun and earth, is reflecting little light from the sun to the earth, and so is faintly, if at all, seen. The moon is then said to be "new." As the moon rotates round the earth, more and more of the moon is reflecting light from the sun, and so more of the moon is seen, until after fourteen days the whole of the one side of the moon facing the sun is reflecting light to the earth, *i.e.* the moon appears round and is said to be "full." Gradually there is the reversal of the change till new moon happens again (cycle shown in Fig. 96).

It should be clear that *a total or partial eclipse of the sun is only possible at, or near, new moon. Eclipses of the moon* are produced when the earth is between

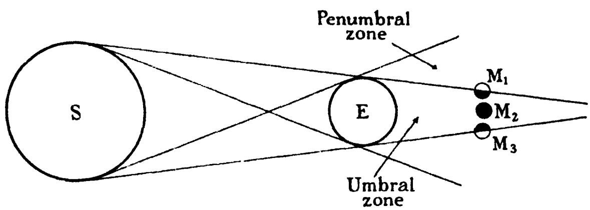


FIG. 97.—Eclipses of the Moon.

the sun and the moon and so prevents the moon receiving and reflecting light from the sun to the earth. This is shown in Fig. 97. In positions M_1 and M_3 part of the moon is in the penumbral zones and so the eclipse is partial. When the moon is in M_2 position, it is in the umbral zone and so the eclipse is total. It should also be noted that the occurrence of a lunar eclipse can only be at or about the period of full moon.

The principle that light travels in straight lines is used in the construction of the **pinhole camera.** Take a box and pierce a small pinhole P in the centre of one end, leaving the opposite end open. Inside fix a screen of tissue paper or ground glass parallel to the opening and about one-quarter of the length away from it (Fig. 98). Rays of light from an object AB, such as a window, proceed as shown in the diagram and give an inverted representation A′B′, known as an *image*, or likeness, of the bright

object. It is interesting to have the pinhole camera made of two tubes, the screen being fixed to one and the pinhole in the other. The effect of varying the distances between hole and screen can be seen (and should be tested by drawing and compared with the results of the experiments on umbra and penumbra formation, on p. 200, etc.).

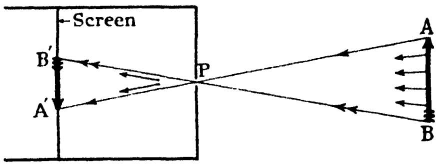


FIG. 98.—Pinhole Camera.

Try also enlarging the pinhole and observe that when it is much larger than $\frac{1}{8}$-inch diameter, the image becomes blurred, though brighter, as shown in Fig. 99, rays from A proceeding to the screen between T and Q, whilst rays from B reach the screen between R and S. Rays from anywhere along AB behave similarly, thus giving

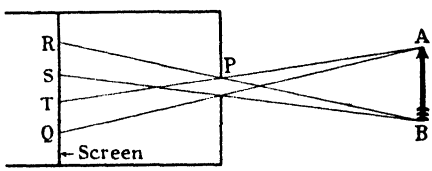


FIG. 99.

an indistinct image, or, as we say, poor definition. In the early days of photography, before the advent of the modern camera, the pinhole camera was used successfully with a small pinhole and a sensitive plate instead of the screen.

Photometry (from Gk. *photos*, of light+*metron*, a measure).—Owing to the development of the use of artificial lighting in the home and in the street, a subject which has come to the fore in recent years is that of photometry, which deals with the brightness or intensity of

light sources and of lighted surfaces. In our study of this we shall consider chiefly—

(*a*) the *illuminating power* of a source of light;
(*b*) the *intensity of the illumination* of a surface due to a source of light.

Illuminating Power.—Sources vary in their power of illuminating surfaces, and we endeavour to compare the illuminating powers of different sources. For this purpose we need a standard of comparison, *i.e.* a unit, and much time and money have been spent in an endeavour to obtain a satisfactory one. So far, however, success has not been complete, and we still base our work on the *candle-power* (or 1 c.p.) which was set up under the Metropolitan Gas Act of 1860 as the official standard for the purpose of testing London gas. It was defined as the illuminating power of a standard candle—a sperm candle, six candles weighing 1 lb., which burned 120 grains per hour. Other units which have been suggested and used are:

(*a*) Vernon Harcourt's Pentane Lamp (1877) which, for practical purposes, superseded the standard candle, and was equal to 10 c.p.

(*b*) The German Hefner lamp (1884) using amyl acetate.

(*c*) The French standard—the Carcel lamp, burning colza oil.

(*d*) The Violle Platinum unit (1881), a surface of 1 sq. cm. at its melting point, and which was accepted at the International Standards Congress, 1890. In recent years, however, this has been found to vary with atmospheric conditions, and better results have been obtained by using 1 sq. cm. of a totally black surface maintained at a temperature of the melting point of platinum.

All of the above present difficulties with regard to the purity of the combustible substances, wick adjustments in the lamps, air variations in moisture and carbon dioxide present. Thus electric lamps are used as "secondary standards," being based on the primary "international" candle-power, the power of an old

standard candle. Obviously electric lamps are more convenient as a standard, providing that electrical conditions are maintained uniform, for they can be carried with safety and the objections regarding wick and atmospheric conditions do not hold. In using the lamps, a public electric supply cannot be employed owing to its variation. An accumulator, which has been used for some time to work it into condition, should be used when it is discharging at 2 volts per cell (and can then be depended on to keep steady at that electric pressure).

Intensity of Illumination of a Surface.—A surface receives illumination from a source of light, and appears bright or dark according to its reflecting (or absorbing) power and also its colour. But apart from this, the intensity of illumination depends on (1) the illuminating power of the source of light—the more powerful the source, the more intense is the illumination; (2) the distance of the surface from the source of light. With regard to the latter, it was shown on p. 173 that when energy is being radiated in all directions, the "Inverse Square Law" holds. Thus **the intensity of illumination at a point varies inversely as the square of the distance from the source of light.**

Thus the intensity of illumination at a point $\propto$ illuminating power of source (I),

and $$\propto \frac{\mathrm{I}}{(\text{distance, } d)^2}$$

We choose our units so that intensity of illumination $= \dfrac{\mathrm{I}}{d^2}$

In English units we measure I in candle-power and d in feet; the intensity of illumination is then in *foot-candles*.

In C.G.S. units we measure I in candle-power, and d in cms. or metres; the intensity of illumination is then in *cm.-candles*, or *metre-candles*.

In the English system, unit intensity is thus at 1 foot from the standard candle, whilst at a distance of 10 feet from a source of 50 c.p. the intensity of illumination

$$= \frac{\text{candle-power}}{\text{distance}^2} = \frac{50}{10^2} = \tfrac{1}{2} \text{ foot-candle.}$$

It must, of course, be noticed that these relationships only hold for light energy radiated in all directions and for a surface perpendicular to the rays. They therefore do not hold for searchlights where devices are used to confine the energy in an almost parallel beam, whilst in the case of a surface not perpendicular to the rays a correction must be made. This will be treated at the end of this chapter. For school classrooms on the reading plane the standard necessary is now being accepted as 5·5–6·0 foot-candles.

Photometric Measurement.—Consider two surfaces, A and B, illuminated respectively by sources of illuminating powers I_1 and I_2 (Fig. 100) at distances d_1 and d_2.

Then intensity of illumination at A due to $I_1 = \frac{I_1}{d_1^2}$ units

and intensity of illumination at B due to $I_2 = \frac{I_2}{d_2^2}$ units.

When these are equal $\frac{I_1}{d_1^2} = \frac{I_2}{d_2^2}$ or $\frac{I_1}{I_2} = \frac{d_1^2}{d_2^2}$,

i.e. the ratio of the powers is equal to the ratio of the squares of their distances from the surfaces where illumination is the same.

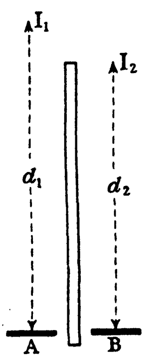


Fig. 100.

This, then, gives us a method of comparing the illuminating powers of different sources—providing we can find when two surfaces are equally illuminated. The eye has to be used to tell when they are, the positions of the sources being adjusted to obtain this effect. A device set up to enable the operation to be carried out is called a **Photometer.** In working, it will be found that a constancy of results is not obtained owing to the following sources of error:

(*a*) The sensation produced is not proportional to the light falling on to the eye, *i.e.* to the intensity of illumination.

(*b*) The eye tends to adjust itself, by movement of

the iris, as light intensity values change (*e.g.* the eye of a cat).

(*c*) The eye becomes fatigued.

(*d*) The illuminations are often differently coloured owing to differences in the sources.

(*e*) There is no permanent standard of comparison.

These difficulties are fairly obvious in most of the types of photometers described

(1) **Bunsen's Grease-Spot Photometer.**—A grease-spot on a piece of absorbent paper appears dark, for it reflects little light compared with the white paper around it. But when held up to a bright light, a grease-spot appears to be lighter than the surrounding paper, for it transmits light, whereas the paper does not. Bunsen mounted a piece of paper, with a grease-spot in the middle, and held it between two sources of light. He moved the photometer (the mounted grease-spot) to and fro till the latter was in such a position that

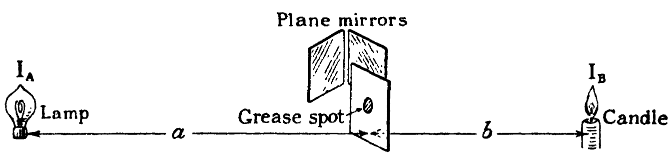


FIG. 101.—Using Bunsen's Grease-Spot Photometer.

it appeared like the rest of the paper. Naturally, the photometer was viewed from one side and so the intensity of the light from one source transmitted by the grease-spot was equal to that of the light from the other source reflected from the white paper. Thus the intensities (at the photometer) due to the two sources were equal. Thus in Fig. 101, $\frac{I_A}{a^2} = \frac{I_B}{b^2}$, and so the ratio $\frac{I_A}{I_B} = \frac{a^2}{b^2}$.

In carrying out such an experiment a series of readings should be taken and entered in a table, each time altering the distance $a+b$ and then carefully moving the photometer to the correct position. Readings should be taken viewing from both sides of the photometer in turn, the mean value being obtained. If possible, it should be done in a darkened room to avoid errors due to the unequal effects of other light sources. Often, when using this photometer, two mirrors are placed, as

shown in the diagram, so that both sides of the photometer can be seen at once. This obviates the necessity of working from each side in turn.

Table for results (with this and other photometers).

Distance of source $I_A = a$ cms.	Distance of source $I_B = b$ cms.	$\frac{I_A}{I_B} = \frac{a^2}{b^2}$

Mean value =

(2) **The Ritchie Wedge Photometer** (1826).—In this form, two dull surfaces MM are arranged, as shown in Fig. 102, so that they diffuse light, from the two sources I_A and I_B which are to be compared, on to a piece of translucent paper P. The positions of the two sources are so adjusted that the two patches of reflected light, close together at P, appear equally bright. Unglazed paper or a plaster-of-Paris surface was used for MM.

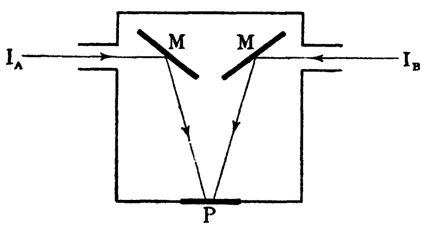


FIG. 102.—Ritchie Wedge Photometer.

The Conroy photometer (1883) is also a modified Ritchie photometer. Two surfaces of dull white celluloid (rubbed with pumice), or dull white paper which has been wetted and then dried, are used in this form. These, A and B, are arranged as shown in Fig. 103 at about 55° with the incident light, so that

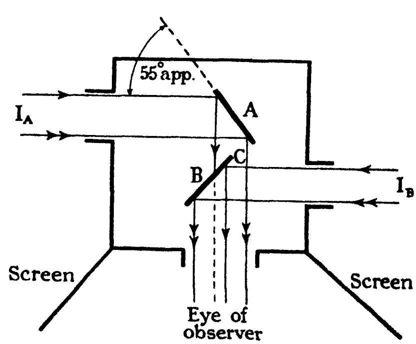


FIG. 103.—Conroy Photometer.

no light is received at the eye by regular reflection (see p. 220) due to any polish left on the surfaces. When the two surfaces are equally illuminated, the edge C of B is lost in the background of A.

(3) **Joly's Paraffin Wax Slab Photometer** consists of two pieces of paraffin wax (from $\frac{1}{4}$ to $\frac{1}{2}$ inch thick) separated by a piece of tinfoil which serves to reflect back the light from sources placed on either side. When the intensities of illumination at the photometer due to the two sources are equal, the pieces of wax on both sides of the tinfoil appear equally bright; otherwise, one piece appears darker. The slabs should be cut from one large piece of wax, so that they are of the same colour.

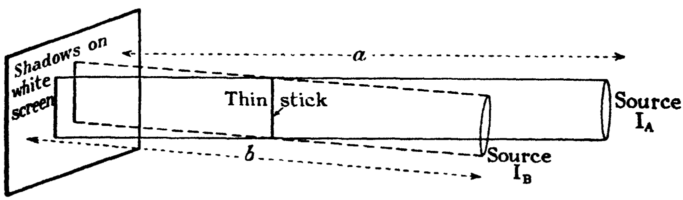


FIG. 104.—Using Rumford's Shadow Photometer.

(4) **Rumford's Shadow Photometer (1792).**—With this the two sources of light to be compared are placed behind a stick, placed in front of a white screen, so that each source throws a shadow of the stick on the screen, but illuminates the rest of the screen except where it casts the shadow (Fig. 104). The sources are moved until the shadows (arranged very close together for ease of comparison) are of equal intensity. But each shadow, due to the fact that the stick cuts off rays from one source, is illuminated by the other source. Hence, when the shadows are of equal intensity the screen, at the shadows, is equally illuminated by the sources, and so the illuminating powers of these sources are directly proportional to the squares of their distances from the screen, and *not* from the stick. Thus $I_A/I_B = a^2/b^2$. If the sources are of different colours (or even shades of colour) there is great difficulty in deciding when the shadows are of equal intensity. An advantage of this method is that it can be used outside a dark room, for external sources of light affect equally both shadows.

(5) **The Flicker Photometer (1896)** alternately presents to the eye the two surfaces to be judged, and at such a speed that the eye cannot separate them (owing to "persistence of vision" in the eye for approximately one-tenth of a second). Thus a uniformly light surface is seen when the two surfaces are equally illuminated, and a flicker when they are not so. Commonly one surface M, in the form of a cross, is rotated by a motor in front of a fixed surface F (Fig. 105). The observer looks through a tube on to X and sees the surfaces, illuminated by the sources, alternately. One source is moved till no flicker is discernible.

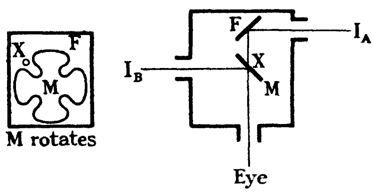


FIG. 105.—Flicker Photometer.

It is left as an exercise for the student to work out how it is possible by photometric methods to—

(*a*) verify the Inverse Square Law ;

(*b*) verify the fact that the intensity of illumination at a point is directly proportional to the illuminating power of the source ;

(*c*) measure the percentage of light reflected from a surface, or transmitted by a plate of transparent substance.

Numerical Examples :

EXAMPLE 1, which introduces the *Cosine Law* (or Lambert's Law).

A lamp is 3 ft. directly above one end of a 4-ft. table.

Compare the intensities of illumination at the two ends of the table.

The difficulty here is that one end of the table receives the light normally (*i.e.* the light falls perpendicularly on it), but the other does not, and so its illumination is reduced. *Lambert*

worked out this problem as follows. In Fig. 106 surface ABCD is illuminated by light falling on it at an angle of incidence θ°. Suppose ABCD to be a hole so that the light, previously falling on the surface there, passes through and falls on a surface which is normal to it. The area illuminated (A′B′C′D′) by the same light is smaller than that on the inclined surface ABCD, and so the intensity is greater.

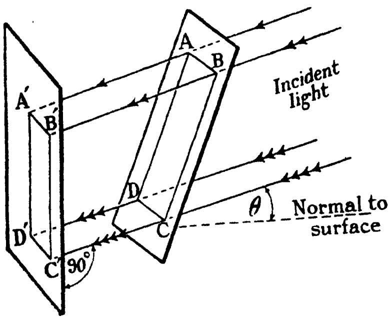


FIG. 106.

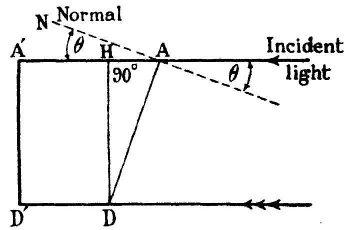


FIG. 107.

In Fig. 107, DH is drawn perpendicular to the incident light.

Thus $N\hat{A}A' = \hat{\theta} =$ the complement of $H\hat{A}D = H\hat{D}A$

$$\therefore \cos\theta = \cos H\hat{D}A = \frac{HD}{AD} = \frac{A'D'}{AD}, \quad \text{or} \quad AD = \frac{A'D'}{\cos\theta}$$

It is obvious that, in the figure, $A'B' = AB$ and $C'D' = CD$.

Thus area

$$ABCD = AD.AB = AD.A'B' = \frac{A'D'}{\cos\theta}\,A'B' = \text{area}\ \frac{A'B'C'D'}{\cos\theta}$$

or
$$\frac{\text{area } A'B'C'D'}{\text{area } ABCD} = \cos\theta.$$

Suppose surface A′B′C′D′ is moved so that, although still normal to the incident light, D′C′ coincides with DC.

Then intensity of illumination at surface A′B′C′D′

$$= \frac{\text{I of source.}}{(\text{distance})^2}$$

and so intensity of illumination at surface ABCD (the light which was falling on A′B′C′D′ now falling on ABCD)

$$=\frac{\text{I of source}}{(\text{distance})^2}\times\frac{\text{area A}'\text{B}'\text{C}'\text{D}'}{\text{area ABCD}}$$

$$=\frac{\text{I of source}}{(\text{distance})^2}\cdot\cos\theta.$$

It must be noticed, however, that θ=angle of incidence of the light on the surface

=(90°—angle between the surface and the incident light).

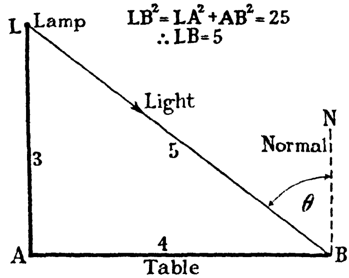


FIG. 108.

Hence, in the example, if I=illuminating power of lamp in c.p., intensity of illumination on table at A directly under lamp

$$=\frac{I}{3^2}\text{ foot-candles,}$$

and intensity of illumination at other end (see Fig. 108)

$$=\frac{I}{LB^2}\cdot\cos\theta=\frac{I}{LB^2}\cdot\cos A\hat{L}B=\frac{I}{LB^2}\cdot\frac{AL}{LB}=\frac{I\times3}{5^3}\text{ foot-candles.}$$

$$\therefore\text{ Ratio of intensities}=\frac{\frac{I}{3^2}}{\frac{I\times3}{5^3}}=\frac{5^3}{3^3}=\frac{125}{27}=\underline{4{\cdot}63}$$

EXAMPLE 2.—Compare the intensity of illumination produced by a 50 c.p. lamp at a distance of 6 ft. with that produced by a 100 c.p. lamp at 8 ft. At what distance should the latter

be placed so as to give the same intensity of illumination as the former? (*L.G.S.* 1924.)

1. Intensity of illumination at 6 ft. due to 50 c.p. lamp

$$=\frac{50}{6^2}=\frac{50}{36} \text{ foot-candles.}$$

2. Intensity of illumination at 8 ft. due to 100 c.p. lamp

$$=\frac{100}{8^2}=\frac{100}{64} \text{ foot-candles.}$$

$$\therefore \text{Ratio } \frac{1}{2}=\frac{50/36}{100/64}=\frac{50}{36}\times\frac{64}{100}=\frac{8}{9}$$

Suppose the 100 c.p. lamp must be at x ft. to give the same illumination as 50 c.p. lamp, *i.e.* 50/36 foot-candles.

Then the intensity of illumination at x ft. from 100 c.p. lamp

$$=\frac{100}{x^2} \text{ foot-candles.}$$

$$\therefore \frac{100}{x^2}=\frac{50}{36} \text{ or } x^2=72, \text{ whence } x=6\sqrt{2} \text{ or } \underline{8{\cdot}48 \text{ ft.}} \text{ approx.}$$

Modern illuminating engineering treats the subject of photometry more fully, considering the characteristics—

(1) brightness,
(2) colour,
(3) the *flux*—the flow of light energy from the source in all directions,
(4) the intensity of the light in one direction, and which we have already fully studied as the intensity of illumination at a point on a surface.

(1) **Brightness,** or brilliance.—For this we consider the illuminating power of a source and the illumination of a surface placed normally.

It is measured in candle-power per sq. in. or sq. mm.

In dealing with illumination we consider the light falling on a surface, but in dealing with brightness we consider the

light returned from or emitted by a surface. Some approximate values of brightness are given below.

Sun: 800,000 candles per sq. in.	Electric arc: 100,000 candles per sq. in.
Good searchlight: 400,000 candles per sq. in.	Paraffin lamp: 5–8 candles per sq. in.
Half-watt gas-filled lamp: 5,000 candles per sq. in.	Candle: 2–2·5 candles per sq. in.
Inverted gas burner: 50 candles per sq. in.	Full moon: 2 candles per sq. in.

(2) **Colour.**—The consideration of this is of practical importance, for artificial lights are mostly yellow, as compared with the bluish-white of the sky. Daylight is imitated, for colour matching and for picture-galleries and studios, by passing light from electric lamps through bluish-green glass, or by reflecting it from similarly coloured reflectors, to cut off the superfluous red and yellow rays.

(3) **Luminous flux** (or flow) deals with the light radiated in all directions, and is measured by the rate of passage of radiant energy—by the quantity falling per sec. on a surface of unit area placed at right angles to the flux.

The unit of flux is the *lumen* which is thus the flux per sec. through unit surface due to 1 c.p. Thus if a standard candle is at the centre of a sphere of 1 cm. radius, each 1 sq. cm. surface of the inside of the sphere would receive 1 lumen of light energy per sec.

But the surface area of the sphere $=4\pi \times \text{radius}^2=4\pi$.

$\therefore$ Total flux $=4\pi$ lumens, and the total flux from a source of illuminating power I c.p. $=4\pi$I lumens.

(4) **Illumination.**—In dealing with this we consider the light falling on a surface, *i.e.* the luminous flux per unit area at the point.

The practical unit of illumination is the *lux*, which is the illumination at a surface distant 1 metre from a standard candle.

$$\text{Now } \frac{\text{the total flux from a source of illuminating power I}}{\text{area of sphere, radius } d\text{, receiving the flux}}$$

$$=\frac{4\pi \mathrm{I}}{4\pi d^2}=\frac{\mathrm{I}}{d^2}$$

=intensity of illumination at a point on the sphere.

$$\therefore \text{ Intensity of illumination} = \frac{\text{flux}}{\text{area}},$$

$$\text{and when in foot-candles} \quad = \frac{\text{flux in lumens}}{\text{area in sq. feet}},$$

or, flux in lumens=intensity of illumination at point in foot-candles $\times$ area in sq. ft.

Exercises on Chapter XIII

1. What evidence is there that light travels in straight lines? Explain the formation of shadows, referring particularly to eclipses of the sun and moon.

2. Draw diagrams illustrating the production of various kinds of shadows by obstacles and sources of light of various sizes. Indicate in particular, in your diagrams, the regions where eclipses corresponding to partial, total, and annular eclipses of the sun may be seen. [*L.G.S.* 1919.]

3. Describe some simple phenomena which indicate that light travels in straight lines in a homogeneous medium, and show how the action of the pinhole camera depends on this fact. [*L.G.S.* 1920.]

4. A piece of white cardboard is stood upright on a table in a darkened room. Describe and explain the appearance of the card when (1) a lighted candle is placed in front of it, (2) an opaque screen with a small hole is placed between the cardboard and the candle. In the latter case what would be the effect of (1) moving the cardboard farther from the perforated screen, (2) increasing the size of the hole in the screen? [*L.G.S.* 1923.]

5. Draw diagrams illustrating the formation of shadows by an opaque obstacle, and state what may be deduced therefrom concerning the propagation of light. Draw, and explain fully, diagrams illustrating partial and total eclipses of the moon. [*C.W.B.* 1924.]

6. How is the intensity of illumination of a surface related to its distance from the source of light? Describe an experiment in verification. Two sources of light are placed on opposite sides of a grease-spot photometer so as to give equal illumination. A sheet of partially opaque paper is interposed between lamp and photometer on one side, and the other lamp has then to be moved to three times its former distance in order to obtain equal illumination again. What fraction of the light is being cut off by the paper? [*L.M.* 1926.]

7. How would you measure the candle-power of an ordinary electric lamp? Compare the intensity of illumination produced on a screen by 3 candles at 4 ft. distance with that produced by 4 candles at 3 ft. [*C.W.B.* 1926.]

8. Describe an experiment by which you could compare the illuminating powers of two sources of light. A 20-candle-power lamp is placed 5 ft. above a table. At what distance from the table must a 30-c.p. lamp be placed to give the same illumination at a point on the table directly below? [*L.M.* 1928.]

9. Two lamps of 16 c.p. and 24 c.p. are 5 ft. apart. A screen is placed between them 2 ft. from the weaker lamp. Compare the illuminations of the two sides of the screen. Where must it be placed to be equally illuminated on both sides?

10. What is the law of inverse squares used in photometry? How

would you verify it experimentally if you were provided with one electric lamp and three similar candles? Describe the form of simple photometer you consider most suitable for use in the experiment, giving reasons for your choice. [*L.M.* 1923.]

11. If I can just see to read small print held 10 ft. from a 50 c.p. lamp, how near must I hold it to a single candle? Explain the theory on which you base your calculation, and point out how the surroundings might affect the actual result. [*L.G.S.* 1926.]

12. Explain clearly the action of a grease-spot photometer. Define the term *foot-candle*. Lamps of 16 c.p. and x c.p. are placed 50 ins. apart on a bench. A screen held vertically between the lamps is illuminated equally by both when it is 22 ins. from the 16 c.p. lamp. What is the c.p. of the other lamp? [*L.G.S.* 1928.]

13. In what manner does intensity of illumination depend on the distance of a source of light? How do you justify your statement? A lamp is 8 ft. directly above one end of a 6-ft. table. How do the intensities of illumination at the two ends compare with one another? [*J.M.B.* 1926.]

14. Upon what two factors does the intensity of illumination of a surface due to a small source of light depend? A horizontal footpath is illuminated by a lamp fixed at a height of 12 ft. above the ground. Compare the intensity of illumination of the path immediately below the lamp with that at a point 16 ft. farther along the path. [*L.M.* 1927.]

15. If you were provided with a grease-spot photometer and six similar lamps (electric), what experiments would you carry out to verify the law of inverse squares and how would you record them? A 50 c.p. lamp is suspended 5 ft. above a drawing table. Find the illumination of the table top directly below the lamp in foot-candles (a foot-candle is the illumination produced on a surface by a source of 1 candle-power at a distance of 1 ft.). [*L.M.* 1924.]

16. A room is lighted by two lamps, each of 20 candle-power, which are 8 ft. apart and 6 ft. above a table on which a book lies directly beneath one lamp, A. How does the illumination of the book when both lamps are alight compare with that given by A only? [*J.M.B.* 1926.]

17. Outline two methods for comparing the relative intensities of two sources of light, and explain the practical values of such comparison. [*J.M.B.* 1927.]

18. How could you find out if a lamp said to be one of 50 candle-power really gives that illumination? [*J.M.B.* 1923.]

19. Describe a simple form of photometer and explain its use. A lamp, with a concave reflector behind it, illuminates a screen. How would you compare the intensity of light on the screen with that produced by the lamp alone? [*C.W.B.* 1929.]

20. Define *candle-power* and *intensity of illumination*. Show how these quantities are connected and explain units in which each can be measured. A 20 candle-power source is placed 6 ft. from an opaque white screen and on the other side a 40 candle-power source is placed at a distance of 12 ft. Compare the intensities of illumination of the two sides of the screen. [*L.M.* 1929.]

21. Describe and explain the action of a pinhole camera. What important fact concerning light can be deduced from its action? Describe other experiments to illustrate the same fact. What is the effect of (*a*) increasing the size of the pinhole; (*b*) making a number of pinholes; (*c*) moving the screen upon which the image is formed when the positions of the object and the pinhole are kept fixed? [*L.M.* 1929.]

CHAPTER XIV

REFLECTION OF LIGHT AT PLANE SURFACES

IF a piece of wood, tissue paper and glass, etc., are held in succession to a source of light, it is seen that some substances allow no light to pass through them, whilst other substances do so to varying degrees. Hence we apply the terms—

opaque, to substances which allow no light to pass through them,
transparent, to substances which allow much light to pass through them,
translucent, to substances which allow a smaller portion of light, falling on them (or as we say, *incident upon them*), to pass through.

The light which passes through is called *transmitted* light. But what happens to the light falling on an opaque body ? Since the body is visible it must throw off, or *reflect*, light just as a wall or floor throws off, or reflects, a ball thrown on it. But there is a difference, for whereas a ball is only reflected in the one direction, light must be reflected in all directions. An object can be seen from all parts of a room, despite the fact that only a single source of light is illuminating it. This process by which a body *reflects incident light in all directions* is called *irregular or diffuse reflection*, or *scattering*, of light, and it is due to this process that we ordinarily see objects.

There is, however, a special form of reflection of light—reflection from a polished surface, *i.e.* a surface so smooth that the surface particles are all in the same plane, and so all the rays in a parallel beam falling on

them are reflected in parallel paths. In this case we shall see that there are definite laws regarding the reflection of the light, and so we call it *regular reflection.* The reception of a regularly reflected beam of light from a surface gives the impression of a brightness at the surface, which is thus said to be *polished*. Surfaces specially made to reflect light regularly are *mirrors*, and the usual forms are a piece of metal, made smooth, and glass "silvered" on the back. Ordinary mirrors are made by pressing mercury between glass and tinfoil. A mercury-tin amalgam adheres to the glass and, in time, hardens. It is coated with shellac varnish and backed with red oxide of iron paint, to preserve it. For very good mirrors a fine layer of silver is deposited on specially smooth glass by a chemical or electrolytic process.

It should be noticed that even a mirror is not a perfect regular reflector, *i.e.* there is a loss of intensity on reflection. If a beam of light be directed on to a mirror in an otherwise dark room, the mirror is visible from all parts of the room, *i.e.* it does scatter some of the light.

Methods of Studying Reflection of Light.—There are several common experimental methods of studying reflection, using a bright source of light such as a carbon arc, a "pointolite" lamp (an electric lamp with a very small metal filament, so that a powerful light is given from almost a point source), or an ordinary gas-filled electric lamp, preferably with a frosted globe. A parallel beam is obtained using a lens, by a method explained on p. 292, and reflected from a mirror, held in a clamp and thus rotatable, and the results studied. A more convenient method is to use a smoke box; this contains the mirrors, etc., a powerful beam of light being used, and the smoke particles, reflecting light, show the paths of the rays. Another device is *Hartl's Optical Disc* (Fig. 109). This is a large circular metal plate round which is a semi-circular metal screen containing an aperture for the admission of light. Into slots in the screen, metal slides can be fitted with slits, so that the incident light may be separated into narrow beams. Any number of the slits can be covered by metal plates

or coloured glasses. The plate and the screen can be rotated separately in a vertical plane. The edge of the plate is divided into degrees, and two dark lines across the plate, and through its centre, are at right angles. Mirrors and other optical apparatus can be fixed to the disc by thumbscrews.

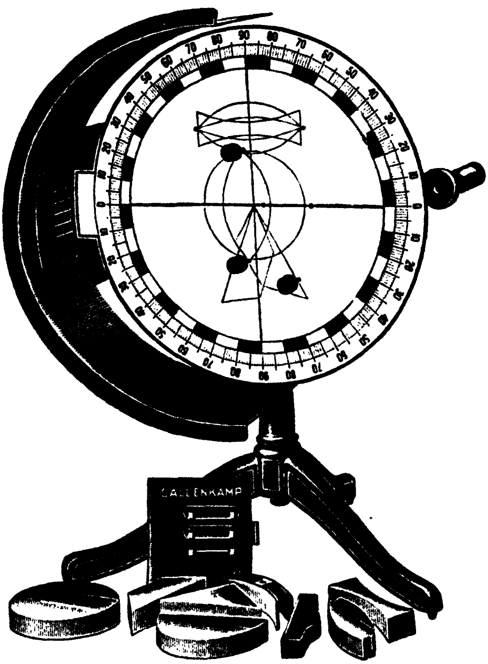


FIG. 109.—Hartl's Optical Disc.

If the apparatus be suitably arranged, a very narrow beam of light (which can be considered as a ray) is made to fall on a plane (or level) mirror and to be reflected off—the paths in each case being observed on the surface of the disc. The line drawn perpendicularly to the mirror, and in the plane containing the incident ray, is called the *normal* to the surface. The angle between the incident

ray and the normal (drawn where the ray meets the surface) is called the *angle of incidence* (Fig. 110).

The **laws of regular reflection** are:

(1) **the reflected ray lies in the same plane as the incident ray, but is on the opposite side of the normal to the incident ray;**

(2) **the angle of reflection** (i.e. **the angle between the reflected ray and the normal**) **is equal to the angle of incidence.**

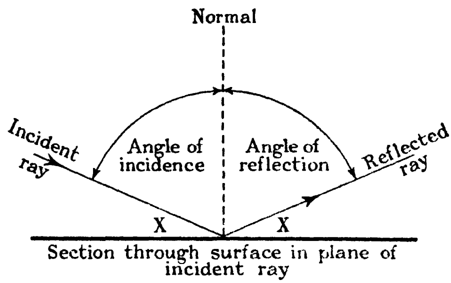


FIG. 110.—Regular Reflection at a Plane Mirror. (Note that angles X are equal.)

The first law is an obvious one, and the second can be easily verified with the optical disc, by rotating the disc and observing the angles of incidence and reflection of the narrow beam. The mirror should be placed so that its normal is one of the black lines drawn on a diameter of the disc.

It can also be verified by using pins to mark out ray directions. A small piece of plane mirror about 3 ins. by 1 in. is held vertically on a piece of drawing-paper, by a wooden frame or a wooden burette clamp removed from its stand. The back of the mirror (the reflecting surface) should coincide with a straight line, AB in Fig. 111, drawn across the paper, to which, from one point P on it, a normal PN, and other lines, PC, PD, etc., at angles of 10°, 20°, 30°, etc., have been drawn on one side of the normal. On one of these lines, PF for example, fix two pins P_1P_2 into the paper and standing upright.

PD represents an incident ray. Light from P_1 is reflected from the mirror and is visible between PN and PB. Rays of light from P_2 along direction FP are stopped by P_1, and so it is quite easy to identify rays of light reflected from the mirror *after* travelling along FP, for the eye when receiving this reflected beam cannot see P_2—it is obscured by P_1. Thus pins P_3

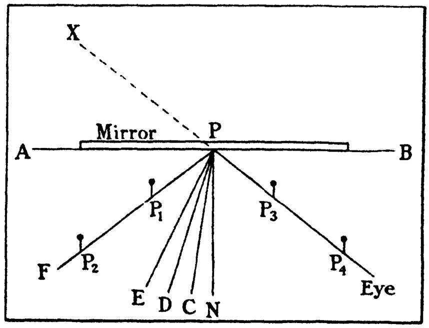


FIG. 111.—Verifying the Law of Regular Reflection.

and P_4 are fixed upright in the paper so that they are in line with P_1 when the latter is obscuring P_2. A line through P_3P_4 must represent the reflected ray, and should meet AB at P. By means of a protractor it can be shown that the angle of reflection, $P_4\hat{P}N$, is equal to the angle of incidence $F\hat{P}N$. This process is repeated with the other lines PC, PD, etc., and the law thus verified for several positions.

Formation of the Image in a Plane Mirror.—The counterpart, or likeness, seen in a plane mirror when an object is held in front of it is called an **image**. In Fig. 111 a ray of light FP is reflected, from the mirror on AB, along PP_3P_4 and so appears to come from somewhere along the line XPP_4. In this direction and apparently behind the mirror is this optical counterpart of the pin P_1. Where exactly is this image? The eye, being of appreciable size, receives a diverging beam, each ray in the beam taking a different path. Consider the case shown in Fig. 112, which is a section drawn perpendicularly to the mirror placed along AB. In this plane, a beam,

bounded by rays OC, OE, proceeds from the object O to the mirror, and is then reflected to the eye, the beam now being bounded by the reflected rays CD, EF, such that $O\hat{C}L = L\hat{C}D$ and $O\hat{E}N = N\hat{E}F$. CL, EN, are normals to the mirror. The image must therefore lie along DC, and FE, produced. These lines are produced

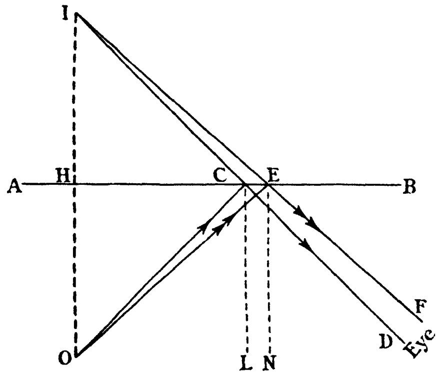


FIG. 112.

backwards to meet at I, which must, therefore, be the image of O—the beam appearing to diverge from I, whereas it really comes from O and is reflected at the mirror. Join IO and let it cut AB at H.

Now since $O\hat{C}L = D\hat{C}L$, their complements $H\hat{C}O$, $B\hat{C}D$ are equal,
and since $O\hat{E}N = F\hat{E}N$, their complements $H\hat{E}O$, $B\hat{E}F$ are equal.

Thus $H\hat{C}O = B\hat{C}D =$ vert. opp. $H\hat{C}I$,
and so the supplementary $O\hat{C}E =$ supplementary $I\hat{C}E$ . . (*a*)
Also $H\hat{E}O$ which is $C\hat{E}O = B\hat{E}F =$ vert. opp. $C\hat{E}I$ . . (*b*)

Thus the triangles OCE, ICE are congruent.

For (1) CE is common,
(2) $O\hat{C}E$ has been shown $= I\hat{C}$, ... (*a*)
and (3) $C\hat{E}O$ has been shown $= C\hat{E}I$. . . . . (*b*)

Thus IC=CO.

Hence triangles HCO, HCI are congruent,
for (1) HC is common,
(2) IC=CO (just shown),
and (3) included $H\hat{C}O$ has been shown $=H\hat{C}I$.

Thus HO=HI and $O\hat{H}C=I\hat{H}C=1$ right angle,

i.e. **the image in a plane mirror is as far behind the mirror, perpendicularly, as the object is in front.**

Since no light passes through the point I, but light only appears to diverge from it, it is called a *virtual image*. To construct the path by which light is received by the eye is now quite a simple matter. Suppose AB is the position of a plane mirror (Fig. 113) and O a "point" object near it. From O drop a perpendicular OD on to AB and produce it to I so that DI=OD. Then I must be the position of the image. Let E represent

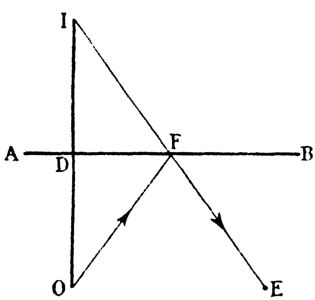


FIG. 113.

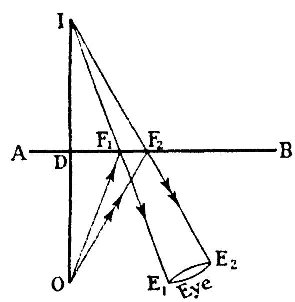


FIG. 114.

the eye observing the image, apparently along the path EI. Let EI cut AB at F. Join OF. The light from O travelled to E along FE, and so its path must have been O to F and then, after reflection, to E. Fig. 114 treats the eye as of material width, and a double construction, similar to the above, shows the divergent beam, bounded by OF_1E_1 and OF_2E_2, received by the eye.

Practical verification of position of image.

(*a*) *Sighting Method.*—Place a pin P in front of a mirror AB (see Fig. 115), and then place pins P_1P_2 so that they appear to be in line with the image of P seen in the plane mirror. In a slightly different position place pins P_3P_4 so that they appear to be in line with the image of P. Draw lines through the positions of P_1P_2 and P_3P_4 and produce them to meet at I. Then I must be the position of the image. Join IP and let it cut AB at H. Show by measurement that $IH = HP$ and $I\hat{H}B = P\hat{H}B = 1$ right angle. This should be repeated for different positions of P.

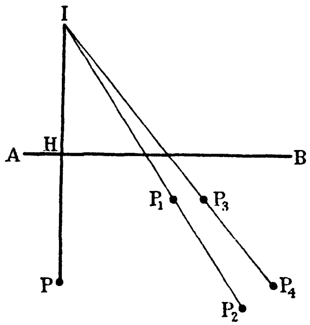


FIG. 115.

(*b*) *Parallax Method.*—This is based on the principle that two things held together at a distance always appear to be together from whatever position they are viewed. But two things which are not together, but are at a distance and in the same straight line from the observer, appear to move relative to one another if the observer moves. This is known as parallax

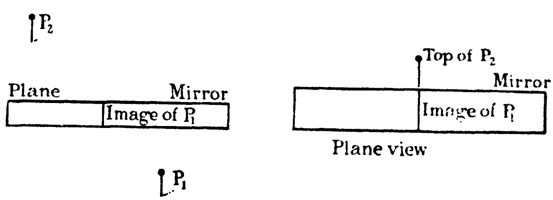


FIG. 116.—Parallax Method of finding Position of an Image seen in a Plane Mirror.

motion. When the objects are together, and there is no motion, they are said to be in the position of no parallax, or parallax is eliminated. This idea is used to find the position of an image in a plane mirror as follows: Fix a pin P_1 in front of a low mirror (about 1 in. high) standing on a piece of drawing-paper (Fig. 116). Now place a taller pin P_2 behind the mirror so that part of it can be seen *above* the mirror, in the same line as the image of P_1 *in* the mirror. Adjust the position of P_2 until, on moving

from side to side, the " same straight line " view is maintained, *i.e.* the part of P_2 seen and the image of P_1 *keep* together and are thus in the position of no parallax, and thus the image of P_1 is at P_2. By measurement it is shown that P_1 and P_2 are equidistant from the back surface of the mirror, and the perpendicular relationship also holds.

Eye specialists and opticians, working in a limited space, often make use of the fact that the image of an object, seen in a plane mirror, is as far behind the mirror as the object is in front. For eye-testing, cards having rows of letters varying in size are placed at a fixed distance. In a room half the required size, these cards can be placed above the head of the person being tested, so that he sees the image of them in a plane mirror fastened to the opposite wall. This image is the width of the room behind the mirror.

At the same time, another point has to be considered. In detective stories use is often made of the fact that if a piece of used blotting-paper be held in front of a plane mirror, much of the markings on the paper appears, in the image, to be legible. This is due to the fact that accompanying reflection in a plane mirror is *lateral inversion.* When moving one's right hand, whilst looking into a mirror, the left hand of one's image appears to move. The effect is clearly seen by placing a letter F as the object in a plane mirror and tracing out the positions of the images of the parts. By applying the relation between the positions of image and object the result shown in Fig. 117 is obtained. Thus the cards used by opticians, etc., in the special way described above, must carry the " laterally inverted " images of the letters to be seen in the plane mirror. This, of course, has the merit of preventing deception by the learning of groups of letters.

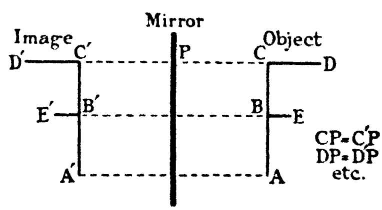


FIG. 117.—Lateral Inversion.

Another point to notice is that *the virtual image of*

an object produced by a plane mirror is the same size as the object

This can be verified experimentally as follows: (1) Use the front surface of a well-polished piece of clear plane glass as a reflecting surface. Place a pin an inch or so in front of it so that an image can be seen, apparently behind the glass. At the back put an exactly similar pin to coincide with this image (parallax method). This pin and the image of the first pin will be seen to be the same size. (2) Fix, in each of two pieces of wood, two pins so that the distance between them is the same in each piece of wood and they are at corresponding heights. Place one in front of a piece of plane mirror, fixed vertically, so that the images of the pins come to the edge of the mirror. Place the other wood and pins behind and outside the mirror (see Fig. 118) so that there is no parallax motion between the pairs of pins. The length of the image of an object AB thus=CD which=AB, and so the image in a plane mirror is the same size as the object.

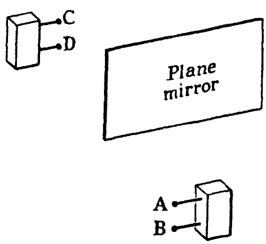


FIG. 118.

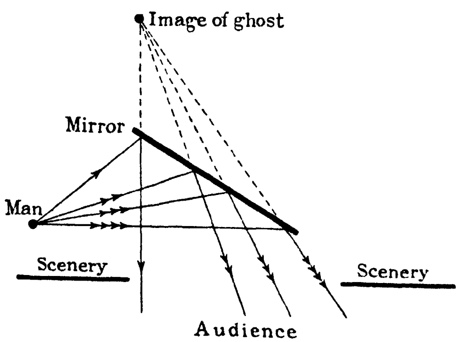


FIG. 119.

Mirrors are often used in theatrical plays to produce images, to represent ghosts, visible to the audience, as shown in Fig. 119. A mirror is lowered on to a darkened

stage when the ghost is to appear. As a man walks towards the mirror so the ghost (his image and so the same size) appears to walk towards the front of the stage.

It is becoming a common practice to place mirrors at an angle of $45°$ to a road at dangerous cross-roads, to enable motorists on one road to observe the traffic approaching from a side road and otherwise obscured from view.

It is also interesting to note that a person can see his own full-length image in a suitably-placed mirror half his height. This is clearly shown in Fig. 120.

M_1M_2 must obviously $= \frac{1}{2}$ $E'F'$ and thus $\frac{1}{2}$ EF, for $EM_1 = M_1E'$ or $EM_1 = \frac{1}{2}EE'$

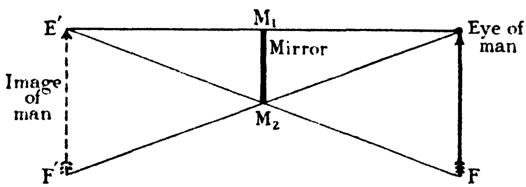


FIG. 120.

Successive Reflection at Two Mirrors.—When an object is placed between two mirrors, several images are produced, varying in number with the angle between the planes of the two mirrors. Light from the object may be received at the eye after 1, 2, 3, 4, etc., reflections, and in the cases given below care should be taken to study the paths of the rays in producing the several images. The loss in intensity at each reflection is shown by the increasing faintness of the images formed by an increased number of reflections.

The simplest case is where two mirrors are at right angles. In the section, Fig. 121, M_1 and M_2 are the mirrors, A their point of intersection, and O the object. I_1 and I_2 are the images in the plane mirrors M_1 and M_2 respectively, their positions following the rules already shown.

Now triangles ABO, ABI_1, are congruent,
for (1) AB is common,
(2) $OB = BI_1$,
(3) included $A\hat{B}O$ = included $A\hat{B}I_1 = 1$ right angle.

Thus $AI_1 = AO$, and in a similar manner it can be shown that $AI_2 = AO$.

But there is yet another image visible. Image I_1 (in mirror M_1) behaves as if it were an object, *i.e.* as a virtual object, to mirror M_2, giving rise to an image I_3 in

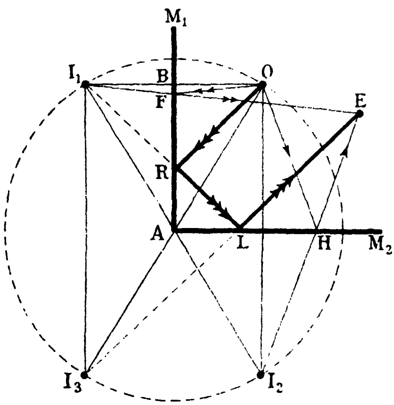


FIG. 121.

a position where I_1I_3 is perpendicular to, and bisected by, M_2A produced. Also AI_3 must $= AI_1$ and so $= AO$.

Thus the object and the images lie on the circumference of a circle of which the centre is the point of intersection of the mirrors and the radius is the distance of this point from the object.

Again, I_2 acts as a virtual object to mirror M_1, giving rise to an image. By a simple geometrical method the student can prove for himself that this image must be at I_3.

Now let us consider the actual path of the rays to an eye placed at E. The paths of the light received after one reflection are obtained by the construction shown in Fig. 121, and are OFE and OHE

To construct the path of the light giving rise to the image I_3, treating it as due to the virtual object I_1, join I_3E so that it cuts AM_2 at L. Thus the light which appears to travel from I_3 to E only came from L to E. Its path to L is obtained by joining L to the virtual object I_1, cutting AM_1 in R. The real path can only have been from R to L, for I_1 is the image due to the object O. Thus OR was the path of the light, and it travelled from O to the eye E by the path ORLE, undergoing two reflections. The student should also work out the other possible path, where light from O is reflected at M_2, then at M_1, passing to the eye E. This is shown for a diverging beam in Fig. 122 (E then being between O and M_1).

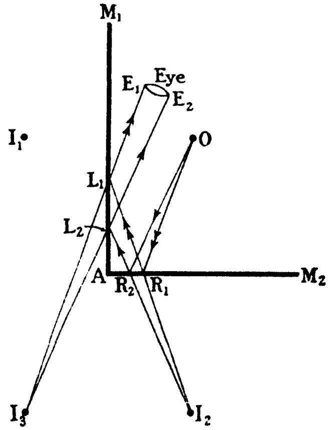


FIG. 122.

The student should now arrange two mirrors at different angles, observe the images and then endeavour to work out their positions and the ray directions. The following are positions which may be taken :

Angle between mirrors			120°	giving	2	images.
,,	,,	,,	60°	,,	5	,,
,,	,,	,,	45°	,,	7	,,
,,	,,	,,	30°	,,	11	,,

There is a simple rule connecting this—

$$\frac{360^\circ}{\text{angle between mirrors}} = \text{number of images} + 1 \text{ (the object).}$$

An interesting example of multiple image formation is that of the kaleidoscope. This is a long tube with two or three mirrors, inclined at 45° or 60°, and loose pieces of coloured glass between. One end, held to the light, is covered with ground glass which is translucent and

so allows light to pass without objects being seen. Through a small hole at the other end, an observer sees the effect of multiple reflection of the coloured objects.

Parallel Mirrors.—In a restaurant, or hairdresser's, where there are large mirrors on both side-walls of the room, many images of an object, *e.g.* an electric lamp, are seen. When an object is between two parallel reflecting surfaces, theoretically an infinite number of images should be formed $\left(\frac{360^\circ}{\text{angle between mirrors}} = \frac{360^\circ}{0^\circ} = \infty\right)$. But in practice this is not so, for, as will be explained, an increasing number of images is due to an increasing

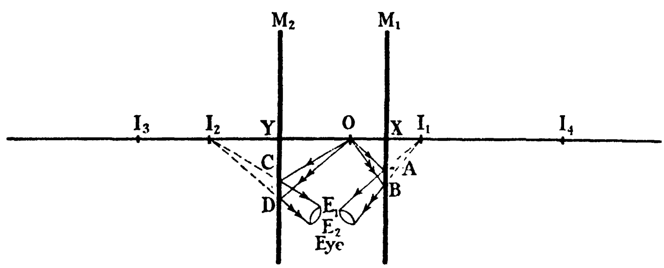


FIG. 123.—Images seen in Parallel Mirrors.

number of reflections. But at each reflection there is a loss of intensity, and so, when an object is between two parallel mirrors, the images seen to be getting more distant are more faint. (Part of the loss in intensity is due to some reflection from the front of the glass of the mirror, whereas most of the light enters the glass and is reflected from the silvered back surface.)

Let us now consider the formation of the images seen in parallel mirrors and the paths of the rays. In Fig. 123 O is the object between mirrors M_1M_2, so that I_1 is the image formed in M_1, and I_2 in M_2, in accordance with the usual rules ($OX=XI_1$, and $OY=YI_2$). The paths of divergent beams to the eye are as shown—looking at image I_1, the extreme paths of the beam are OAE_1

and OBE_2, whilst when viewing I_2 the extreme paths of the beam are OCE_1, ODE_2.

Now I_1 acts as a virtual object relative to mirror M_2 giving rise to an image I_3 such that $I_1Y = YI_3$. Similarly,

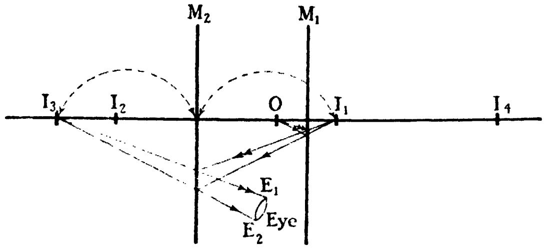


FIG. 124.

with M_1, I_2 gives rise to an image I_4 where $I_2X = XI_4$. Fig. 124 shows the path of the rays from O to the eye when I_3 is viewed. The student should trace the paths of the rays for the formation of image I_4, noting that two reflections take place.

Fig. 125 extends this procedure to the fifth and sixth

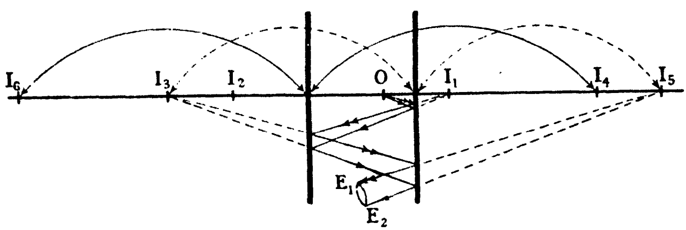


FIG. 125.

images, *i.e.* the images formed after three reflections at mirror surfaces, it being left for the student to trace out the paths of the rays received when viewing image I_6.

By repeated reflections at the mirrors, more images are formed, each reflection resulting in a loss of light, so that the distant images get more and more faint. To

observe this with two small mirrors, slightly tilt them out of the vertical plane; the images are then seen one above the other.

TABLE SHOWING PERCENTAGE OF LIGHT REFLECTED FROM SOME SURFACES

White drawing-paper	82	Good white reflectors used in illumination	80
Good white paint	75	(65 per cent. scattered, 15 per cent. regularly reflected)	
Yellow wall papers	40–45		
Dark-brown papers	12–14		
Black cloth	1–1·5		

Deviation of Light by Reflection.—In Fig. 126 the reflection of the incident beam AB, along BC, means that it is deviated by $D\hat{B}C$ which $=2x$ (where

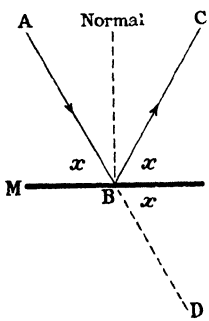


FIG. 126.—Deviation of Light on Reflection.

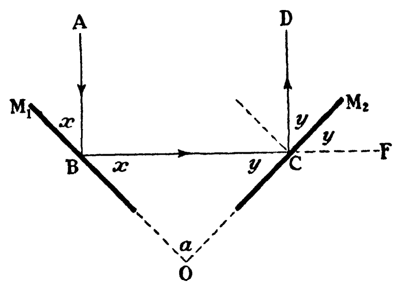


FIG. 127.—Deviation due to Two Reflections. (Diagram shows the two mirrors at right angles.)

$x=90°$—the angle of incidence). If two mirrors are used, as shown in Fig. 127, the angle between the two mirrors being $B\hat{O}C$ (or a), then the deviation on reflection at $M_2=F\hat{C}D=2y$, where $y=180°-a-x$.

∴ Total deviation of ray AB after reflection at the two mirrors

$$=2x+2y=2x+2(180°-a-x)$$
$$=360°-2a.$$

Thus deviation$=180°$ when $a=90°$, or a ray of light, after successive reflection from two mirrors at right angles, travels in a direction which is parallel and opposite to its original direction.

Again, the deviation$=0$ when $2a=360°$ or $a=180°$. Thus if *a ray of light is reflected, in turn, from two mirrors which are parallel, it is then travelling parallel to, and in the same direction as, the original direction.*

This is the principle of the simple *periscope*, used for viewing scenes in the presence of crowds, for observing enemy movements in trench warfare without danger to the observer, and on submarines. Fig. 128 explains the principle of the periscope, and shows how the double reflection overcomes the lateral inversion at a single reflection, with the consequence that a true view of the objects looked at is obtained.

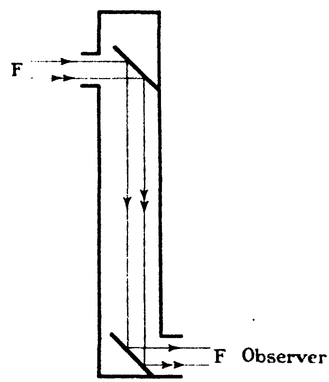


FIG. 128.—Simple Periscope.

Rotation of a Mirror (effect on reflected light).— In the use of the optical disc to verify the laws of reflection, the relation between the movement of the mirror and the reflected ray may have been noticed. If a ray is made to meet the mirror normally, the reflected beam is along the same path. If now the disc, to which the mirror is attached, is rotated so that the angle of incidence of the light ray is 45°, the angle of reflection is also 45°, so that the angle between the incident and reflected rays is 90°. Thus the mirror is rotated 45° and the reflected ray moves $2 \times 45°$. In this manner it is seen that, as the mirror is rotated from the original position indicated, the reflected ray moves twice as fast (or twice the angle of rotation of the mirror). This

relation is easily proved geometrically using Fig. 129. In this a ray OP is incident on a mirror AB (normal$=PN_1$) and is reflected along path PR so that $O\hat{P}N_1=R\hat{P}N_1$ (and thus $A\hat{P}O=B\hat{P}R$).

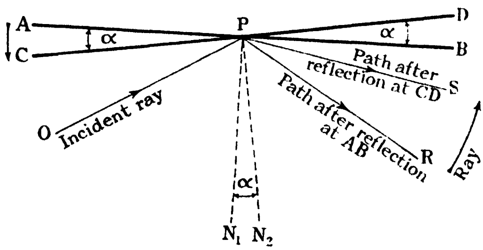


FIG. 129.

The mirror is rotated $\alpha°$ to position CD (normal now PN_2).

The new path of the reflected ray is PS where $O\hat{P}N_2=S\hat{P}N_2$ (and thus $C\hat{P}O=D\hat{P}S$).

The rotation of reflected ray

$$\begin{aligned}&=R\hat{P}S=R\hat{P}D-D\hat{P}S\\&=(\alpha+R\hat{P}B)-C\hat{P}O\\&=\alpha+A\hat{P}O-C\hat{P}O\\&=\alpha+A\hat{P}C=2\alpha\\&=\text{twice the rotation of the mirror.}\end{aligned}$$

The **Sextant,** an instrument used to measure the horizontal angle at a place between two distant objects, is based upon this principle. It is used by mariners to measure the angular altitude of the sun (and of certain stars) above the horizon, in order to determine their latitude at the time. A sextant consists of two mirrors, B fully silvered, A silvered only on the top half, and a telescope T (Fig. 130). Mirror B can be rotated and is fixed at one end of a rod, the other end of which is a

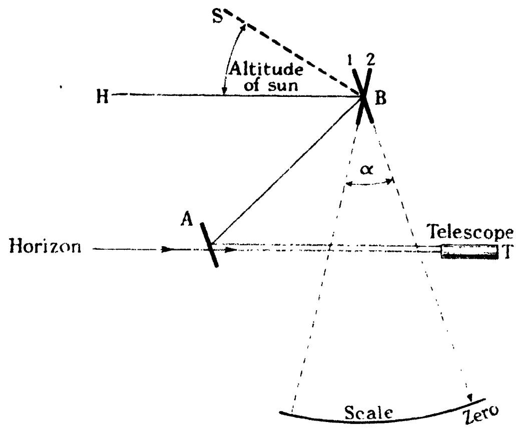


FIG. 130.—Principle of Sextant.

pointer which moves over a circular scale. Mirrors A and B are first set parallel, B being temporarily fixed with the pointer at zero on the scale. To obtain this parallel position, mirror A is rotated till an image of the sun (or the horizon) formed, after reflection at B and then A, in the telescope, and the image seen directly through the unsilvered portion of A, coincide Mirror A is now securely clamped. In many sextants this parallel position is set by the makers and A cannot be moved.

To measure the altitude of the sun, the sextant is held so that the horizon is sighted directly through the unsilvered portion of A. Mirror B is then rotated (through, say, α°) until rays from the sun, reflected from B to A and then to the telescope, produce an image of the sun which coincides with the view of the horizon seen directly through A. Obviously, then, in the parallel position, a ray HB would, after reflection at B and then A, travel along AT. In the second case, when the mirror rotated α°, a ray SB follows that path. Thus, considering reverse paths, a ray AB, reflected by B along BH, is rotated to path BS when B is rotated α°. Thus $S\hat{B}H = 2\alpha$.

Then the angular altitude of the sun $= 2\alpha$.

For convenience, the scale is marked off in double values (sextant degrees), *i.e.* 5° is marked as 10°, etc., and the sun's altitude read off directly. Dark glasses are provided, to be rotated between the mirrors, to reduce the intensity of the image of the sun. A ship's movement makes no difference to the position of the horizon, but much practice is necessary before a sextant is operated with precision and speed.

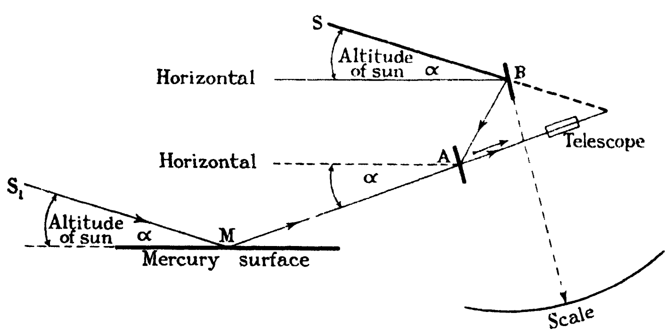


FIG. 131.

The horizon on land is not necessarily at the same level as that at the place of observation, and so is not suitable for use with the sextant. To obviate this difficulty, a mercury surface, horizontal owing to gravitation, is used. The instrument is held so that the image of the sun reflected from the mercury surface is seen in the telescope. Mirror B is then rotated from its zero position till the direct rays of the sun travel from B to A and to the telescope, and the images, coincide (Fig. 131). Then obviously the angle between MA and $BS=2\alpha$, where α is the altitude of the sun. Thus the movement of mirror B and the pointer is, in this case, twice the altitude of the sun.

Small Angular Deflections.—In some scientific instruments the deflection (small) of a suspended or pivoted system

has to be measured. For this a small mirror is fixed to the suspension, and a narrow, parallel beam focussed on to it, and from there is reflected on to a distant scale (*e.g.* 100 cms. away). This reflected beam is rotated through twice the angle of rotation of the mirror—the movement along the scale being observed. Thus, in Fig. 132, a beam SA falls on mirror in

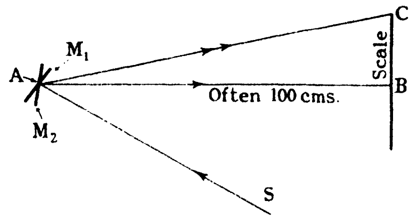


FIG. 132.

position M_1 and is reflected along path AB to the scale at B. When the mirror is rotated into position M_2, the reflected beam moves to AC, its angle of rotation being $C\hat{A}B$.

Then $\tan C\hat{A}B = \frac{BC}{AB} = C\hat{A}B$ for small angles.

Thus rotation of mirror (from position M_1 to M_2) $= \frac{1}{2}\frac{BC}{AB}$, and if BC is measured to the nearest millimetre, an accurate value for the rotation of the mirror (and the suspended system) is obtained.

EXERCISES ON CHAPTER XIV

1. State the laws of reflection of light. Prove that the rays from a point source of light which fall upon a plane mirror proceed, after reflection, as though they diverge from a single point. What is the point called and what is its position?

2. State the chief facts relating to images formed by plane mirrors. How would you verify each by actual experiment?

3. Describe an experiment for finding the positions of the image formed by a plane mirror of a pin placed in front of it. What difference would it make whether you were using a polished metal surface or a piece of thick glass silvered at the back? [*L.M.* 1923.]

4. What is meant by the *image* of a luminous point? Such a point is placed in front of a plane mirror. Determine the position of its image,

using the laws of reflection of light. How would you verify your result experimentally?

5. Explain the nature of an image, and show clearly why the images of things seen reflected in the surface of a still lake appear upside down. [*J.M.B.* 1927.]

6. State the laws of reflection of light. An object 3 ins. high is placed 8 ins. in front of a plane mirror. What is the nature, position and size of the image? How would you verify your statements experimentally? [*C.W.B.* 1929.]

7. State the laws of reflection of light. Explain, with diagrams, (*a*) the action of a periscope, (*b*) the position of the image of an object seen in a plane mirror. [*J.M.B.* 1928.]

8. State the laws of reflection of light at a plane surface. Show, by means of a carefully-drawn diagram, how a man, height 6 ft., could place a mirror, length 3 ft., flat against a vertical wall, so that he could see a full-sized image of himself in it. [*J.M.B.* 1924.]

9. A plane mirror 2 ft. high is fixed on one wall of a room, the lower edge being 4 ft. 6 ins. from the ground. If the opposite wall of the room is 14 ft. distant and 10 ft. high, draw a diagram to show from what point a man must look in order to see reflected in the mirror the whole height of the opposite wall from floor to ceiling. [*L.G.S.* 1923.]

10. A ray of light is reflected from two plane mirrors in succession. Find the angle between the mirrors for the ultimate path of the light to be at right angles to its original path. [*L.M.* 1925.]

11. The angle between two plane mirrors is one-fifth of four right angles, and a luminous point is placed midway between them. Make a diagram showing the images that are formed, and explain carefully how each is formed. Trace a ray by which an eye, placed midway between the mirrors and twice as far as the luminous point from their intersection, sees an image formed by two reflections.

12. Two mirrors, A and B, are inclined to each other at 45°. An object, represented by the letter P, is placed with its lowest point ½ in. from A and 1 in. from B. Show in a diagram the position of the image of the P formed by two reflections, first at mirror A and then at mirror B. Draw, also, the paths of the rays by which an eye at Q, 1½ ins. from each of the mirrors, will see a point on this image. [*L.G.S.* 1924.]

13. Show how to arrange two plane looking-glasses so that a person, by looking straight before him, could get a side view of his own head. [*J.M.B.* 1922.]

14. Two plane mirrors are fixed parallel to one another on opposite walls of a room. Explain the formation of the series of images which are seen when an object is placed between them. Draw a diagram showing the path of the rays by which an eye sees the third image in one of the mirrors. [*L.G.S.* 1921.]

15. Show that when a plane mirror is turned through a given angle, a ray reflected from the mirror is turned through twice that angle. Describe any one practical application of this.

16. A horizontal beam of light AB falls normally on the centre B of a plane mirror placed in a vertical position. It is required to produce a reflected beam BC such that BA=BC=1 metre, and AC=2·5 cms. Find, *approximately*, the angle through which the mirror must be rotated. State concisely the optical principles you have employed in solving this problem. [*L.G.S.* 1925.]

17. A horizontal beam of light from a lamp falls on a mirror and is reflected at right angles horizontally, forming a spot of light on a screen 1 metre distant from the mirror. How must the mirror be placed? Through what angle must it be rotated to make the spot of light move 10 cms. on the screen horizontally? Justify the truth of the theorem which you employ in solving this problem. [*L.G.S.* 1926.]

18. Show how it is possible to arrange two plane mirrors so that we can "see ourselves as others see us."

19. The image formed by a single reflection at a plane mirror is said to be *laterally inverted* or *perverted*. Explain this and show how by using two plane mirrors a non-perverted image of an object can be seen.

CHAPTER XV

REFLECTION AT CURVED SURFACES

MIRRORS, whose surfaces are curved and not plane, are commonly used as shaving mirrors, back reflectors on lamps and searchlights, etc., etc. In many cases the mirror used is part of the surface of a sphere and is called a *spherical mirror.* Spherical mirrors are either concave or convex.

A *concave* mirror is one in which the reflecting surface is curved away from the source or object placed in front of it ; whilst a *convex* mirror is one in which the curvature

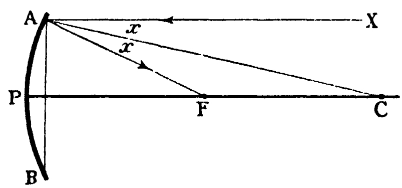


FIG. 133.

of the reflecting surface is towards the source or object placed in front of it. The centre of the sphere of which the surface of the mirror forms a part is called the *centre of curvature* of the mirror.

Fig. 133 is a section of a concave spherical mirror through the centre of curvature C, and it is known as a *principal section of the mirror.* The extreme width AB of the mirror is called its *aperture,* and the middle point P of the reflecting surface is the *pole* of the mirror. A line CP, drawn in a principal section of a spherical mirror,

through the pole and centre of curvature, is called a *principal axis*.

By means of the optical disc it can be shown that **reflection occurs at a spherical mirror, as if the small element of mirror,** from which a ray is reflected, **were plane and tangential to the curved surface at the point.** A narrow beam is allowed to pass through a single slit and fall on the mirror at P (Fig. 133), returning along the same path PC (a dark line on disc). The disc is then rotated, when it is seen that the incident and reflected rays are equally inclined to this dark line, which is thus the normal to the mirror at the point P (the mirror at P behaving as if it were plane). **In general, the normal to a spherical mirror surface at a point is the line joining the point to the centre of curvature.**

Consider a ray XA parallel to the principal axis PC. Then AC is the normal to the surface at A and the ray XA is reflected along AF, such that $F\hat{A}C = X\hat{A}C$ (Fig. 133).

But since XA and PC are parallel, $X\hat{A}C = F\hat{C}A$.

$\therefore$ $F\hat{A}C = F\hat{C}A$, and so $FA = FC$.

If the mirror is of small aperture, we can consider A as being sufficiently near to P for FA to be treated as if it were equal to FP.

Then FP must equal FC, or F is the middle point of CP.

Thus it follows that **all rays parallel to the principal axis of a concave mirror of small aperture are reflected through a point** which is half-way between the pole and the centre of curvature of the mirror. This effect is known as "focussing" of the rays, and this special point F is called **the principal focus** of the mirror (the rays "converge" on F).

FP, the distance of the principal focus from the pole, is called the *focal length*, and is usually represented by f.

Thus $f = \frac{1}{2}r$, or $r = 2f$, where r represents the radius of curvature of the mirror.

Using the optical disc, a series of parallel narrow beams can be directed on a concave mirror parallel to the principal axis, and the focussing effect observed (Fig. 134). It can also be shown that a narrow aperture

is essential—if the outermost rays are shut off, the focussing is more precise.

A simple method of finding the focal length of a mirror should be obvious. Rays from the sun at a point on the earth are considered to be parallel, and thus, when a concave mirror is held up to the sun, it focusses rays, so giving a bright image of the sun which can be shown on a piece of paper, which must thus be at the principal focus of the mirror.

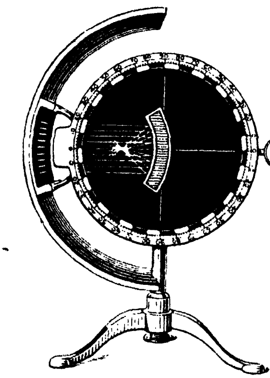


FIG. 134.

It should also be realised that, if a source of light be placed at the principal focus, rays from the source, after striking the mirror, travel parallel to the principal axis. In such a manner, then, a parallel beam of light can be obtained. The use of this will be dealt with later.

Image Formation by Reflection in a Concave Mirror.—Consider a point source of light at O on the principal axis of a concave mirror and outside the centre of curvature. What happens to the beam of light from O after reflection at the mirror? Consider the case of any ray OA incident upon the mirror. This ray will be reflected along AI such that $I\hat{A}C = O\hat{A}C = x$ (Fig. 135).

Let $A\hat{I}P = i$, $A\hat{C}P = c$, and $A\hat{O}P = o$.
Then exterior angle $i = c + x$ in triangle ACI,
and exterior angle $c = o + x$ in triangle ACO.
Hence, subtracting, $i - c = c - o$,
or $i + o = 2c$.

Under the same conditions as before, *i.e. the aperture of the mirror is small*, these angles are small, and the tangents of the angles are approximately the same as the circular measure of the angles (see p. 20).

Thus we can consider $\tan i + \tan o = 2 \tan c$.

Further, the perpendicular AN($=y$) is, under the same conditions, so close to P that N and P approximately

coincide, and so when trigonometrical ratios are substituted for tangents,

$$\frac{y}{\mathrm{IP}}+\frac{y}{\mathrm{OP}}=\frac{2y}{\mathrm{CP}}$$

The following conventional symbols are used :—

u for the object, or source, distance OP,
v for the image distance (shown below to be IP),
and r for the radius of curvature CP.

Thus, $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}=\frac{1}{f}$. (Since $f=\frac{1}{2}r$ or $\frac{2}{r}=\frac{1}{f}$.)

If the object O is fixed so that u is constant, since f is also a constant for the mirror, v must be constant,

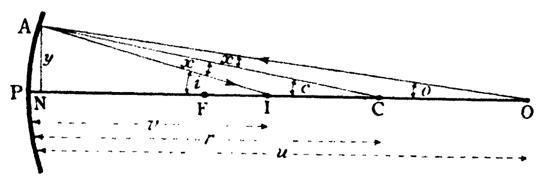


FIG. 135.

and so this relationship holds for all rays leaving the point source O. Rays proceeding from O pass, after reflection at the concave mirror, through a fixed point I on the principal axis. The focussing of the rays produces a *real image* at I—the word *real* being applied in contrast to "virtual," which was applied to the image formed by reflection in a plane mirror. If a screen were held at I in such a manner that it did not prevent rays from O reaching the mirror, a bright image would be produced on the screen. Thus, real images can be shown on a screen (the rays pass through the image), whilst virtual images cannot be shown on a screen (rays only appear to pass through the image).

Conversely, also, a source of light at I would give rise to an image at (*i.e.* focus rays to) O. Thus the points O and I are called *conjugate foci*.

Reflection at a Convex Spherical Surface.—The terms defined at the beginning of this chapter for concave mirrors apply equally to convex mirrors. But the behaviour of a convex mirror is different. If it is held up to the sun's rays, it will not focus them on to a piece of paper. Consider a ray XA parallel to the principal axis CP of a convex mirror (Fig. 136), C being the centre of curvature. Then CA produced (*i.e.* AN) is the normal to the mirror surface at A.

Hence reflected ray is AD, where $X\hat{A}N = D\hat{A}N$.
Produce AD backwards to meet CP at F

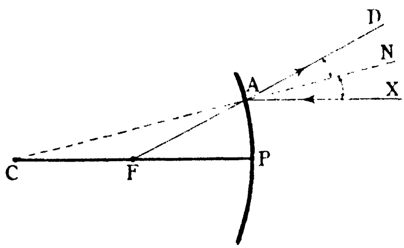


FIG. 136.

Since XA and CP are parallel, $X\hat{A}N = A\hat{C}F$.
Also $D\hat{A}N$ = vert. opp. $C\hat{A}F$.
Thus $A\hat{C}F = C\hat{A}F$, and so FA = FC.
Considering mirror of small aperture, FA = FP approx.,
and so FC = FP, or F is midway between C and P.

Thus all rays parallel to the principal axis of a convex mirror are reflected, in a diverging beam, so that they appear to come from this point F, behind the mirror. This point is the principal focus of the convex mirror, but, unlike the principal focus of a concave mirror, is virtual.

Now consider the behaviour of the mirror towards a point source O on the principal axis (Fig. 137). Let OA be any ray which, on striking the mirror at A, is reflected

along the path AD, where $D\hat{A}N = N\hat{A}O = x$ (CA being produced to N). Produce AD backwards to cut CP in I

Let $A\hat{O}P = o$, $A\hat{I}P = i$, and $A\hat{C}P = c$.
$C\hat{A}I =$ vert. opp. $D\hat{A}N = x$.
Thus exterior angle $i = c + x$ in triangle ACI,
and exterior angle $x = c + o$ in triangle OCA.
Thus $i = 2c + o$,
or $i - o = 2c$.

And since the aperture is small, these angles are small, and so we can consider $\tan i - \tan o = 2 \tan c$ (p 20).

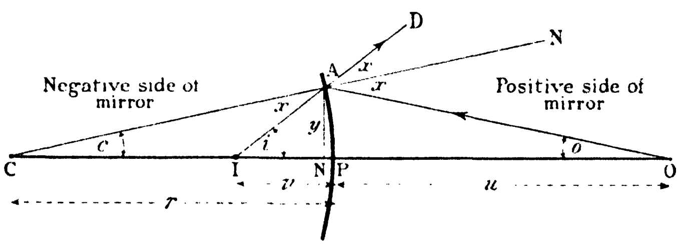


FIG. 137.

If this be compared with a similar kind of relationship on p. 244, it will be seen that there is a difference in sign, and which would entail a difference in sign in the equation derived, from the above, for a convex mirror as compared with that obtained for a concave mirror.

Hence, although we represent, as before,

the object or source distance OP as u,
the distance of IP as v,
and the radius of curvature CP as r,

we now introduce the **sign convention**, which is an agreed method of treatment of the difference in behaviour of concave and convex mirrors. *By this convention*,

(i) **all distances are measured from the pole of the mirror,**

(ii) **one side of the mirror is considered the positive side and the other the negative side. Distances measured on the positive side are positive, and those on the negative side are negative.**

In this book the source is always drawn on the right-hand side of the mirror, and this side is called positive, the left-hand side thus being negative. Thus distances measured (from pole of mirror) towards the source are positive, and away from it are negative.

Now for the convex mirror we have seen $\tan i - \tan o = 2 \tan c$.

Thus, substituting trigonometrical ratios, the perpendicular being AN($=y$), and approximately coinciding with P,

$$\frac{y}{\text{PI}} - \frac{y}{\text{PO}} = \frac{2y}{\text{PC}}$$

But, using the sign convention, and symbols similar to those used with concave mirrors, $\text{PI} = -v$,
object or source distance, $\text{PO} = +u$,
and radius of curvature, $\text{PC} = -r$.

$$\therefore \frac{y}{-v} - \frac{y}{+u} = \frac{2y}{-r}$$

$$\therefore \frac{1}{v} + \frac{1}{u} = \frac{2}{r} = \frac{1}{f} \text{ as before,}$$

f being the focal length of the mirror.

Thus, since v is constant if u is constant, all rays from O, after reflection at a convex mirror, appear to diverge from a fixed point I, which is thus the image of the object O, but is a virtual one.

Conversely, a beam of rays converging on I would, after reflection at the convex mirror, converge on O. Thus I and O are also called *conjugate foci*.

General Consideration of Reflection at Convex and Concave Mirrors.—The formula $\frac{1}{v} + \frac{1}{u} = \frac{2}{r}$ or $= \frac{1}{f}$, using the symbols previously stated, holds for both types

of mirror, provided the appropriate signs are used. In using this formula the signs must only be introduced with numerical values. It should be noticed that the focal length and radius of curvature of a concave mirror are positive, and so a concave mirror is sometimes called a positive mirror. The corresponding constants are negative in the case of a convex mirror, which is thus called a negative mirror. It is possible to treat the problem of the variation in image position with variation in object position by two methods, (*a*) by application of the general formula, and (*b*) diagrammatically; examples will be given of the use of both of them.

In *drawing diagrams*, there are *two important rules* to remember.

(1) **Any ray parallel to the principal axis of a spherical mirror passes, or appears to pass, after reflection through the principal focus,** and, conversely, a ray through, or in line with, the principal focus is, after reflection, parallel to the principal axis.

(2) **Any ray passing through, or in line with, the centre of curvature of a spherical mirror is perpendicular to the small element of mirror at the point of contact, and so is reflected back along its original path.**

To illustrate the behaviour of spherical mirrors some simple experiments should be carried out in a semi-darkened room. A suitable arrangement is shown in Fig. 138. A wooden box has a frosted electric lamp mounted from the top (ventilation holes being bored in the latter). On two parallel sides, metal plates, with a suitably shaped pattern about $1\frac{1}{2}$ ins. high cut in them, are fixed opposite the centre of the bulb and serve as objects. The lamp, when connected to the lighting supply, serves as a source for students working on either side.

Starting with a concave mirror (radius of curvature 12–24 ins.) a good distance from the source, obtain the image on a screen and measure the object and image distances. Gradually move the mirror nearer to the object, adjusting the screen position to get a sharply defined image. In several positions measure the object and image distances, and also the height of the image, and enter them in a table of the form

given below. (A ruler or millimetre graph paper mounted on cardboard can conveniently be used as a screen.) Notice that the image, always inverted, varies in size—sometimes it is larger than the object (*i.e.* is magnified) and sometimes smaller (diminished)—and that when the mirror and object are

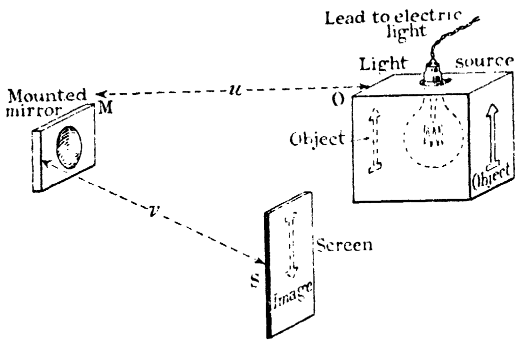


FIG. 138.—Reflection at a Concave Mirror.

fairly close together it is impossible to obtain an image on the screen.

Now try the same procedure with a convex mirror—you will find that you can never obtain an image on the screen. If you look again at Fig. 137 you will see the reason for this —no real image is possible, for a diverging beam is always produced by reflection at a convex mirror.

The table for the results with a concave mirror is :

Height of object= cms.

Object distance MO $=u$ cms.	Image distance MS $=v$ cms.	uv	$u+v$	$f=\dfrac{uv}{u+v}$	Height of image.	$\dfrac{\text{Height of image}}{\text{Height of object}}$	$\dfrac{v}{u}$
						(X)	(Y)

Several values for the focal length of the mirror are calculated For since $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, $\frac{u+v}{uv}=\frac{1}{f}$, and so $f=\frac{uv}{u+v}$.

The mean value of f is then calculated.

A similarity of results is seen between columns indicated (X) and (Y). The ratio of

$$\frac{\text{height of a line in image}}{\text{height of a corresponding line in object}}$$

and thus $\dfrac{\textbf{height of image}}{\textbf{height of object}}$ is called the **magnification** of the image, and it is equal to $\dfrac{\textbf{image distance}}{\textbf{object distance}}=\dfrac{\mathbf{v}}{\mathbf{u}}$.

Geometrical Proof for concave and convex mirrors (see Figs. 139, 140).—The usual lettering is used and AB represents the object. Rays AX from A parallel

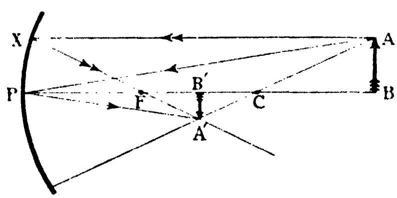


FIG. 139.—Reflection at Concave Mirror, giving a Real Image.

to the principal axis pass, or appear to pass, after reflection through the principal focus F. Rays from A through the centre of curvature C are reflected back along their original path. The intersection of these lines gives the position of the image A′ of A. All rays from A must thus be reflected through A′ (or as if they came from A′).

Thus, in the case of the concave mirror, incident ray AP must be reflected along PA′, and thus $A\hat{P}B$ must $=A'\hat{P}B'$. In the case of the convex mirror, if A′P be produced to D, the incident ray must be reflected along PD (*i.e.* as if it came from A′) and $A\hat{P}B=D\hat{P}B=$vert.

opp. $A'\hat{P}B'$ (Fig. 140). The image of B must be at B′, along the axis and at the foot of the perpendicular from A′ on to the principal axis.

In both cases, then, triangles A′PB′, APB are similar, for (1) $A'\hat{P}B'$ has been shown $=A\hat{P}B$,
(2) $A'\hat{B}'P=A\hat{B}P=1$ right angle.

$$\text{Thus } \frac{A'B'}{AB}=\frac{PB'}{PB'},$$

and thus $\frac{\text{height of image}}{\text{height of object}}=$magnification and numerically$=\frac{\text{image distance}}{\text{object distance}}$.

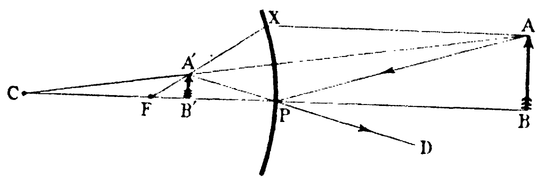


FIG. 140.—Reflection at Convex Mirror, giving a Virtual Image.

Concave Mirrors.—Let us now consider the change in the position of the image as an object is moved from infinity towards the mirror.

(1) *Object at Infinity.*—Rays reaching the mirror must be parallel to the principal axis and so, after reflection, pass through the principal focus (*i.e.* a point image).

Or from $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ (usual symbols)

$u=\infty$ and thus $\frac{1}{u}=0$. Hence $\frac{1}{v}=\frac{1}{f}$ or $\underline{v=f}$,

i.e. image is at the principal focus.

As the object is moved nearer to the mirror a finite object is formed as shown in Fig. 139. This image is obviously real, inverted, and diminished.

(2) *Object at Centre of Curvature.*—Obviously all rays through this point are reflected along their original

paths. In the case of an object as shown in Fig. 141, the image of B at C is B′ at C also. The image of A is obviously on the other side of the normal PC, and is at

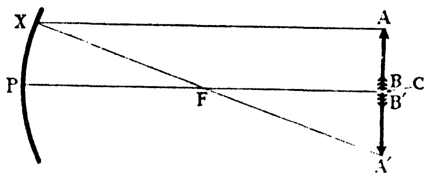


FIG. 141.

A′, such that B′A′=AB. (The student should show this geometrically.)

$$\text{Or, since } u=r,\ \frac{1}{v}+\frac{1}{u}=\frac{2}{r} \text{ becomes } \frac{1}{v}+\frac{1}{r}=\frac{2}{r}$$

$$\text{or } \frac{1}{v}=\frac{1}{r} \text{ and } \underline{v=r}.$$

Thus the image is at the centre of curvature and is *the same size as the object* ($u=v$) and is real and inverted.

If the object is moved nearer to the mirror than the

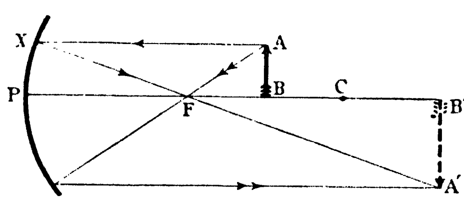


FIG. 142.

centre of curvature an image is formed, as shown in Fig. 142, outside the centre of curvature, and is thus a real, inverted, and magnified image

(3) *Object at Principal Focus.*—The image is then, of course, at infinity, for $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ and $\frac{1}{u}=\frac{1}{f}$.

Thus $\frac{1}{v}=0$, or $v=\infty$.

(4) *Object between the Principal Focus and the Mirror.*—This is a very interesting case.

If $u<f$, then $\frac{1}{u}>\frac{1}{f}$

Since $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$, and so must be negative.

Thus v must be negative, *i.e.* the image must be a virtual one and not able to be received on a screen. (Compare this with the result in the experiment described on p 250.)

Fig. 143 shows this case, where AB is the object. Ray CAD is normal to the mirror at D and so is reflected

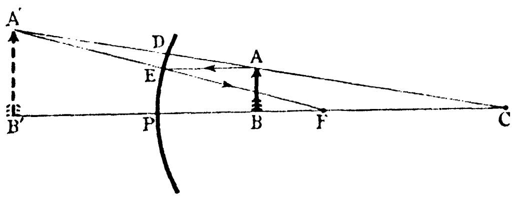


FIG. 143.

along its original path. Ray AE, parallel to the principal axis, is reflected along path EF. These reflected rays, DC and EF, are obviously diverging from the mirror, since FC (equal to the focal length) is greater than AE.

Produce CD and FE to meet at A′. This is then the virtual image of A, and A′B′, perpendicular to the principal axis and meeting it at B′, is the virtual image of AB seen in the mirror.

Also since $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$, $\frac{1}{v}$ is $<\frac{1}{f}$

Thus v is $>f$, whilst u is $<f$, and so v is always $>u$

Thus the image is always a magnified one—except when the object is at P, the image then being at P also, and the same size as the object. The virtual image is also always erect, and so it is in this position that a concave mirror is used as a shaving mirror to give an enlarged image of the face.

The following is a summary of the results worked out :—

Object.	*Image in a Concave Mirror.*	
Infinity	Principal focus	
↓	↓	Real, Inverted Diminished Image
Centre of Curvature	Centre of Curvature	(Image same size as object)
↓	↓	Real, Inverted Magnified Image
Focus	Infinity	
↓	↓	Virtual, Erect Magnified Image
Pole of Mirror.	Pole of Mirror.	

Convex Mirrors.—The case of a convex mirror is very simple, since, as can be seen from the one case shown in Fig. 140, the image must always be a virtual one, the limiting positions being :

(*a*) at infinity, image is at principal focus,

(*b*) object at pole of mirror, image is at pole.

Since $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, and f is negative, $\frac{1}{v}+\frac{1}{u}$ must be negative

u is positive, and so v must be negative, and $\frac{1}{v}$ must be numerically $>\frac{1}{u}$.

Hence v is always $<u$ and negative.

Thus the image in a convex mirror must always be virtual and diminished, and its position can only be between the pole and the principal focus of the mirror. From this it should be clearly seen why a convex mirror is used on motor vehicles to enable a driver to see traffic overtaking him. With such a mirror (and *not* with a plane or concave mirror),

(i) the image is always upright,

(ii) the change in size and position of the image of an overtaking car is very small, until the latter is actually passing, and so is not disturbing.

Determination of the Optical Constants of Mirrors.

(1) *Concave Mirrors.*—The following are some methods for finding the position of the principal focus (or centre of curvature):

(*a*) Simple method—in which the sun's rays are focussed on a paper.

(*b*) A method in which a luminous source is used, as given on p. 250.

(*c*) A quick method entails placing a pin so that the image coincides with the pin—*i.e.* there is no parallax motion between the pin and its inverted image. The pin is then at the centre of curvature of the mirror.

(2) *Convex Mirrors.*—The simplest method is to use a pin as an object (distance u cms.) in front of a convex mirror, and to find the position of the image (distance

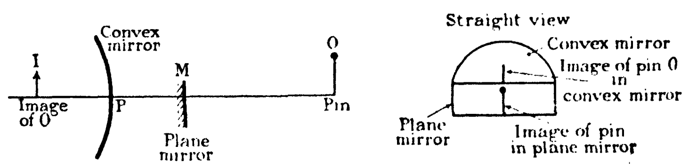


FIG. 144.

v cms.) by means of a narrow piece of plane mirror placed a little in front of the convex mirror and partially covering the image (Fig. 144). There is then also an image of part of the pin in the plane mirror, which is so adjusted

that this image is in line with the image of part of the pin seen in the convex mirror. The position of the pin is adjusted till these two images coincide in position, *i.e.* there is no parallax motion between them.

Then by reflection in the plane mirror OM=MI.

$\therefore$ Image distance$=v=$PI=MI−PM=MO−PM.

OP=the object distance, and the values for u and v (the latter being negative) can be substituted in the equation $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, and f calculated.

Spherical Aberration.—It is important to remember that, in dealing with spherical mirrors, the relationships obtained depend on the assumption that a narrow-apertured mirror is used. It is seen when

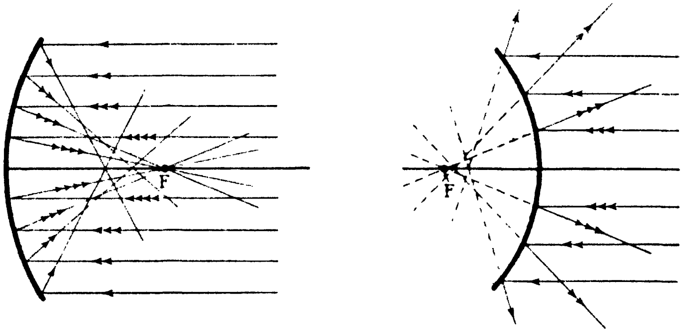


FIG. 145.—Spherical Aberration on Reflection at Spherical Mirrors of Large Aperture.

using an optical disc that the central rays of a parallel beam converge, in the case of a concave mirror, towards a single point, but the rays reflected from the margin of the mirror do not. This wandering of the marginal rays from the focus of the central rays is called *spherical aberration* (see Fig. 145), and can be shown graphically as follows: On a large sheet of drawing-paper draw a semicircle to represent a concave mirror. Draw a series of parallel lines to represent a beam parallel to the principal axis and as wide as the aperture of the mirror

(*i.e.* the diameter of the semicircle). Construct the reflected ray directions by drawing the normals, at the points where the parallel lines meet the semicircle, by joining the points to the centre of the semicircle, and making the angle of reflection equal to the angle between the ray and the normal, in each case. These reflected "rays" from the central portion of the semicircle will be found to pass through a single point, but the more outer ones do not. These, the intersections of adjacent pairs of reflected "rays," will be found to give a peculiar double curve, with its vertex at the principal focus (*i.e.* half-way between the pole of the semicircle and its centre of curvature); such a curve is called a *caustic* curve (the vertex is called the cusp). The effect can be shown to be similar with a beam of rays diverging from a point, instead of a parallel beam, except that the vertex is displaced; it is easily seen on looking into a glass half filled with milk, a cup of tea, or a saucepan, when a light is somewhere above.

To limit spherical aberration it is possible to cut off the marginal rays by the use of a diaphragm or stop. This causes a sharp definition of image, but reduces its intensity. A better method is to use a paraboloidal mirror. With this, parallel rays are focussed to a point —or a source of light at that point, called the focus, will give rays which are reflected in a parallel beam. This method is often used in

(*a*) lighthouses, searchlights, railway lamps, etc., to throw out a parallel beam;

(*b*) large telescopes to converge parallel rays from a very distant star or object to a point, where the image is then observed and studied in several ways.

In practice, of course, a point source of light cannot be obtained; and so, for example, searchlights, in which a powerful arc or electric light is used, throw out a beam which is slightly diverging.

NUMERICAL EXAMPLES—mirror problems.

EXAMPLE 1.—Where must an object be placed in front of a concave mirror of focal length 8 cms., so that the image

may be magnified twice, (*a*) when the image is real, (*b*) when it is virtual?

$$\text{In each case } \frac{\text{image distance}}{\text{object distance}} = \frac{v}{u} = 2,$$

$$\text{or } v = 2u.$$

(*a*) In this case image is to be real, and so both u and v are positive, as is the focal length, f.

$$\text{Thus} \quad \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \text{ becomes } \frac{1}{+2u} + \frac{1}{+u} = \frac{1}{+8},$$

$$\therefore \frac{1+2}{2u} = \frac{1}{8} \left\{ \begin{array}{l} \text{and } u = +12 \text{ cms.} \\ \text{or } v = +24 \text{ cms.} \end{array} \right.$$

(*b*) In this case the image is to be virtual, *i.e.* v is negative (and so $= -2u$), and u is positive, as is f.

$$\text{Thus} \quad \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \text{becomes} \quad \frac{1}{-2u} + \frac{1}{+u} = \frac{1}{+8},$$

$$\therefore \frac{-1+2}{2u} = \frac{1}{8} \quad \text{or} \quad \underline{u = +4 \text{ cms.}}$$

EXAMPLE 2.—A virtual image $2\frac{1}{2}$ times the size of an object is formed by a concave mirror at a distance of 10 ins. from it. What is the radius of curvature of the mirror? (Student should make a diagram in working this out.)

$$\frac{\text{Image distance}}{\text{Object distance}} = \frac{v}{u} = 2\tfrac{1}{2} \quad \text{or} \quad \frac{5}{2}$$

$$\text{Thus} \qquad u = \frac{2}{5}v = \frac{2}{5} \times 10 = 4 \text{ ins.}$$

Since image is virtual, $v = -10$ ins., whilst $u = +4$ ins.

$$\text{Thus} \quad \frac{1}{v} + \frac{1}{u} = \frac{2}{r} \quad \text{becomes} \quad \frac{1}{-10} + \frac{1}{+4} = \frac{2}{r}.$$

$$\therefore \frac{-2+5}{20} = \frac{2}{r} \quad \text{or} \quad r = \frac{+40}{3} = +13\tfrac{1}{3} \text{ ins.}$$

EXAMPLE 3.—*An object 1 in. high is place 8 ins. from a concave mirror of radius of curvature 5 ins. Obtain the position of the image and its magnification by a graphical method.*

Fig. 146 shows the principal axis and section of the mirror. From P mark off, according to a suitable scale, PC to represent 5 ins., so that C is the centre of curvature. Bisect PC at F, which is thus the principal focus. Mark B 8 ins. from P and draw AB perpendicular to PC, 1 in. high to represent the object.

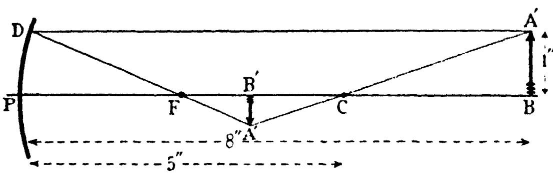


FIG. 146.

From A draw AD parallel to PB and cutting the mirror. Join DF and produce it to cut a line drawn through AC at A'. From A' drop a perpendicular A'B' meeting PB at B'. Then A'B' represents the image, and its height relative to AB can be measured, and so the magnification is obtained (0·45 approx.). PB' also gives the image distance (3·6 ins. approx.).

EXERCISES ON CHAPTER XV

1. According to what laws is light reflected from polished surfaces? Define the centre of curvature and focus of a concave mirror, and show that the radius of curvature is equal to twice the focal length.

2. What is meant by the "focal length" of a concave mirror? A source of light is placed at a distance of 45 cms. from a concave mirror of radius of curvature 30 cms. Calculate the position and kind of image. Illustrate your answer with a diagram. [*J.M.B.* 1925.]

3. The radius of curvature of a concave spherical mirror is 4 ins. A luminous object is placed on, and at right angles to, the principal axis at a distance of 6 ins. from the mirror. Find, *by geometric construction* (full size), the position of the image and give any further information you can concerning it. State clearly what optical principles you have used in your construction. [*L.G.S.* 1928.]

4. Illustrate by means of diagrams the formation of images by a concave mirror. If an object placed 2 ft. from a concave mirror gives rise to a real image 10 ft. from the mirror, calculate the focal length of the mirror. [*L.G.S.* 1920.]

5. Prove the relation between the focal length and the radius of curvature of a concave mirror. Where would you place a concave mirror of focal

length 1 ft. so as to throw an image of a candle flame on a wall distant 10 ft. from the candle? [*L.G.S.* 1924.]

6. A clearly lighted body stands upon the axis of a concave mirror, focal length 3 ins., at a distance of 10 ins. from the surface. Work out the details of magnification, position, and nature of the image that this information allows, and draw a diagram to illustrate your answer. [*J.M.B.* 1924.]

7. What are the centre of curvature and focus of a concave mirror, and how are they related? Explain how a concave mirror can be used to produce (*a*) a divergent beam, (*b*) a convergent beam, (*c*) a parallel beam, from a small source of light. Give diagrams in illustration.

8. A small glowing electric lamp is set up in two positions in turn on the axis of a spherical concave mirror whose radius of curvature is 12 ins. The first position is 4 ins., the second is 20 ins. Find the magnification in each case and describe the characteristics of the image. [*J.M.B.* 1922.]

9. You are given a concave spherical mirror of focal length 10 cms. What changes would you observe in the image formed by reflection at its surface as the object is brought nearer to the surface, starting at a distance of 100 cms.? Explain your answer by accurate geometrical drawings. [*J.M.B.* 1923.]

10. What is the principal focus of a concave mirror? Describe how you would obtain a series of readings from which the position of the principal focus of a concave mirror may be calculated. [*L.G.S.* 1920.]

11. Distinguish between a real and virtual image. Where must an object be placed in front of a concave mirror of focal length 20 cms. in order that the image may be magnified threefold (*a*) when the image is real, (*b*) when it is virtual? [*L.G.S.* 1921.]

12. Obtain a relation connecting the distance of an object from a concave mirror, the distance of the image and the radius of curvature of the mirror. An object 3 cms. in height is placed perpendicular to the axis of a concave mirror of 10 cms. focal length and 4 cms. from the mirror. Show where the image is formed. If an observer's eye is 25 cms. from the mirror, what is the least diameter of mirror necessary for the whole image to be visible at once? [*L.M.* 1928.]

13. A concave mirror has a focal length of 20 cms. Where must an object be placed so as to form an image twice the size when the image is (*a*) real, (*b*) virtual? Draw clear diagrams in illustration. [*L.G.S.* 1923.]

14. Distinguish between real and virtual images. A virtual image three times the size of the object is formed by a concave mirror at a distance of 12 ins. from it. What is the radius of curvature of the mirror? [*L.M.* 1925.]

15. Explain the use of a concave mirror to enable a person to see an enlarged image of his own face. Draw a diagram showing the position and size of the image if a 12-in. focal length mirror 8 ins. from the eye is used. How would the apparent size of the image compare with that obtained by a plane mirror in the same position? [*L.M.* 1924.]

16. State the laws of reflection of light. Show why a concave mirror produces an image of an object placed on its axis. Under what circumstances is this image (*a*) real, (*b*) virtual, (*c*) erect, (*d*) inverted, (*e*) enlarged, (*f*) diminished? [*C.W.B.* 1926.]

17. Explain how a concave mirror is used to obtain a magnified, erect image of an object. Draw the *pencil* of rays by which a person would see a point on his chin by means of a concave mirror of 12 ins. radius of curvature held 3 ins. in front of his face. (Put the object point 2 ins. below the axis and the eye 3 ins. above and make the diagram half size.) [*L.G.S.* 1929.]

18. How would you employ a concave spherical reflector and a point source of light in order to make (*a*) a parallel beam, and (*b*) a divergent beam of light? If the radius of curvature of the mirror was 2 ft., and the light was placed 1 ft. 4 ins. away from it, to what point would the beam converge after reflection? [*J.M.B.* 1926.]

19. Assuming the laws of reflection, show why a parallel beam of light after falling directly on a convex mirror is converted into a diverging pencil of rays. Why do images seen in a convex mirror always appear diminished in size? [*J.M.B.* 1926.]

20. Why is a convex mirror used as a "driving mirror" on a motor-car in preference to a plane or a concave mirror?

21. An object is (*a*) 40 cms., (*b*) 20 cms., (*c*) 5 cms., in front of a convex mirror of 20 cms. radius of curvature. Where is the image in each case, and what is the magnification?

22. Define the terms: centre of curvature, principal axis, aperture and focal length of a concave mirror. Being provided with some pins and a measuring rod describe how you would determine the focal length of a concave mirror. [*L.M.* 1930.]

CHAPTER XVI

REFRACTION AT PLANE SURFACES

We have been considering the movement of light in a single medium—air. Let us now see what happens when light enters another medium, *e.g.* glass or water. This can be studied with the Optical Disc, by directing a single narrow beam on a piece of glass with parallel faces. If the beam is made to fall perpendicularly on a glass face, it passes through the glass undeviated. If the disc is gradually rotated so that the beam is incident obliquely on the glass, it is then bent, or as we say, *refracted* towards the normal to the surface. It should also be noted that all the light does not enter the glass—a little is reflected at the surface into the air—and a faint reflected beam is seen (Fig. 147). As the angle of incidence of the beam on the glass (*i.e.* the angle between the incident ray and the normal to the surface) is increased, the angle of refraction (the angle between the refracted ray and the normal) gets larger too, but the angle of refraction, for light travelling from air into glass, is always less than the angle of incidence, *i.e.* the ray is bent towards the normal. It should also be noticed that as the angle of incidence is made greater, the more bright becomes the reflected ray, thus showing that less light is entering the

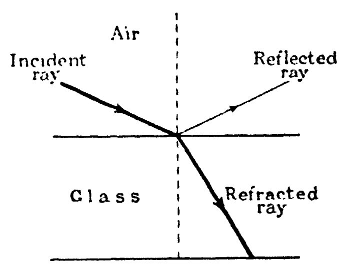


FIG. 147.—Refraction and Partial Reflection at an Air-Glass Surface.

glass. Further, observe the behaviour of the beam, travelling in glass, when it reaches the glass-air surface (parallel to the surface at which the light entered the glass from air). Some of it is seen to be reflected internally, whilst most is seen to travel out into the air, being bent away from the normal. This emergent beam will be observed to appear parallel to the original incident beam, although laterally displaced.

When light, entering a second medium from a first medium, is bent towards the normal, the second medium is said to be **optically denser** *than the first medium, but if the light is bent away from the normal the second medium is said to be* **optically rarer** (*or less dense*) *than the first.* Though this property of optical density does appear to be associated with physical density (*e.g.* glass, water, etc., are denser than air, both physically and optically), it is not always so. Thus methylated spirit is less dense physically, but more dense optically, than water. So when considering the idea of density with regard to refraction, the words "optical density" should be used.

Laws of Refraction.—History records that Ptolemy, who lived in the second century A.D., measured the amount of the bending consequent upon light entering certain substances from air. In 1621, Snell discovered **the relation that the ratio $\dfrac{\textbf{sine of the angle of incidence}}{\textbf{sine of the angle of refraction}}$ was a constant quantity** for the light passing from one certain medium into another certain medium. This ratio is called the **refractive index** (or **index of refraction**) from the first medium to the second medium, and is usually represented by μ, or, to be precise, ${}_{\text{air}}\mu_{\text{medium}}$. **This relationship is sometimes called the second law of refraction. The first law states that the refracted ray lies in the same plane as the incident ray and the normal to the surface, but is on the opposite side of the normal to the incident ray.**

A simple experiment to determine the refractive index from air to glass can be carried out with a glass slab, section

PQRS, about 4 ins.×3 ins.×1 in. This is placed with a larger face on the middle of a piece of drawing-paper (Fig. 148), and its outline marked by a fine pencil. In the middle of side PQ a point A is marked, and ANM is drawn perpendicular to PQ, meeting RS at M. AN is thus the normal to the surface PQ. Between AN and AP a series of lines AB, AC, AD, etc., are drawn from A. On line AB two pins, P_1, P_2, are fixed upright some distance apart. Now, whilst looking through the glass slab from side RS, two pins P_3, P_4, are fixed so that they appear to be in the same straight line as P_1 and P_2, seen through the glass. A line KL is drawn, through the pin points P_3 and P_4, meeting SR in K. Then AK must be the path of the ray represented by BA (fixed by pins P_1, P_2 in line) in the glass. Thus for this ray proceeding from air into glass, $B\hat{A}N$ is the angle of incidence, and $K\hat{A}M$ is the angle of refraction.

The procedure is repeated with pins P_1, P_2 on AC, AD, etc., in turn, and the corresponding paths, AV, AX, etc., in the glass found.

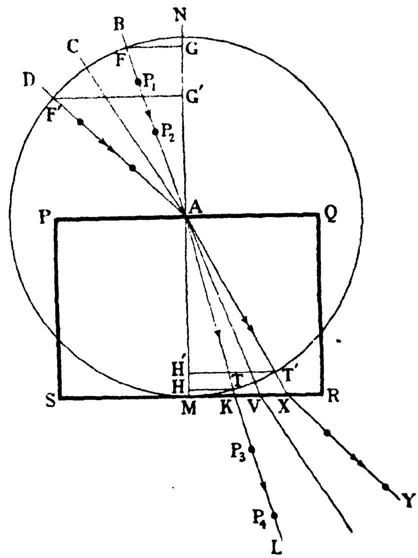


FIG. 148.—Refraction of Light through a Glass Slab.

With centre A, and radius AM, a circle is drawn, and from where the lines, radiating from A, cut this circle, perpendiculars are drawn on to NAM (see Fig. 148).

Now considering the ray BA proceeding along AK, the refractive index of air to glass

$$=\frac{\text{sine angle of incidence}}{\text{sine angle of refraction}}=\frac{\sin B\hat{A}N}{\sin K\hat{A}M}=\frac{FG/AF}{HT/AT}$$

$$=\frac{FG}{HT},\text{ since AF=AT, being radii of the circle.}$$

Thus, by measuring FG, F′G′, etc., and also HT, H′T′, etc., several values of the refractive index are calculated and the average obtained.

In this experiment it should also be verified that, in each case, the emergent ray is parallel to the corresponding incident ray.

TABLE OF REFRACTIVE INDICES

Air to water	1·33	Air to carbon bisulphide	1·67
,, window-glass	1·53	,, methylated spirit	1·36
,, crown glass	1·53–1·61	,, aniline	1·59
,, flint glass	1·62–1·79	,, turpentine	1·47
,, ruby	1·76	,, paraffin	1·44
,, diamond	2·417	,, Canada balsam	1·53

For liquids, μ varies with temperature, decreasing in value as the temperature is raised.

Refraction and Theories of Light.—The earliest theory of light of any importance was, like the theory of heat, a "fluid" theory, and was accepted at the same time as the Caloric Theory of Heat, when "fluid" theories were fashionable. The so-called **Corpuscular Theory of Light** considered light as due to small particles, or corpuscles, of fluid thrown off by luminous bodies. Here, however, a difficulty arose in explaining refraction, as illustrated by the well-known coin experiment. In this a penny was placed in an empty bowl, and a tube was supported, just out of line with the penny, so that, on looking down the tube, an observer failed to see the coin, until water was carefully poured into the bowl to cover the penny to a good depth without moving it. Fig. 149 makes this clear. Light scattered from the penny and entering the air, from the water, is bent away from the normal. Thus rays in direction PA are bent along direction PE, and so the observer sees the penny, receiving light from all parts of it in this manner.

The corpuscular theory (with which theory Newton's name is chiefly associated) explained refraction as due to the attraction of the molecules of the substance for light corpuscles, and reflection by surfaces as due to their repulsion. (The great difficulty was to explain

partial refraction and reflection—the normal occurrence.) In 1676 Römer showed that light had a finite velocity (and crossed the earth's orbit in 22 minutes). Newton considered that light corpuscles, incident obliquely, with this velocity, on a surface, *e.g.* air–water surface, had resolved velocities parallel to and perpendicular to the surface. The component parallel to the surface was unaffected when the corpuscles entered the other medium, but the component perpendicular to the surface was increased. Thus if the length AO represented the velocity in air of the incident corpuscles along that line

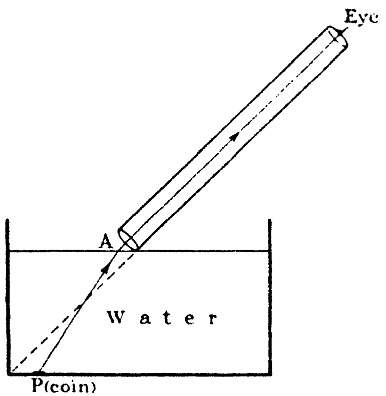


FIG. 149.

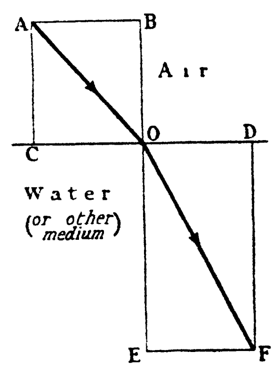


FIG. 150.

(in Fig. 150), the lengths AB and AC represented the considered components. In the second medium the component parallel to the surface remained the same, *i.e.* OD=AB. But the component perpendicular to the surface was increased, and so this was represented by OE, which was greater than OB. Hence OF, the diagonal of the rectangle of which OD - so the are adjacent sides, represented the velocity really is. This the second medium. This obviously m kly at minnows or the normal—*i.e.* the light was [illegible] nearer the surface normal. Further, since FO>AO,

in the second medium was greater than the velocity in air. This result seems to have been accepted without much question, and at that time terrestrial methods of measuring the velocity of light in different media were not known.

About 1800 the fluid, or corpuscular, theories were practically abandoned, and a wave theory of light became accepted. A luminous body generated waves, in the "æther" of space, which travelled outwards with a definite velocity in air, but which must be reduced in other media. Consider a parallel beam of light, wave-front AB (see p. 171), travelling in air and meeting the surface of separation from water, AS (Fig. 151). When A meets the surface, B is some distance from it. When the beam is observed in water it is found to be travelling in a different direction (bent towards the normal). Produce EB to meet the air–water surface at C. Let CH and AG be the direction of the beam in the water, so that CD, which is perpendicular to them, represents the wave-front in water. Thus, whilst part of the wave-front AB is travelling from B to C in air, the part A must travel from A to D in the water,

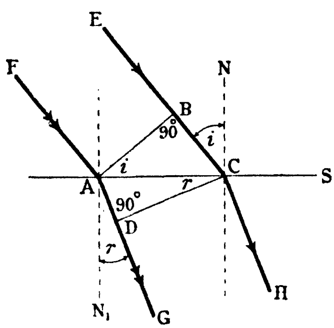


FIG. 151.

or $$\frac{\text{Velocity of light in air}}{\text{Velocity of light in water}} = \frac{\text{distance BC}}{\text{distance AD}}.$$

AB is perpendicular to EC and FA.
-he corp
name is chief complement of $B\hat{C}A = B\hat{C}N$ = angle of
to the attraction cidence, i;
light corpuscles, aement of $D\hat{A}C = N_1\hat{A}G$ = angle of
their repulsion. (Ttion, r.

Now $\sin i = \sin B\hat{A}C = \frac{BC}{AC}$, and $\sin r = \sin A\hat{C}D = \frac{AD}{AC}$,

and so $BC = AC \sin i$, whilst $AD = AC \sin r$.

$$\therefore \frac{\text{Velocity of light in air}}{\text{Velocity of light in water}} = \frac{BC}{AD} = \frac{AC \sin i}{AC \sin r} = \frac{\sin i}{\sin r} = {}_a\mu_w$$

The refractive index of air to another medium is found to be always greater than 1 (light entering another medium from air is always bent towards the normal); and so light must travel faster in air than in any other medium. This, then, is a vital point of difference between the corpuscular and wave theories of light. It was not till 1850 that Foucault showed experimentally that light did travel slower in water than in air, in the ratio of the refractive index of air to water, and so confirmed belief in the wave theory. It should be noticed that this relationship for the refractive index from air to another medium, which expresses the change of speed of light when entering the medium from air, is of more importance than the mere relationship of sines of angles.

Images by Refraction.—*Apparent Depth of Water.*—A favourite example of the phenomenon of refraction is that of the "bent stick." If a ruler or stick is partly immersed in a tank of water, it is observed to appear bent below the water. Fig. 152 explains this. Rays from the bottom C of the stick ABC do not reach the eye by a direct path, but, owing to refraction away from the normal when travelling out of water into air, travel along CD in water and DE in air. Thus C appears to be along ED produced, at H. Similarly other parts of BC appear raised, and so the stick appears to be ABH, instead of ABC as it really is. This explains the difficulty in grasping quickly at minnows or tadpoles in water; they appear to be nearer the surface than they really are.

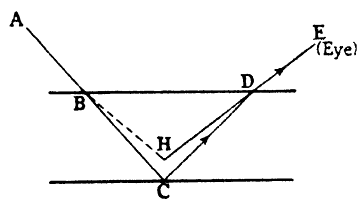


FIG. 152.

Consider a point O at the bottom of water of depth OA (Fig. 153). Let a ray of light from O travel to the eye, its path being OB in water and BE in air. Produce EB to cut AO in I. The observer at E thus sees O as if it were at I, *i.e.* I is the apparent image of O.

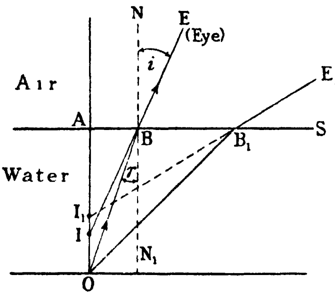


FIG. 153.

Conversely, a ray of light travelling from EB in air would be refracted along BO in water.

$$\text{Hence} \qquad {}_{\text{air}}\mu_{\text{water}} = \frac{\text{sine of angle of incidence}}{\text{sine of angle of refraction}}$$

$$= \frac{\sin E\hat{B}N}{\sin O\hat{B}N_1},$$

where NBN_1 is perpendicular to surface AS.

But since NB is parallel to AO, $E\hat{B}N = B\hat{I}A$, and $O\hat{B}N_1 = B\hat{O}A$.

$$\therefore \ {}_a\mu_w = \frac{\sin B\hat{I}A}{\sin B\hat{O}A} = \frac{AB/IB}{AB/OB} = \frac{OB}{IB}.$$

If the observer is practically above A, BO=AO approx., and IB=AI approx.

$$\text{Thus } {}_a\mu_w = \frac{AO}{AI} \text{ approx.} = \frac{\text{real depth}}{\text{apparent depth}} = \frac{4}{3} \text{ for water.}$$

That is to say, an object in water, observed from above, appears to be only three-quarters of its real depth. If the eye is seeing an object obliquely, the latter seems to be still nearer to the surface than it really is—*e.g.* the eye at E_1 sees the object O as if it were at I_1. The student should test this by drawing OB 4 units of length and BI 3 units. If OB_1 is then drawn 8 units and BI_1 marked 6 units for I_1 to be on OA, then I_1 will be found to be

above I. This explains why the lines on the floor of a swimming bath, when viewed from about the middle of the bath, appear to curve upward, even though the bottom of the bath is sloping downward towards the deep end.

The altitude of a star or planet appears to be greater than it really is, for light from either is refracted in its path to the earth, owing to a variation in optical density of air layers. As the physical density of the air increases, or decreases, so its optical density increases, or decreases. The denser air being usually nearer the surface of the earth, the path of light when received there is more oblique than a direct line from the heavenly body to the earth (as shown in Fig. 154).

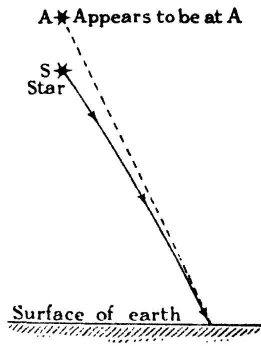


FIG. 154.—Apparent Altitude of a Star (exaggerated).

The apparent flattening of the sun and moon when near the horizon is due to refraction. Rays from the lower part of the sun or moon travel at greater length through the earth's atmosphere than rays from the higher part, and so are more refracted. Thus the lifting-up effect is greater for the lower part of the sun or moon than for the top. The extremities of the sides are equally affected, and so the moon appears elliptical instead of circular.

Stars are a great distance away from the earth, and so a very small proportion of the light radiated by them reaches us. Light received from a star travels along paths which are very variable, with a consequent variation in time taken. Thus the light received from a star by an observer on the earth is not uniform in intensity; at one instant little light is received, at another, light is received by more than one path, and so we get a twinkling effect. Planets are of greater angular size than stars, *i.e.* they subtend at the eye a much larger angle than do stars; thus more light is received at any instant from a planet by an observer on the earth. Refraction effects are therefore inappreciable and so planets do not twinkle. This is borne out by the fact that stars, as seen in large telescopes, do not twinkle, for the amount of light collected by the telescope and then viewed is approximately constant.

The displacement of the image of an object seen through a refracting material with parallel faces is used to measure the refractive index of the material. A very simple method for a slab of glass similar to that used in the experiment on p. 265 is as follows (Fig. 155). A pin A is fixed in drawing-paper so that it touches the back of the glass slab. A drawing-pin is placed upside down on the top of the slab, and its position adjusted, when looking into the slab (and above it, at the same time), so that there is no parallax motion between the image of A seen through the glass and the point of the drawing-pin seen in the air.

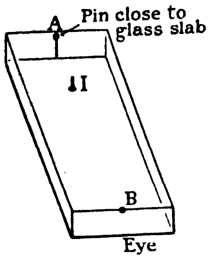


FIG. 155.

$$\text{Then } {}_{\text{air}}\mu_{\text{glass}} = \frac{\text{real thickness of glass}}{\text{apparent thickness of glass}} = \frac{AB}{BI}.$$

A more precise method of finding the position of the image of pin A is to use a piece of plane mirror, not so tall as the glass slab, and placed in front of it. Another pin P is placed in front of the plane mirror and adjusted so that there is no parallax motion between the image of A, seen through the glass slab, and the image of P in the plane mirror (see Fig. 156).

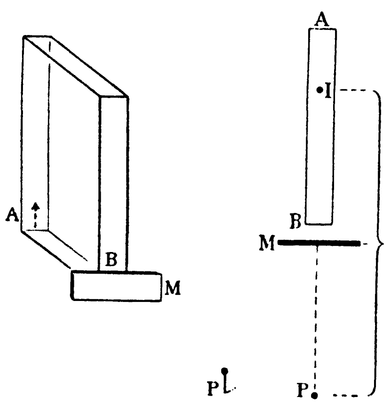


FIG. 156.

By reflection in plane mirror, PM = MI.
From diagram, BI = MI − MB = MP − MB.

$$\text{Thus } {}_{\text{air}}\mu_{\text{glass}} = \frac{\text{width of glass}}{\text{apparent width}} = \frac{AB}{BI} = \frac{AB}{MP - MB}.$$

This method can be used for a liquid, contained in a rectangular glass tank

Construction of the Refracted Path of Light.—The following is the method for constructing the path of

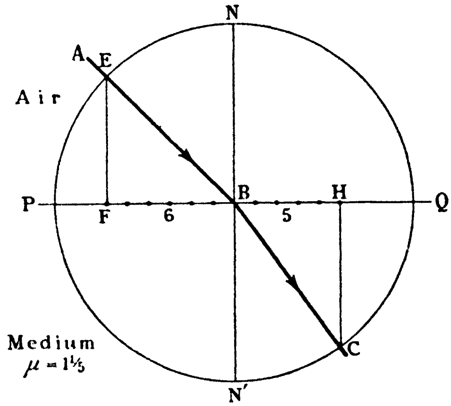


FIG. 157.—Construction of Refracted Ray.

light which is incident, along any path AB in air (Fig. 157), upon a medium of refractive index μ (*e g.* $1\frac{1}{5}$): Draw a normal NBN′ to the surface PQ at B. With centre B and a suitable radius describe a circle cutting BA at E. From E draw a perpendicular to surface PQ meeting it at F. *Divide FB into μ equal parts* and mark off, along PQ on the other side of FB, *BH equal to one of these parts* (in our case FB is divided into six parts and BH=5 of these parts, since $\mu : 1 = 6 : 5$). From H draw HC perpendicular to PQ, and on the opposite side of it to AB, cutting the circle at C. Then *BC represents the refracted ray.*

For, angle of incidence from air into medium=ABN =alt. $B\hat{E}F$, and angle of refraction, if the construction is justified=$N'\hat{B}C$=alt. $B\hat{C}H$.

$$\text{Whence } \frac{\text{sine angle of incidence}}{\text{sine angle of refraction}} = \frac{\sin B\hat{E}F}{\sin B\hat{C}H} = \frac{FB/EB}{BH/BC}$$

$$= \frac{FB}{BH} \text{ for } EB=BC, \text{ being radii of circle,}$$

$$= \mu \text{ by construction.}$$

Thus BC *is* the refracted ray.

By a reversal of this method it is possible, given the refracted ray and the refractive index, to construct the incident ray. BH is made 1 part and FB μ parts.

Critical Angle and Total Internal Reflection.—Consider a ray of light BA meeting an air surface, when travelling in an optically denser medium (Fig. 158). On entering the air the ray is bent away from the normal and along the direction AH. Suppose the angle of incidence of ray BA is increased (position now CA). Then it will be even more refracted, its path now being AK in air. It is clear from the diagram that a limiting position is reached when the incident ray DA is

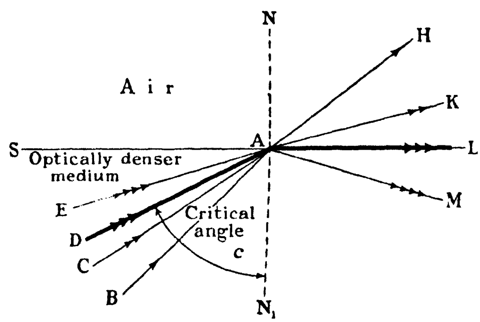


FIG. 158.—Refraction of Light when passing into an Optically Denser Medium.

just refracted along the surface AL. A ray EA in the medium, at a greater angle of incidence, cannot be refracted into the air, and so must be reflected back at the surface. It is so along path AM such that $E\hat{A}N_1 = N_1\hat{A}M$. This can be illustrated by means of the optical disc, which has suitable parts for showing the phenomenon in glass and water (Fig. 159). It will be noticed that a ray BA (Fig. 158) is not completely refracted along AH, part of the light being reflected internally at the surface SAL. Further, as the angle of incidence of the ray is increased, less light is refracted and more reflected.

Eventually the position is reached where the light is refracted to just skim the surface of separation. This is called the critical position, and the angle of incidence then ($D\hat{A}N$ in Fig. 158) is called the **critical angle.** As the angle of incidence is increased above this value, the light is seen to be completely reflected internally, inside the optically denser medium, according to the laws of regular reflection. Such reflection is called **total internal reflection** to distinguish it from all other cases of reflection which are, as we have already shown, only partial.

It is thus obvious that refraction, from one medium into a less optically dense medium, can only occur when

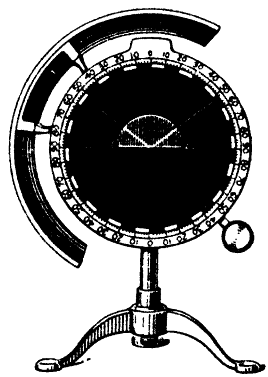


FIG. 159.—Showing Total Internal Reflection by the Optical Disc.

the angle of incidence is less than the critical angle. If the angle of incidence is greater than the critical angle, then the light is reflected back into the denser medium. Now, theoretically, a ray travelling in air along LA (Fig. 158) would, in the denser medium, travel along AD.

$$\text{Hence } {}_{\text{air}}\mu_{\text{medium}} = \frac{\sin N\hat{A}L}{\sin D\hat{A}N_1} = \frac{\sin 90^\circ}{\sin \text{critical angle } (c)}$$

$$= \frac{1}{\sin c} = \text{cosec } c.$$

μ for air to glass $=1 \cdot 5$ approx., and so critical angle is about 42°.
μ for air to water $=1 \cdot 33$ approx., and so critical angle is about $48\frac{1}{2}^\circ$.

Thus an observer under water sees (*a*) objects outside as if they were in a cone of semi-vertical angle $48\frac{1}{2}^\circ$, and lifted up, and (*b*) objects in the water away from

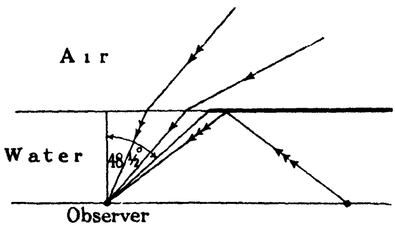


FIG. 160.—Viewing Objects from under Water.

him by total internal reflection (see Fig. 160). If a glass of water be held in the air above the level of a bright light, the water surface, viewed from below, appears like one of mercury or silver, and a coin or spoon in the water can be seen in this surface by total internal reflection.

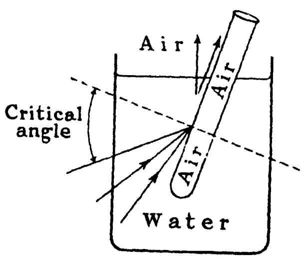


FIG. 161.

To illustrate the above phenomenon, perform the following experiment. Hold an empty test-tube slant-wise in water and look at it from above (Fig. 161). Light travelling in water towards the test-tube at an angle of incidence greater than the critical angle cannot pass into the air in the test-tube and so is reflected upwards. The total reflection causes the test-tube to appear as if silver-coated. Pour water into the test-tube; the effect disappears. The bright metallic appearance of cracks in plate-glass windows when looked at obliquely is due to such total internal reflection in the glass at the cracks.

Mirages.—Pools of water appear to be on the surface of a tarred road or a stone promenade on a hot afternoon. This occurs when the sun is shining along the direction of either of them and is at a certain height. Rays of light, travelling through air layers of varying optical density,

gradually change in direction. As a result they strike some layers at angles greater than the critical angle.

In hot countries the resultant effect is more common. The layers of air near the earth become very hot and rare,

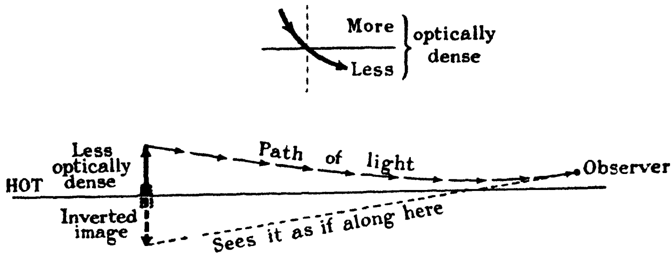


FIG. 162.—Mirage produced in Hot Regions.

and, at the same time, less optically dense than the air above. Thus some rays of light from a distant object travel as shown in Fig. 162, giving an inverted image.

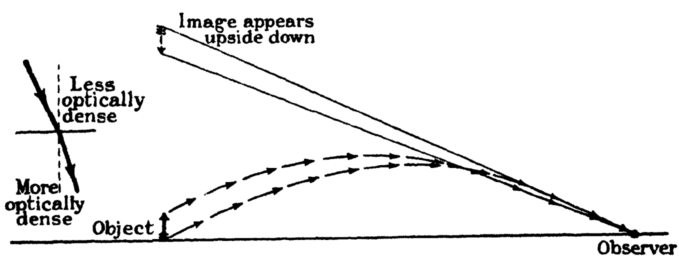


FIG. 163.—Mirage produced in Very Cold Regions.

Some light, when at the critical angle position, travels along parallel to the ground and gives the appearance of bright pools of water. This optical illusion is called a mirage.

In arctic regions, and over the sea in hot weather, mirages are seen in the sky. The air near the ground is cooler and more optically dense than the air above, and the effect is as shown in Fig. 163.

Refraction of Light through a Prism.—We have so far only considered the passage of light through a medium bounded by parallel faces. A prism, however, is any portion of a medium lying between two faces which are not parallel. The intersection AD of these two faces P and Q (Fig. 164) is called the *edge* of the prism. It is usual to consider the effect of the prism on light travelling in a plane, or section, which is perpendicular to the edge of the prism. This section, which is called a *principal section*, is usually represented in diagrams as the plane of the paper. The angle between the faces is the *refracting angle*, and it is measured by the corresponding plane angle (B$\hat{A}$C) of the principal section. For ordinary work triangular prisms are used, commonly with the principal section approximately equilateral. Prisms with the two faces at right angles have a special use, as will be shown later.

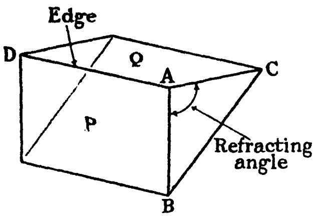


FIG. 164.—Refracting Prism.

The paths of rays of light through a prism can be studied by using the optical disc, and the effect of altering the angle of incidence of a ray falling on the first face should be carefully observed. The path of a ray through a prism can also be traced out, using pins, as follows: Place two pins, P_1 and P_2, on a line DE (Fig. 165),

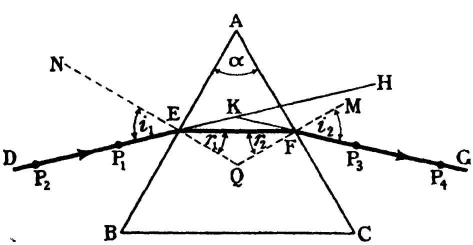


FIG. 165.

drawn at an angle to one face of a glass prism standing on a sheet of drawing-paper. Looking through the prism from the other face place two pins, P_3 and P_4, so that they appear to be in the same straight line with the image of P_1 and P_2 seen through the prism. Draw a line through these pin-points to meet the face in F (and the side AC of the principal section). Join EF. Then the path of a ray DE must be EF in the prism, and FG on emerging into the air. Produce DE to H, and let GF produced meet it at K. Then angle HKG is called for obvious reasons, the *deviation* of the ray DE. Draw normals, NEQ and MFQ respectively, intersecting at Q.

Let $N\hat{E}D=i_1$, $Q\hat{E}F=r_1$, $E\hat{F}Q=r_2$ and $M\hat{F}G=i_2$.

Then the angle of deviation $D=H\hat{K}G=$sum of int. opp. $K\hat{E}F+K\hat{F}E$

$$\begin{aligned}&=(K\hat{E}Q-r_1)+(K\hat{F}Q-r_2)\\&=(N\hat{E}D-r_1)+(M\hat{F}G-r_2)\\&=i_1-r_1+i_2-r_2=\underline{i_1+i_2-(r_1+r_2)}.\end{aligned}$$

But the refracting angle of the prism is A or α, which=the supplement of angle Q, since angles AEQ, AFQ are both right angles.

$$\therefore \alpha=r_1+r_2.$$

$$\text{Hence } \underline{D=i_1+i_2-\alpha.}$$

Thus the angle of deviation varies with the initial angle of incidence i_1 of the ray DE on face AB.

To study the relationship a very important extension of the experiment, just described, should be carried out. Having marked the outline ABC of the prism on the paper, from E, and between EN and EB, draw a series of lines at angles 20°, 30°, 40°, 45°, 50°, 55°, 60°, 70° and 80° to the normal NE. Place the prism in position and then in turn place two pins on each of these lines to represent incident rays, and trace the paths of the corresponding emergent rays on the other side of AC by the method just described. Hence obtain the paths of the rays through the prism, and in each case measure the

angle of deviation and the angle of emergence by a protractor. Enter the results in a table :

Angle of incidence on face AB $= i =$ N$\hat{\text{E}}$D.	Angle of deviation $=$ D $=$ H$\hat{\text{K}}$G.	Angle of emergence $= e =$ M$\hat{\text{F}}$G.
20°		
⋮		
80°		

These results should be plotted in graphical form—two curves being drawn on the same paper as in Fig. 166,

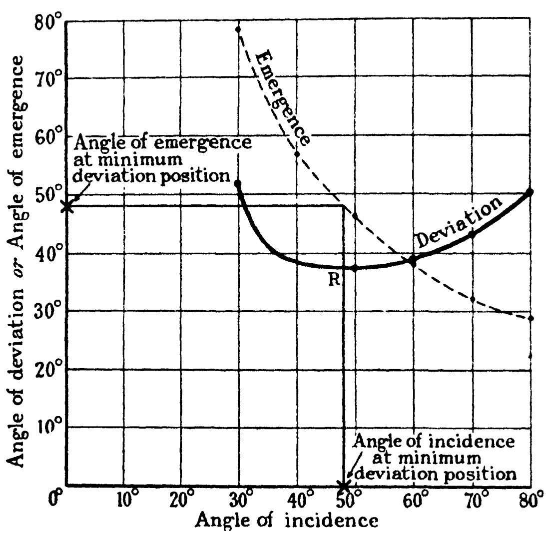


FIG. 166.

which shows the results of an experiment with a prism. The curve showing the variation of the angle of deviation with the angle of incidence indicates a minimum value at R. The corresponding value of the angle of emergence is found to be equal to the angle of incidence, *i.e.* **in the position of minimum deviation, the angles of incidence and emergence are equal.**

Thus, in Fig. 165, in the position of minimum deviation, $i_1 = i_2 = i$, and so r_1 must $= r_2 = r$.

Hence angle of minimum deviation,

$$D_M = i_1 + i_2 - \alpha = 2i - \alpha, \quad \text{or} \quad i = \frac{D_M + \alpha}{2}.$$

Also $$r_1 + r_2 = 2r = \alpha \quad \text{or} \quad r = \frac{\alpha}{2}.$$

Thus air μ glass for the prism $= \dfrac{\sin i}{\sin r} = \dfrac{\sin \dfrac{D_M + \alpha}{2}}{\sin \dfrac{\alpha}{2}}$

D_M, i, and r are found from the graph (or, without going through the whole experiment, the position of the prism for minimum deviation can be found by trial, and the values determined), and if α is measured the refractive index can be calculated.

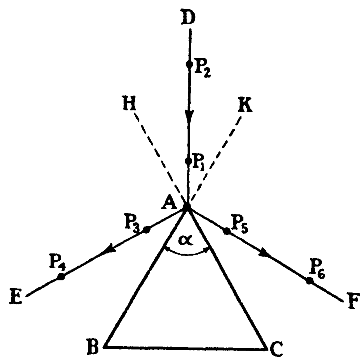


FIG. 167.

A simple method for finding the angle of the prism is as follows: Place the prism on drawing-paper as before, and draw a line AD such that DA produced would lie between B and C (Fig. 167). Place pins P_1, P_2 on this line. Now, looking into face AB, place pins P_3, P_4 in line with the reflected image of P_1 and P_2. Similarly, looking into face AC, place pins P_5, P_6 in line with the reflected image of P_1 and P_2. Through the pin points draw lines EA and FA respectively. Produce BA and CA to K and H respectively.

Since a ray DA is reflected to AF from surface AC, $D\hat{A}H = F\hat{A}C$,

also a ray DA is reflected to AE from surface AB,

$\therefore D\hat{A}K = E\hat{A}B$.

Hence $F\hat{A}C + E\hat{A}B = D\hat{A}H + D\hat{A}K = H\hat{A}K =$ vert. opp. $B\hat{A}C = \alpha$.

$\therefore E\hat{A}F = 2\alpha$, *i.e.* the angle of the prism is half the angle between the two reflected rays.

Total Reflection Prisms.—Since the critical angle for glass is about 42°, rays in glass meeting an air surface at an angle of incidence greater than 42° undergo total *internal reflection, with no loss of intensity. Thus a prism can be used as a mirror. This is often done,* particularly when *it* is required *to turn rays* through 90°. A prism used for such a purpose has a refracting angle of 90°, the other angles being 45°. As shown in Fig. 168, the light is caused to enter one of the faces containing

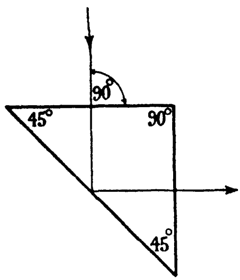


FIG. 168.—Ray turned through 90° by a Right-angled Prism.

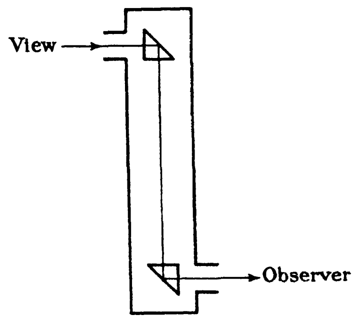


FIG. 169.—Prism Periscope.

the right angle perpendicularly, so that it is undeviated and meets the face opposite the refracting angle at 45° (>the critical angle, 42°). It is thus reflected at 45°, *i.e.* is turned through 90°, and meets the other face perpendicularly, passing out undeviated. As shown in Fig. 169 this principle is incorporated in a form of periscope. Another use of such a prism is to invert a beam of light. For this, the light, when travelling parallel to the base of the prism,

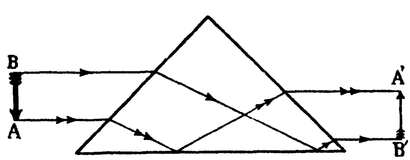


FIG. 170.—Right-angled Prism used for Inversion.

is directed on a refracting face. The result is shown in Fig. 170, total internal reflection taking place at the base

of the prism. (The student should insert the angles in the figure and see that it is so.) Such a prism is often used with an optical lantern to project on a screen an erect image of experiments performed. As we shall see later, such a lantern ordinarily gives an inverted image of an erect picture. Since prismatic glass has been manufactured, another use of the prism to reflect light through 90° is developing. As shown in Fig. 171, light from above is directed into a room instead of being partially scattered or reflected downward by the window. This is particularly valuable for basement rooms and rooms with windows opening on alleys.

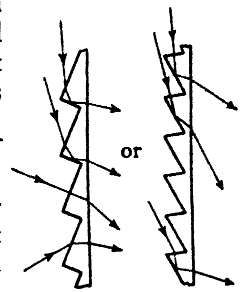


FIG. 171.—Action of Prismatic Glass for Windows.

Path of Light through Prisms (constructional method).—The methods given on p. 273 can be used to solve the path of a ray of light into and out of a glass prism. Several questions are given involving these constructions.

EXERCISES ON CHAPTER XVI

1. Define "refractive index." Indicate briefly what you consider to be the cause of the refraction of light. How would you measure the refractive index of a liquid? [*J.M.B.* 1928.]

2. Describe an experiment to determine the index of refraction of glass. Show that when things are viewed obliquely through a window they do not appear in exactly their true position. [*J.M.B.* 1926.]

3. Light is said to be refracted when it passes from one medium to another. What is meant by this? How does it explain the phenomenon of total reflection? Give a practical example of the application of total reflection. [*J.M.B.* 1928.]

4. Explain carefully the terms *refraction* and *refractive index*. Describe any natural phenomenon due to refraction. [*L.G.S.* 1920.]

5. Rays of light from an electric lamp strike the surface of water in a tank. Explain with the help of diagrams what happens to the rays (*a*) when the lamp is in the air above the surface of the water, (*b*) when the lamp is in the water. [*J.M.B.* 1929.]

6. Show why the image of a stationary object, as seen in a plane mirror, always appears in the same place, from whatever position it is viewed. Why is not the same thing true of the apparent positions of things seen lying at the bottom of a pool of water? [*J.M.B.* 1927.]

7. Explain the term "optical image" and distinguish between "real" and "virtual" images. Describe one method of determining the position of the image of a pin lying at the bottom of a bowl of water, and show how to deduce the refractive index of water. [*L.M.* 1923.]

8. Explain the term *refractive index*. How would you find the refractive index of water as accurately as possible? [*L.G.S.* 1924.]

9. Explain (*a*) how it is that a pool of water looks shallower than it really is, (*b*) the distorted appearance of objects viewed through a window glazed with cheap glass. [*L.G.S.* 1920.]

10. Explain the term *refractive index*, and show why, if the refractive index from air to water is $\frac{4}{3}$, that from water to air is $\frac{3}{4}$. Describe some method of measuring the refractive index of water. [*L.M.* 1920.]

11. Explain as briefly and clearly as you can why the apparent depth of water is three-quarters of its real depth when the observer is looking straight down, and how the apparent depth changes with a change in his point of view. [*L.M.* 1923.]

12. Explain, by the aid of carefully drawn diagrams, the apparent bending of a stick partly immersed in water, and the apparent raising of the bottom of a vessel containing liquid. Show how this phenomenon may be used in finding the refractive index of a liquid. [*L.M.* 1919.]

13. If the moon had an appreciable atmosphere, a star, which was actually just behind the edge of the moon with regard to an observer on the earth, would actually be visible outside the moon. Explain this fact by the aid of a diagram.

14. *When a clean empty test-tube is pushed* down into a bowl of water the submerged part often appears as though coated with silver. Why is this? Describe any experiment performed in the laboratory for finding the conditions under which the phenomenon occurs. [*J.M.B.* 1923.]

15. Explain the statement that the refractive index from air to glass is 1·5. Give a careful diagram of the path of a ray of light in passing from air through a rectangular block of glass 3 ins. thick when the angle of incidence in air is 60°. [*L.G.S.* 1925.]

16. State the laws of refraction of light. Two incident rays of light enter a cube of glass, side 2 ins., at the middle point of one side, making angles of incidence 15°, 30°, respectively. Draw the path of these rays inside the cube and after they again emerge into the air. The refractive index of glass is 1·5. [*J.M.B.* 1923.]

17. Draw as accurately as you can the path of three pencils of rays from a point under water (refractive index $\frac{4}{3}$) to three different eyes above the surface, the angles of incidence of the central rays of the pencils in the water being 0°, 30°, 45°. By means of your figure explain the change in the apparent depth of the water as the position of the observer changes. [*L.M.* 1923.]

18. A ray of light travelling in glass strikes its surface, making an angle with the normal of (*a*) 30°, (*b*) 60°. Show by diagrams the subsequent path in each case, explaining fully. Refractive index of glass = 1·5. [*C.W.B.* 1928.]

19. State the laws of refraction of light, and add sufficient explanation to make their meaning clear. An object rests on the bottom of a swimming bath 11 ft. from the side in water 8 ft. deep. Draw on squared paper the paths of three rays from the object meeting the surface 3, 5 and 7 ft. respectively from the point on the surface immediately above the object. (Refractive index of water=$\frac{4}{3}$.) From your diagram determine approximately the apparent position of the object as seen by a boy standing at the water's edge, his eye being 4 ft. above the surface. [*L.G.S.* 1929.]

20. A ray of light is incident at an angle of 45° on a prism whose angle is 60° and refractive index $\frac{3}{2}$. Construct the path of the ray through the prism and measure the deviation produced. How would you determine this deviation experimentally? [*L.M.* 1926.]

21. State the laws of reflection of light, and explain how they may be applied to determine the angle of a glass prism. [*L.M.* 1920.]

22. What is meant by *refraction* and *refractive index?* A ray of light travelling parallel to the base of an equilateral glass prism strikes one side. Trace the path of the ray through the prism by construction, and find the deviation. Refractive index=1·5. [*C.W.B.* 1926.]

23. What do you understand by *refracting angle* of a prism and the *angle of deviation?* Show how you would carry out an experiment with a given prism with the object of finding out how the angle of deviation varies with the angle of incidence. What important conclusions would you draw from the result of your experiment? [*J.M.B.* 1922.]

24. Explain what is meant by the refractive index and critical angle for light incident on a transparent material. A ray of light falls normally on one face of a prism whose refracting angle is 30°. If the refractive index of the material be 1·5, draw carefully a diagram showing the path of the ray through the prism, and find the deviation of the ray. [*L.G.S.* 1921.]

25. What is meant by the *critical angle* of a substance? Light falls normally on the face of a prism of refractive index 1·5 and emerges from the second face parallel to the surface. Determine graphically the angle between the faces. [*L.M.* 1925.]

26. What is meant by the refractive index of a transparent substance? Describe how it may be determined for glass in the form of a prism. [*L.M.* 1925.]

27. A ray of light is incident at an angle of 60° on a prism of 60° of refractive index 1·5. Draw a diagram to scale showing the path of the ray through the prism, and measure the deviation produced. Find *graphically* the *minimum deviation* produced by the prism. (All constructional details must be shown and fully explained.) [*C.W.B.* 1924.]

28. State the laws of refraction of light at a plane surface. The cross-section of a glass prism is an equilateral triangle ABC. In passing through the prism a ray of light is parallel to BC. By means of a careful diagram determine the angle between the original incident ray and the final emergent ray (refractive index 1·5). [*L.G.S.* 1925.]

29. Explain what is meant by the refractive index of a transparent medium. A ray of light making an angle of 30° with the normal to the plane surface of a piece of glass enters to meet a second surface making an angle of 60° with the former. If the refractive index of glass is 1·5, trace the path of the ray.. [*L.M.* 1928.]

30. *A beam of light from a distant source falls upon the surface of a* solid cylindrical block of glass whose axis is at right angles to the direction of the beam. Trace the course of a single ray which does not pass through the axis. Under what circumstances would a pencil undergo no deviation whatever on passing through the glass? (Refractive index of air to glass=1·5.) [*J.M.B.* 1922.]

31. Describe an experiment to verify the law of refraction, using a rectangular slab of glass and some pins. Explain how it is sometimes difficult to see a crack in a glass tumbler when the eye is directly in front of it, but quite easy in other positions. [*L.G.S.* 1921.]

32. When light has to be reflected through 90°, a right-angled prism is frequently used instead of a silvered mirror. Explain with diagrams how this is done, and why this method is preferred. [*L.G.S.* 1924.]

33. State the laws of refraction of light. A ray of light is incident at an angle of 60° on one face of a glass cube of 3 ins. side. Give a geometrical construction for tracing the path of the ray through the cube and find how much it is displaced laterally. (Refractive index of glass=1·5.) [*L.M.* 1930.]

CHAPTER XVII

REFRACTION AT CURVED SURFACES—LENSES

IT is common knowledge that a piece of lentil-shaped glass, called a *lens*, can be used to focus rays of the sun on a piece of paper, which becomes charred or burned. Such a piece of glass was called a burning-glass, and, by the use of one, Priestley in 1774 directed the rays of the sun on to some mercuric oxide in a closed tube and

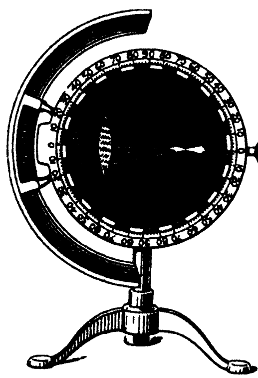


FIG. 172.—Action of a Converging Lens.

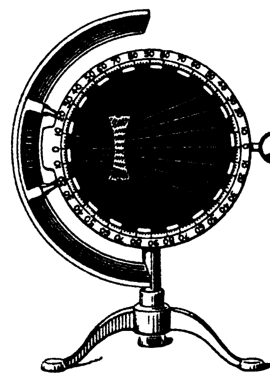


FIG. 173.—Action of a Diverging Lens.

so was the first person to isolate and study oxygen. This action of focussing rays from the sun, *i.e.* parallel rays, on a spot, is an important one.

A lens is a portion of a refracting medium (*i.e.* one that transmits and deflects light) bounded by two spherical surfaces, or by one spherical and one plane surface. The latter kind is, of course, only a special

case of the general class, the radius of the plane surface being infinity.

Thus lenses are of two types—those which are thicker in the centre and those which are thinner in the centre. By means of the Optical Disc it is easy to see that only lenses of the first type focus rays of light. If the aperture, with several slits, is used, a series of narrow beams of light can be directed on one of each type of lens in turn. It is then seen that the lens which is thicker at the centre converges the light (Fig. 172), and so it is called a *converging lens*. It is also called a *convex lens* because the surfaces (except a plane one) appear to be convex to an observer.

But a lens which is thinner at the centre will be found to produce a divergent beam, and so we call it a *diverging lens* (Fig. 173). It is also called a *concave lens*, because the surfaces (except a plane one) appear concave to an observer.

Lenses are classified as shown below (Fig. 174).

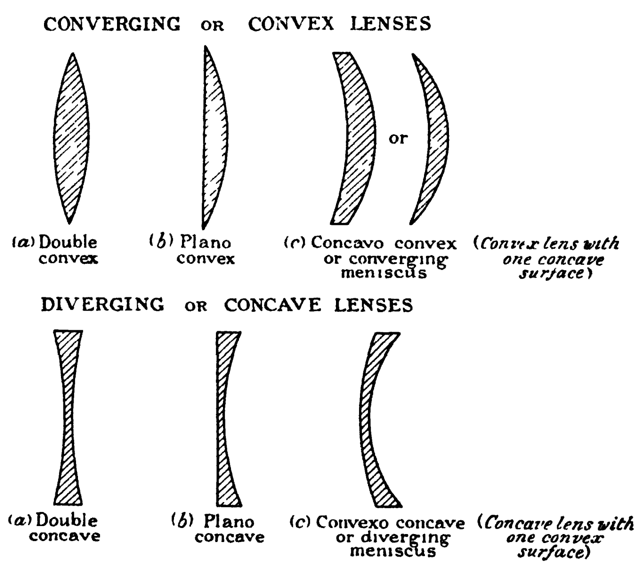


FIG. 174.

For ordinary work, kind (*a*) of each class is used.

The straight line joining the centres of curvature of the surfaces (or, if one surface is plane, the straight line through the centre of curvature of the curved surface and perpendicular to the plane surface) is called the **principal axis of the lens.**

In diagrams dealing with lenses in section it is usual to show a *principal section*, which is in a plane through the principal axis.

The point to which a parallel beam of light is focussed by a converging, or convex, lens is called the principal focus of the lens. The Optical Disc experiment showed that it was a **real focus. But with a diverging, or concave, lens** a diverging beam is obtained. This appears to diverge, or come, from a point which is **the principal focus** and **is,** of course, **a virtual focus.** The distance of the principal focus from the surface of the lens is called, in each case, the *focal length* of the lens.

Action of a Lens compared with the Action of Prisms.—A further study of the results of the experiment described on p. 280 shows that, working away from the minimum deviation position, there is an increase in the deviation, no matter whether the angle of incidence of the light is smaller or larger. Consider a series of truncated prisms (*i.e.* prisms each with the pointed end cut off parallel to the base) put together as shown in Fig. 175, the refracting angles of the central prisms being 0° and the others increasing to a maximum for the top and bottom prisms. Suppose a ray PA is striking the shaded prism section in the minimum deviation position, so that it emerges symmetrically on the other face, along BD. Then a ray PE strikes the face of the prism section at E at an angle greater than that required for the minimum deviation position, and so is more deviated (than is the ray PA) in the passage through the prism section. A ray PH, striking a prism section at H, is incident upon it at an angle smaller than that necessary for the minimum deviation position, and so is more deviated. It is conceivable that the change in angles of the truncated prisms,

together with the positions of the latter, might be such that the effect on rays falling on the prism would be to cause them all, after refraction, to pass through the same point D on the line along which a ray, from P, passes undeviated. Now this is what does happen with a convex

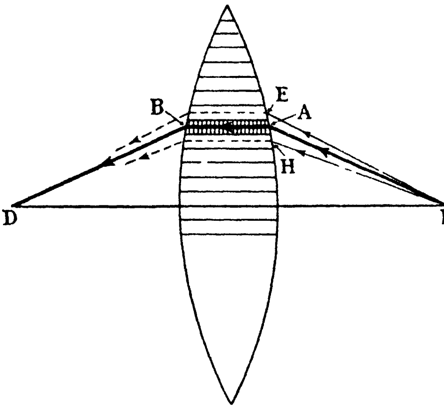


FIG. 175.

lens, which is thin, and the surfaces of which are spherical ; so we can consider a lens as being built up of an infinite number of truncated prisms, of gradually-varying refracting angles.

Similarly, the behaviour of a diverging lens is as if it were built up of a series of truncated prisms, but the rays are not converged, appearing to diverge from a single point (Fig. 176).

It can be shown that if an object AB (Fig. 177) is at a distance of u units of length from a thin converging lens of focal length f units of length, an image is produced at a distance v units of length from the lens, such that $\frac{1}{\mathbf{v}}-\frac{1}{\mathbf{u}}=\frac{1}{\mathbf{f}}$. This relationship holds for all lenses, provided that the *sign convention is adopted* in all cases. This convention is similar to that used for mirrors. Thus :

(1) One side of a lens (usually the right-hand side in

diagrams) on which is the source of light, is called the positive side of the lens, and the other side is called the negative side ;

(2) Distances are measured from the lens, and are

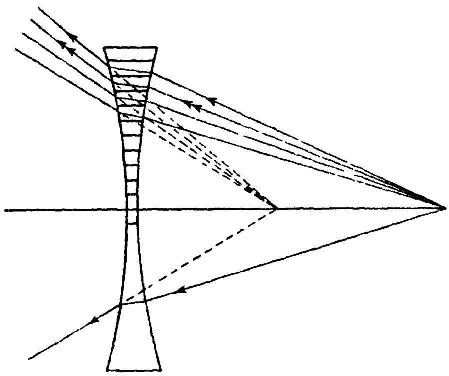


FIG. 176.

positive or negative according as they are measured on the positive or negative side of it (*i.e.* towards or away from the incident light).

Thus the *focal length of a converging lens is negative*,

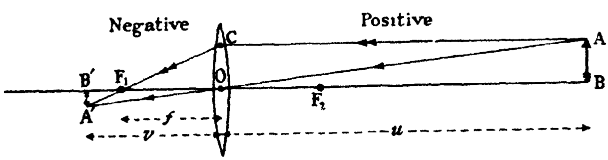


FIG. 177.

and so converging, or convex, lenses are often called negative lenses, *i.e.* they converge light towards their negative side. The *focal length of a diverging lens is positive*, and so diverging, or concave, lenses are often called positive lenses.

Graphical Methods of Studying Image Formation by Lenses.—We have already seen the first general rule

—rays parallel to the principal axis are refracted by a lens so that they converge to (or appear to diverge from) a single point called the principal focus of a lens (Fig. 178). Conversely, if a source of light is at a distance, equal to the focal length, from a converging

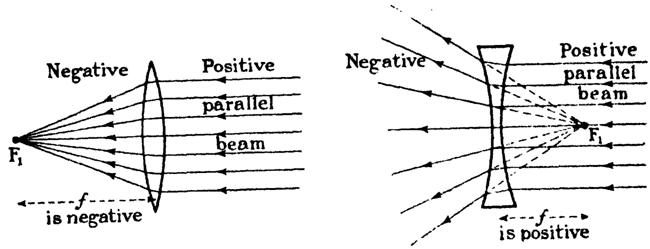
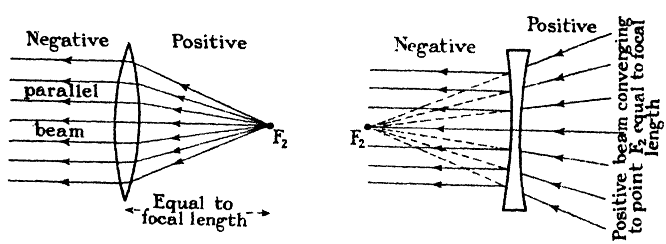


FIG. 178.

lens, rays, after refraction, travel parallel to the principal axis (Fig. 179, *a*). Or, if a beam, incident upon a diverging lens, is converging to a point beyond the lens, at a distance equal to the focal length of the lens, then after refraction by the lens it is parallel to the principal axis (Fig. 179, *b*).



(*a*) FIG. 179. (*b*)

Thus we distinguish the two points equidistant from a lens, and along the axis, at a distance equal to the focal length of a lens, as the *first* (F_1) and the *second* (F_2) *principal foci*. In lighthouses, signal lamps, bull's-eye lanterns, optical lanterns, etc., a source of light is placed at the second principal focus of a large converging lens and a parallel beam thus obtained.

The second general rule for drawing diagrams involves the use of the so-called *optical centre* of a lens. Consider a ray AB incident upon a converging lens at A (Fig. 180) and being refracted to C, such that the inclination of BC to both surfaces is the same. Then the path of the emergent ray CD is, obviously, parallel to AB. Thus the effect of the lens on the ray AB is merely slightly to displace it laterally. It can be shown geometrically that all rays passing through the lens so that the incident and emergent directions are parallel must pass through a fixed point O in the lens. This point O is called the *optical centre* of the lens, and, when the surfaces are of the same curvature, is at the centre of the lens. For a thin lens the lateral displacement is negligible, and so, for our work with lenses, *rays passing through the optical centre are considered to be undeviated. Thus in graphical work it is legitimate to treat rays passing through the optical centre, taken as the intersection of the lens position and the principal axis, as continuing their path undisturbed.*

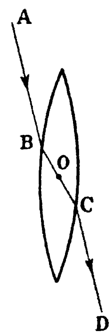


FIG. 180.

Simple Image Formation.—Consider an object AB at a distance $+u$ cms. outside the focal length f of a converging lens. The construction for the image is as shown in Fig. 177. A ray AC, parallel to the principal axis F_1OB, after refraction passes through F_1, the first principal focus of the lens. A ray from A through the optical centre O passes on undeviated. Hence, join CF_1 and AO, and produce them to meet in A′. Then from the relationship $\frac{1}{\text{image distance}} - \frac{1}{\text{object distance}} = \frac{1}{f}$, all rays from A must pass through a fixed point. This point must be A′ where two rays meet. Thus A′ is the image of A. Draw a perpendicular A′B′ from A′ on to the principal axis. B′ is obviously the image of B. (A and B are equidistant from the lens, and so must A′ and B′ be.) The image is inverted and *real* (it could be received on a screen, for the light rays *do* pass through A′B′).

If the lens is a diverging one, the ray AC, parallel to the principal axis BF_1O, is refracted and then appears to have come from the first principal focus F_1 of the lens (Fig. 181). A ray AO through the optical centre O passes through the lens undeviated. Thus, draw lines F_1C and AO to intersect at A′, which is thus the image of

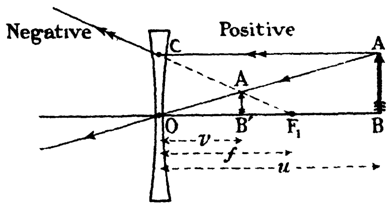


FIG. 181.

A, and the perpendicular A′B′ on the principal axis represents the image of AB. This image is erect but virtual, for an observer receives rays as if they come from A′B′. It cannot, of course, be received on a screen (the rays of light do *not* pass through the image), but an observer sees it by intercepting the diverging beam, preferably when close up to the lens.

In both figures it should be seen that the triangles ABO, A′B′O are similar, for $A\hat{B}O = A'\hat{B}'O = 1$ right angle, and $A\hat{O}B$ is either common or = vert. opp. $A'\hat{O}B'$.

$$\text{Thus } \frac{A'B'}{AB} = \frac{OB'}{OB} = \frac{\text{image distance}}{\text{object distance}} = \frac{v}{u} \text{ (numerically).}$$

But

$$\frac{\textbf{height of image (or a line in the image)}}{\textbf{height of object or (height of corresponding line in the object)}} = \frac{\mathbf{A'B'}}{\mathbf{AB}}$$

is called the magnification of the image.

Thus **the magnification of the image in a lens**

$$= \frac{\textbf{image distance}}{\textbf{object distance}}.$$

This is the same relationship as was obtained with mirrors.

Note also that in either figure OC=AB, and it can easily be shown that triangles $F_1A'B'$, F_1CO are similar.

Thus $$\frac{A'B'}{OC}=\frac{F_1B'}{F_1O}.$$

But OC=AB, and so $\frac{A'B'}{AB}$ which $=\frac{v}{u}$ also $=\frac{F_1B'}{F_1O}$ $=\frac{\pm(f-v)}{f}$.

Substituting correct signs according to the sign convention, we have

Converging Lens	*Diverging Lens*
$\frac{-v}{+u}=\frac{-(v-f)}{-f}$.	$\frac{+v}{+u}=\frac{f-v}{+f}$.

$\therefore$ in each case $vf=u(f-v)$, or $vf=uf-uv$.

Dividing by uvf, $\frac{1}{u}=\frac{1}{v}-\frac{1}{f}$, and thus $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, which is the relationship between the image and object distances and the focal length for a lens previously stated.

Obviously an object placed at A′B′ would give rise to an image at AB, *i.e.* the two points, distant u and v from the lens, are conjugate foci (*cf.* spherical mirrors).

Position of the Image produced by a Converging Lens.—This will be treated as in the case of mirrors, *i.e.* an object will be considered to approach the lens from infinity, and the corresponding change in the position of the image found. The characteristics of the image must be studied. These are (1) real or virtual, (2) erect or inverted, (3) magnified or diminished (reduced).

(*a*) *Object at Infinity.*—The rays reaching the lens are then considered to be parallel and so are converged to the principal focus of the lens, giving a real image, theoretically a " point " image.

As the object is taken nearer the lens a real, inverted, diminished image is produced (Fig. 177).

(*b*) When the *object is at a distance from the lens, equal to its focal length.* Then, of course, rays from the object are, after refraction through the lens, parallel to the principal axis, *i.e.* the image is at infinity.

Or $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ becomes $\frac{1}{v}-\frac{1}{+f}=\frac{1}{-f}$,

whence $\frac{1}{v}=0$ and so $v=\infty$.

Between the positions (*a*) and (*b*) the image has changed from a point image to one of infinite size. Somewhere between, the image must be the same size as the object. When this is so the image distance is equal to the object distance. Then u and v are numerically equal, but v is negative, and so $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ becomes $\frac{1}{-u}-\frac{1}{u}=\frac{1}{-f}$.

$$\therefore \frac{2}{u}=\frac{1}{f} \quad \text{or} \quad u=2f. \quad \text{Thus}$$

(*c*) When the *object is at a distance equal to the focal length*, the image is at the same distance on the other side of the lens, is real, inverted and the same size as the object. Further, when the object is between ∞ and $+2f$, the image is diminished; whilst when the object is between $+2f$ and $+f$, the image is magnified.

(*d*) *When the object is between the second principal focus and the lens*, an interesting case appears.

For $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, or $\frac{1}{v}=\frac{1}{u}+\frac{1}{f}$ (which is negative).

Since $u<f$, $\frac{1}{u}>\frac{1}{f}$ and so $\frac{1}{v}$ is positive.

$\therefore$ v is positive, *i.e.* a virtual image is produced

This is shown, diagrammatically, in Fig. 182.

Further, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ which is negative, and so $\frac{1}{v}$ must be $<\frac{1}{u}$ (v being negative and u positive)

Thus v must be $>u$, *i.e.* the image is a magnified one.

Ray AD, parallel to the principal axis, is refracted along DF_1.

Ray AO, through the optical centre O, is undeviated.

Now AD (which=OB) is $<OF_2$ and so is $<OF_1$.

Thus the rays DF_1 and OH do not meet, and so an observer receiving these rays would consider them to have come from A′, the point of intersection of DF_1 and OH produced backwards. Thus A′ is the virtual image

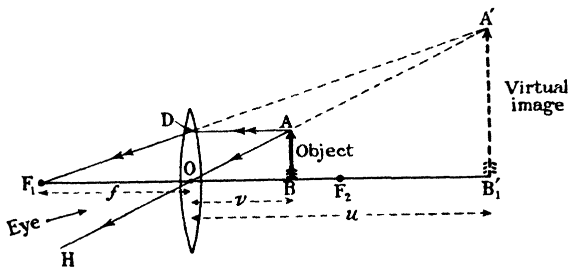


FIG. 182.—Principle of Simple Microscope, or Reading-glass.

of A, A′B′ being the virtual image of AB. Since OB′ must be >than OB, this virtual image is also magnified, but to receive it the eye must be placed fairly close to the lens so as to intercept the diverging beam.

This, then, is the use of a converging lens as a *simple microscope* (Gk. *mikros*, smaller+scope) for reading small print, and vernier scales, for botanical and crystallographic work, and by the watchmaker, etc. The object to be viewed is held close to the lens within a distance equal to its focal length, and a virtual, erect, magnified image of the object is seen.

(*e*) When the *object is at the lens*, i.e. $u=0$, then $\frac{1}{u}=\infty$.

Thus $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ becomes $\frac{1}{v}=\infty+\frac{1}{f}$, and so $v=0$.

Thus the image is at the lens too—this can be seen by gradually bringing such a lens right up to one's face.

We can thus summarise our results for a *converging lens* as follows :

Position of Object.	*Position of Image.*	
Infinity	Principal focus	
↓	↓	real, inverted, diminished image
Twice the focal distance (positive)	Twice the focal distance (negative) Same size as object	
↓	↓	real, magnified, inverted image
Focal distance (positive)	Infinity	
↓	↓	virtual, erect, magnified image
Lens	Lens	

The types of beam which can be obtained with a converging lens are clearly shown in Fig. 183.

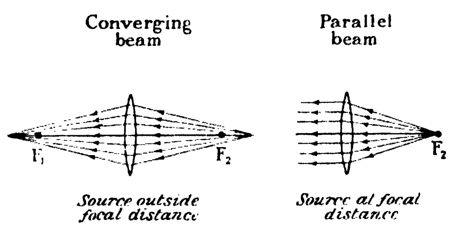


FIG. 183.

Position of the Image produced by a Diverging Lens.—This is much easier to study, there being two extreme positions. The focal length, f, is, of course, positive.

(1) *Object at Infinity*—then the image is virtual and at the focus.

For $u=\infty$, and so $\frac{1}{u}=0$. Thus $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ becomes $\frac{1}{v}=\frac{1}{f}$, i.e. $v=f$, which is positive.

(2) *When the object is at the lens*, the image is at the lens too. For if $u = 0$, $\frac{1}{u} = \infty$,

thus $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ becomes $\frac{1}{v} = \infty + \frac{1}{f}$.

Hence $v = 0$.

(3) *When the object is anywhere between the lens and infinity*, the image is virtual, erect, and diminished, as shown in Fig. 181.

For $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ or $\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$; and u and f are positive.

Thus $\frac{1}{v}$ is positive and $> \frac{1}{u}$.

Hence v is positive and $< u$ (*i.e.* diminished image).

An interesting position is *when* $u = f$, *i.e.* object is at the principal focus.

Then $\frac{1}{v} = \frac{1}{f} + \frac{1}{+f} = \frac{2}{f}$ or $\underline{v = \frac{1}{2}f.}$

This result should be compared with what happens in the case of a converging lens. The results obtained for a diverging lens can be tabulated as under :

Object.	*Image.*	
Infinity	Principal focus	
↓	↓	
Principal Focus	½ Focal length	Virtual, erect, diminished
↓	↓	
Lens	Lens (same size).	

Determination of Focal Lengths of Lenses.—Some methods of measuring the focal length of a lens are given.

Converging Lenses.—(1) The most obvious method is to use a luminous object, and by means of the lens obtain a *real* image on a screen. The object and image distances are measured and the focal length calculated. Since $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}=\frac{u-v}{uv}$, $f=\frac{uv}{u-v}$. In substituting values, it must be remembered that the values of v are negative, and so $f=-\frac{uv}{u+v}$, when u and v are numerical values only.

The experiment should be carried out in a similar manner to that performed with a concave mirror and described on p. 249, and data should be obtained to verify that the magnification=size of image÷size of object. The results should be tabulated as follows:

Height of Object= cms.

Object distance u cms.	Image distance v cms. (negative)	$f=-\frac{uv}{u+v}$ using numerical values.	Height of Image in cms.	$\frac{\text{Height of Image}}{\text{Height of Object}}$	$\frac{\text{Image distance}}{\text{Object distance}}=\frac{v}{u}$
				A	B

Thus the mean value of the focal length is calculated. Parallel values in columns A and B should, of course, be approximately equal.

(2) In another method a plane mirror is employed, and use is made of the fact that when an object is at a distance from a converging lens, equal to its focal length, rays from the object are, after refraction at the lens, parallel to the principal axis. If, then, this beam meets a plane mirror normally, the rays return along their original paths, giving rise to an inverted image at the same place as the object (and, of course, the same size

as the object). Thus the lens is placed flat on a plane mirror, and a pin is held by a clamp above the centre of the lens, and its position so adjusted that there is no parallax motion between the pin and its inverted image. The pin is then at the principal focus of the lens, the focal length of which can be measured.

(3) *Displacement Method.*—This method is introduced because students commonly observe the happening described here, and also because it illustrates the term "conjugate foci."

If in the method (1) given above, the screen and object are placed about a metre apart, it will be found that there are two positions in which the lens can be placed to give a real image (inverted) on the screen.

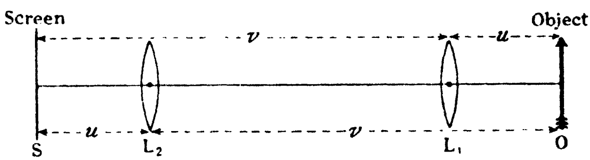


FIG. 184.

When the lens is in position L_1 (Fig. 184) the image is magnified; when the lens is at L_2 the image is diminished. This is, of course, merely a case of the two conjugate positions. If OL_1=the first object distance$=u$, and L_1S=the first image distance$=v$, numerically, then for the second image position, OL_2 must$=v$ and L_2S must$=u$ (both numerically).

Note also that if the magnification, when lens is at L_1, is x, then the magnification, when lens is at L_2, is $\frac{1}{x}$.

Let the distance between the two lens positions

$$=L_1L_2=l=(v-u),$$

and the distance between object and screen

$$=OS=d=(v+u), \text{ signs being neglected.}$$

$$\text{Then from } \begin{array}{r} d=v+u \\ l=v-u \end{array} \left\{ \begin{array}{l} \text{adding, } d+l=2v \text{ or } v=\dfrac{d+l}{2} \\ \text{subtracting, } d-l=2u, \text{ or } u=\dfrac{d-l}{2}. \end{array} \right.$$

Now, using the formula $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$ and introducing signs

$$\frac{1}{-f}=\frac{1}{-v}-\frac{1}{u} \quad \text{or} \quad \frac{1}{f}=\frac{1}{v}+\frac{1}{u}.$$

Thus, introducing the above values, $\frac{1}{f}=\frac{2}{d+l}+\frac{2}{d-l}$

$$\text{or} \quad \frac{1}{f}=\frac{2(d-l+d+l)}{(d+l)(d-l)}=\frac{4d}{d^2-l^2} \quad \text{or} \quad \underline{f=\frac{d^2-l^2}{4d}}.$$

Note that a limiting position is found when the two image positions just coincide, i.e. $l=0$;

$$\text{then} \qquad f=\frac{d^2}{4d}=\frac{d}{4} \quad \text{or} \quad \underline{d=4f},$$

i.e. *the distance between screen and object must be greater than four times the focal distance if two images are to be obtained.*

Diverging Lenses.—The usual method is to determine the positions of an object placed on one side of such a lens, and its image when viewed from the other side of the lens. Since the image is always a virtual one, it cannot be obtained on a

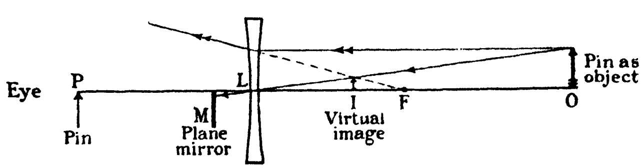


FIG. 185.—Finding Position of Virtual Image produced by a Diverging Lens.

screen. The method is similar to that used for finding the position of the image seen by refraction through a slab of glass (p. 272). A small plane mirror is placed in front of the lower

half of the diverging lens, and a pin placed in front of the mirror. Its position is then adjusted so that there is no parallax motion between the image of this pin seen in the plane mirror and the image of another pin (the object) seen through the top half of the lens. Thus in Fig. 185, PM=MI.

Hence image distance $=LI=MI-LM=MP-LM=v$, and is positive.

Object distance$=LO=u$, and is positive.

Thus $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=\frac{1}{MP-LM}-\frac{1}{LO}$ and is thus determined.

Power of a Lens.—In this book the usual Physics convention is used, *i.e.* the focal length of a convex or converging lens is taken to be negative and that of a concave or diverging lens to be positive. But this is not always adopted, *e.g.* opticians use the opposite signs, considering the focal length of a convex lens to be positive and that of a concave lens to be negative. Further, opticians consider the so-called *focal power* of a lens, which is taken to be the reciprocal of its focal length.

The unit of focal power (or dioptric strength, symbol F) is the *dioptre*, which is the focal power of a lens of 1 metre focal length.

The power in dioptres is measured as $=\frac{1}{\text{focal length}}$ in metres, the sign being opposite to the conventional sign used in this book. This method is very useful for calculations, as curvatures are expressed by the same method, and so the usual formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ becomes $V-U=F$ on the dioptric system. Opticians use a modified spherometer (p. 41) which indicates for a curved surface the curvature as a fraction of unit curvature (taken to be that of a surface of 1 metre radius of curvature).

Spherical Aberration in Lenses.—The formula for lenses which we have used is dependent upon the following conditions :—

(1) The lenses must be thin, *i.e.* the curvature of the lens surfaces is small (the radii of curvature being appreciable).

(2) The lenses must be of narrow aperture.

If these conditions are not fulfilled, then a point source does not give rise to a point image, and a parallel beam is not focussed to a point. The latter is easily shown by using the optical disc, or an ordinary lamp and a large reading-glass. The rays farthest from the centre of the lens are seen to have a shorter focal length, *i.e.* are more refracted.

Thus photographic cameras are supplied with a "diaphragm," or "stop," to shut off the rim of the lens. This "stopping down" has the disadvantage of reducing the amount of light which enters, with the result that a longer exposure is necessary. The diaphragm is made adjustable so that on dull days, or for general "snap" photography, a fair amount of lens is exposed. Whenever possible, it is obviously better, however, to use a small aperture and to give a longer exposure. The risk of movement then necessitates the use of a tripod, with which better results, especially for still objects, or groups, can be obtained.

Large lenses can be made with differently curved surfaces so that spherical aberration is obviated. These, however, are costly. In many optical instruments, as will be shown later, the aberration is reduced by using plano-convex lenses, instead of the ordinary double form, the curved surface being placed to face the incident light. This results in a comparatively equal deviation of a ray at both surfaces, and then the aberration is at a minimum.

Problems concerning Lenses.

EXAMPLE 1.—A lens placed 8 ins. from an object produces a real image, twice the size of the object. Where must the lens be placed if an erect image twice the size is to be produced?

$$\frac{\text{Size of image}}{\text{Size of object}} = \frac{2}{1} = \frac{\text{image distance}}{\text{object distance}} = \frac{\text{image distance}}{8 \text{ ins.}}.$$

$\therefore$ Image distance $= 16$ ins., and since image is real, is negative.

$\therefore$ Thus object distance, $u = +8$ ins., and image distance, $v = -16$ ins.

Thus $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}=\frac{1}{-16}-\frac{1}{8}$, where f is the focal length of the lens.

$$\therefore \frac{1}{f}=-\frac{3}{16} \quad \text{or} \quad f=-\frac{16}{3} \text{ ins.}$$

To produce an erect image, *i.e.* a virtual image, twice the size, if object distance $=+u$, image distance must $=+2u$.

Thus $\quad \frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad$ becomes $\quad \frac{1}{+2u}-\frac{1}{+u}=-\frac{3}{16}$.

Thus $\quad \frac{1-2}{2u}=-\frac{3}{16} \quad$ or $\quad \frac{1}{2u}=\frac{3}{16}$.

Thus $u=2\frac{2}{3}$ ins., *i.e.* object must be $2\frac{2}{3}$ ins. from the lens.

(Diagrams should be drawn by the student to illustrate this example.)

EXAMPLE 2.—A convex lens of 6 cms. focal length gives rise to a real image, of a source of light, 30 cms. away from the lens. What is the position of the source? How far must the lens be moved so that another image of the source is obtained on the screen?

Let u cms. = distance of object from lens.

Then f cms. = focal length $=-6$ cms., since lens is convex and v cms. = distance of image from lens $=-30$ cms., since image is real.

$$\therefore \frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \text{becomes} \quad \frac{1}{-30}-\frac{1}{u}=\frac{1}{-6}.$$

Thus $\frac{1}{u}=\frac{1}{6}-\frac{1}{30}=\frac{4}{30}$ or $\underline{u=7\frac{1}{2} \text{ cms.}}$ when $v=-30$ cms.

i.e. the source of light is $7\frac{1}{2}$ cms. from lens.

Another image can be obtained by moving the lens into the second of the two conjugate positions for image and object. Thus image distance must become $-7\frac{1}{2}$ cms., and the object distance $+30$ cms.

To verify this, $\frac{1}{v}-\frac{1}{u}$ then $=\frac{1}{-7\frac{1}{2}}-\frac{1}{+30}=-\frac{1}{30}-\frac{2}{15}=-\frac{5}{30}$ $=-\frac{1}{6}=\frac{1}{f}$ (or $f=-6$), and so is correct.

EXAMPLE 3.—What must be the focal length of a magnifying glass if it is to give an erect image, magnified five times, of an object placed 3 cms. from it?

$$\frac{\text{Size of image}}{\text{Size of object}}=\frac{5}{1}=\frac{\text{distance of image}}{\text{distance of object}}.$$

Object distance $u=+3$ cms.

$\therefore$ Image distance $v=15$ cms., and is positive, since an erect image is formed (as shown in Fig. 182).

$$\text{Thus } \frac{1}{\text{focal length}(f)}=\frac{1}{v}-\frac{1}{u}=\frac{1}{+15}-\frac{1}{+3}=-\frac{4}{15}.$$

$$\therefore f=-3\tfrac{3}{4} \text{ cms.}$$

EXERCISES ON CHAPTER XVII

1. Name the chief properties of a convex lens. A candle flame is placed at the following distances in turn, from a thin convex lens whose focal length is 10 cms.: (*a*) 15 cms., (*b*) 10 cms., (*c*) 5 cms. State the result obtained in each case. [*L.G.S.* 1925.]

2. How is it that an image can be produced with a convex lens and not with two prisms placed base to base? A convex lens has a focal length of 20 cms. Draw a graph on squared paper showing distances of a real object from the lens along the horizontal axis and distances of the corresponding image along the vertical axis. [*L.G.S.* 1921.]

3. Draw diagrams illustrating the formation of images by a convex lens. How would you determine experimentally the focal length of such a lens? [*L.G.S.* 1919.]

4. Explain clearly the terms divergent, parallel, and convergent as applied to a beam of light. How should a convex lens be used in order to produce beams of these three types?

5. Describe in detail how you would verify by experiment the formula connecting the focal length of a thin convex lens with the distances of any pair of conjugate foci from the lens.

6. What are the focal points of a convex lens and what do you know about their positions? Prove that the magnification produced by a lens is proportional to the distance of the image from one of the focal points. [*L.M.* 1922.]

7. Explain, with the aid of a diagram, how a magnified image of an object can be obtained by means of a convex lens. How does the magnification depend on the focal length of the lens?

8. Describe an experiment you have performed to find the value of the focal length of a convex lens. Point out the precautions you took to obtain a good result. Name three important uses of such lenses, and give diagrams to illustrate them. [*L.G.S.* 1928.]

9. Describe two methods of finding the focal length of a convex lens. How would you place a lens of 1 ft. focal length so as to form a real image of 4 times the linear dimensions of the object? [*L.M.* 1923.]

10. Show by diagrams the formation of real and of virtual images by a converging lens. An erect image of an object at 25 cms. distance is required at 60 cms. distance from a converging lens. What focal length lens is necessary? [*L.M.* 1926.]

11. Why does a convex lens held close to the eye act as a magnifying glass? How does the magnification depend on the lens employed? [*L.G.S.* 1920.]

12. Explain how a converging lens is used as a simple magnifying glass. How far from an object must a magnifying glass of focal length 5 cms. be placed to form a virtual image 25 cms. from the lens? Where should the eye be placed for the image to appear as large as possible?

13. Describe in detail a method of finding the focal length of a convex lens. An image of a luminous object is thrown by a convex lens of 8 ins. focal length on a screen 48 ins. from the object. Find the distance of the lens from the object, and also the distance through which the lens must be moved in order once again to throw an image on the screen. [*L.M.* 1919.]

14. Define the principal focus of a lens. It is required to project an image of an object, magnified threefold, on to a screen 3 ft. away. What kind of lens must be employed, what must be its focal length, and where must it be placed? [*L.G.S.* 1923.]

15. An object is placed in front of a convex lens so that a real image of the same size is formed. It is then moved 16 cms. nearer the lens, when the image, still real, is three times as large as the object. What is the focal length of the lens? [*L.M.* 1922.]

16. What are the two focal points of a convex lens? What do you know about them? A lens placed 6 ins. from an object produces a real image twice the size of the object. Where must the lens be placed to form an erect image also twice the size of the object? [*L.G.S.* 1927.]

17. Where must an object be placed so that a virtual image of it may be formed, by a convex lens of 1 in. focal length, at a distance of 12 ins. from the lens? Where should the eye be placed to see the image best?

18. Show graphically that the magnification produced by a convex lens is inversely proportional to the distance of the object from one of the focal points of the lens. Find, graphically or otherwise, where an object must be placed in order that a convex lens of 15 ins. focal length may give an erect image twice the size of the object. [*L.G.S.* 1929.]

19. Define the term principal focus of a lens. A convex lens forms a real image of three times the linear dimensions of an object on a screen placed 1 metre from the object. Determine the focal length of the lens. How much nearer the screen would the lens have to be placed in order that the image formed on it may be one-third of the linear dimensions of the object? [*L.M.* 1920.]

20. Explain what is meant by the *focal length of a convex lens*. An electric light is 6 ft. from a wall, and it is found that a convex lens held 2 ft. from the light throws a sharp image on the wall. In what other position may the lens be placed so as to throw a sharp image on the wall? Give reasons. Contrast the appearance of the images, and calculate the focal length of the lens. [*C.W.B.* 1924.]

21. Describe the apparatus you would use and the measurements you would make to determine the focal length of a convex lens of approximately 20 cms. focal length. If the lens had a focal length of approximately 100 cms. what alternative method would you adopt? [*L.M.* 1928.]

22. Explain what is meant by the principal axis, optical centre and principal focus of a lens. Light is converging to a point P, and a convex lens of focal length 20 cms. is placed at A in the path of the beam, where AP=30 cms. The beam now converges to Q. Calculate the distance AQ. [*L.M.* 1928.]

23. Explain, with the aid of a diagram, how an image of the sun is formed by a convex lens. Show how to determine by a geometrical construction the diameter of the sun's image formed by a convex lens of 6 ft. focal length, assuming that the sun's diameter subtends, as seen from the earth, an angle of $\frac{1}{2}°$. How does the brightness of the image depend upon the size of the lens and its focal length?

CHAPTER XVIII

COLOUR AND SPECTRA

COLOUR has fascinated mankind of all ages: the infant in a cradle watching a dangling ball; the artist painting a sunset; the Oriental weaving a carpet; or a woman buying a silk dress. The origin of colour has been the subject of much speculation from early times.

Seneca (A.D. 2–66), a tutor of the Emperor Nero, observed a similarity between the arrangement of colours in a rainbow and that seen in the edge of a piece of glass. The formation of a rainbow was a puzzle and by some considered to be due to reflection, from dark clouds, of light from the sun, which was behind the observer looking at the clouds. A later theory of the formation of colours, about 1650, was that light was sent out in pulses, the red being condensed light and blue rarefied light, the sensation of red or blue being received by the human eye according as condensed light or rarefied light arrived first.

In 1666 *Newton discovered that white light was apparently compound, and that by refraction through a glass prism it is split up into its separate parts, which are coloured.* He carried out his experiments in his room at Trinity College, Cambridge. He had a hole, $\frac{1}{3}$ in. diameter, in the shutter outside his window, and allowed a small beam of the sun's rays to pass through it on to a glass prism (Fig. 186). An image of the sun, about $2\frac{1}{8}$ ins. in diameter, was obtained, and it had a coloured band, with an imperceptible change from one colour to another, the order of tints being as in a rainbow. Newton called this a *spectrum* (L. image), in this case a solar spectrum;

the phenomenon of separation of the differently coloured portions is called **chromatic dispersion.**

Newton carried out further experiments. He allowed the rays of coloured light to travel several feet and, by

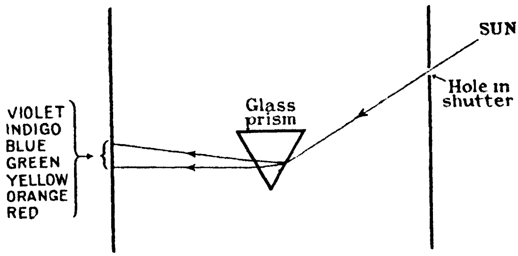


FIG. 186.—Newton's Experiment.

further slits, cut off most of it, leaving a single coloured beam to pass through. He then intercepted this by another prism (Fig. 187), and found that no further colour effect was produced; the light was merely more bent. As with the first prism, blue rays were more refracted, or bent, than the red rays, in passing through the second

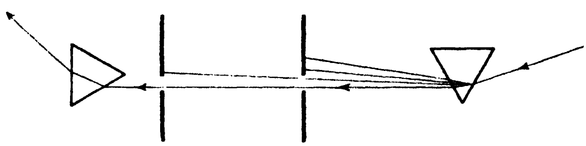


FIG. 187.

prism. Newton thus concluded that the colours of the spectrum are not given to the white light by the prism, but are actually present in white light. The prism merely separates them—the different coloured rays of light are thus differently refracted by the prism.

This conclusion of Newton's is still generally accepted —that white light is compound, and when we receive all the constituents, in their correct proportions, we have

the sensation of white light. When portions are missing we experience the sensation of colour or shade of colour. Some people, however, still believe that white light is simple, and that the prism imposes the colours on it. Our reasons for accepting the theory of the compound nature of white light are based on experiment and are as follows :

(1) In the experiment already described, a second prism fails to produce any effect, other than a further separation of the colours, on light previously dispersed by one prism.

(2) The coloured rays in a dispersed beam can be recombined to form white light. This was shown by

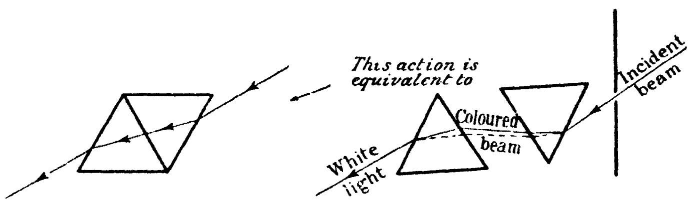


FIG. 188.—Recombination of Rays of a Dispersed Beam to give White Light.

Newton, who used a second prism, exactly similar to the first prism, placed after it in an inverted position. Fig. 188 explains this result, which is as would be expected, for the two prisms are equivalent to a slab of glass with parallel faces. Thus the light is deviated, but not chromatically dispersed.

Newton also showed an experiment, using a *Newton's Colour Disc*, to support the theory. A circular card is divided into many sectors ; these are painted with the different colours of the spectrum, the areas of the sectors being proportional to the amounts of the colours in white light. When the card is rotated rapidly, an eye observing it is stimulated by the different colours in rapid succession. Owing to the "persistence of vision" of the eye—*i.e.* an image in the eye persists for a period of time approaching one-tenth second—the effect is as if the

eye perceives all the colours at once, and so a sensation of white light is experienced. This effect is not always perfect, owing to the difficulty of getting on the card the correct proportions of the constituent colours of white light.

A further method of combining the colours in the spectrum of white light is to reflect them, as formed in an actual spectrum, on to a spot by a series of very narrow mirrors all mounted so that they can be fixed in any plane.

Colours and Refraction.—Accepting the idea of the compound nature of white light, we thus see that its differently coloured portions are differently refracted by a glass prism, the blue light being more refrangible than red light. Thus the refractive index of a medium varies with the nature of the light, *i.e.* with its colour. It is obviously greater for blue rays than for red rays. On p. 269 we saw that ${}_{\text{air}}\mu_{\text{medium}}$ is equal to the ratio of the velocities of light in air and in the medium. It thus follows that the velocity of blue light in the prism is less than that of the red light, and thus the slower-moving blue light is more refracted, or bent, in passing through the prism than is the faster-moving red light.

Methods have been devised to measure the wave-length of light, and it is found that the essential difference in colours is one of wave-length of the wave-motion, in the ether of space, by which the light energy is transmitted through space. The wave-lengths of the different colours change according to their order in the solar spectrum, violet having the shortest and red the longest wave-length.

Regarding the separation of the colours in white light a comparison is often made with a line of soldiers, arranged "tallest on the right, shortest on the left," entering at the same time a piece of ploughed ground. The long-legged soldiers are less affected by the "bad going" than are the short-legged soldiers, and so the direction of the line is turned. So the long wave-length red rays of light are less turned than are the short-wave length blue rays. If a medium is bounded by parallel faces, the waves come out parallel, and so there is no colour effect; with

a prism each colour takes its own path, and so the separation is visible when the light enters air again.

Pure Spectrum.—It was seen in Newton's original experiment on dispersion that an impure image was obtained—the colours overlapped and the demarcation between two colours was indistinct (see Fig. 189). A spectrum in which the colours do not overlap but are separated distinctly into clear bands is called a *pure spectrum*. Newton himself somewhat reduced the diffused effect by the use of a narrow slit instead of a circular hole. He also used a lens to concentrate more light on to the prism, so as to obtain a brighter spectrum.

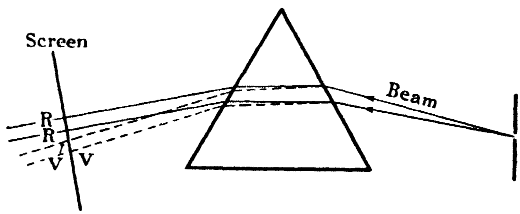


FIG. 189.—Impure Spectrum.

It is a remarkable thing that in all his work Newton did *not* observe anything peculiar about the solar spectrum, and it was left to Wollaston, in 1802, to point out that it was crossed by some dark lines. He saw seven, and considered that most of them were dividing lines between the various colours.

In 1814, Fraunhöfer, a Bavarian scientist and optician, examined the solar spectrum in greater detail by the use of a telescope, and he distinguished about 700 lines. Further, he invented a method of measuring their positions (wave-lengths corresponding), which he found were constant, and so these lines are now called *Fraunhöfer lines*. The use of a lens by Fraunhöfer separated the colours and made a wider and more easily observed spectrum (Fig. 190).

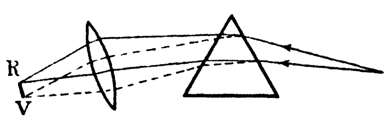


FIG. 190.—Single Lens as used by Fraunhöfer to Improve the Solar Spectrum.

In 1856, Swan (better known for his pioneer work,

with Edison, on electric lamps) introduced the idea of making all the rays fall on the prism at one angle, *i.e.* by using a parallel beam. For this, he allowed the light to pass through a slit which was placed at the principal focal distance from a convex lens. The refracted, parallel beam when incident upon a glass prism was dispersed, similar colour components taking parallel paths, and a lens then focussed these components, giving a pure

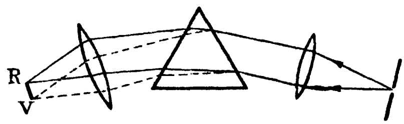


FIG. 191.—Use of Two Lenses to obtain a Pure Spectrum.

spectrum, as shown in Fig. 191. A slight improvement is observed when the prism is placed in the minimum deviation position (see p. 281).

The methods described for producing a pure spectrum are incorporated in the *spectroscope* (Fig. 192), which was perfected by Bunsen and Kirchoff about 1859–1860. It consists of a *collimator*, to give a parallel beam, a prism table and a *telescope* to observe the spectrum. The first is a metal tube with a convex lens at one end, and an adjustable slit at the other end, at its principal focal distance. Thus when a source is behind the slit, rays leave the lens parallel. They are received by the prism on the prism table, which has levelling screws. The dispersed beam is received by a lens in a metal tube, and the resultant image of the spectrum is observed by one or more lenses, arranged as a telescope, the behaviour of which will be described in the next chapter.

By means of a spectroscope and using the principle of the methods given on p. 281, for the measurement of the refracting angle of the prism and the angle of minimum deviation, the refractive index of a prism for light can be determined very accurately.

Spectra.—About 1830 it was pointed out by several observers that a spectrum, obtained from a flame in which a solid body was made luminous (the light being passed through

a prism), differed from the solar spectrum. In these so-called *flame spectra*, there were bright lines, or bands, of colour, each element studied apparently having its own characteristics. These spectra are now called *continuous spectra*, in contradistinction to the discontinuous or line spectrum of the sun. It was at first thought that these bright lines could be produced by several elements, for in most flame spectra there was a very bright yellow line (in good spectroscopes this is seen to be two lines near together), but in 1856 Swan showed this was due to sodium, a very minute amount of it present in any substance observed giving rise to the line. Bunsen and Kirchoff, however, observed many spectra, developing the spectroscope for the

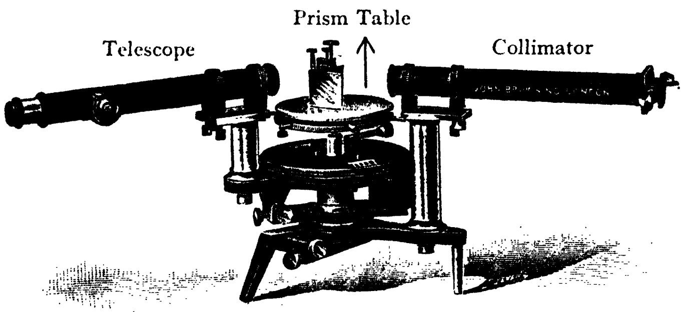


FIG. 192.—Spectroscope.

purpose, and established the fact that the flame spectrum was characteristic of a substance and so could be used for the detection of a substance, *i.e.* in analysis. There followed a period of activity in the study of flame spectra and particularly of metals vaporised at the metal electric arc, and previously unknown elements were discovered. These included Cæsium and Rhubidium, which were found by Bunsen and Kirchoff in 1860, Thallium by Sir William Crookes in 1861, and Indium in 1863 and Gallium in 1875, which were found to occur in zinc-blendes. In 1868, during an eclipse of the sun, the corona of the sun was noticed to give a bright orange line, not corresponding with any line in any known flame spectrum. Lockyer, who made many astronomical observations, suggested it was due to an element in the sun not yet found on the earth, and proposed the name Helium (Gk. *helios*, sun) for it. In 1895 Sir William Ramsay found this, as a gas, terrestrially.

Explanation of the Dark Lines in the Solar Spectrum. —In 1845 Foucault observed that the bright yellow line in the sodium spectrum corresponded exactly with the position of a black line in the solar spectrum, and this correspondence was observed for other bright lines in flame spectra and dark lines in the spectrum of the sun. Foucault also carried out an interesting experiment. He reflected rays of the sun on to a hot arc flame, in which was some sodium, observing both through a prism and lenses. Instead of the yellow line of the sodium spectrum persisting, the corresponding dark line of the solar spectra was found to be more pronounced. Foucault thought that possibly this was because hot gases in the arc could absorb, as well as give out, yellow light.

In 1859 Bunsen and Kirchoff explained the whole of the phenomena after examining, by their spectroscope, the light from an element in a very hot flame after it had passed through a cooler chamber of the same element in gaseous form. They obtained dark lines in the position where the spectra of the elements alone gave bright lines. They thus realised that the dark lines indicated an absorption, by a cooler source, of the energy, from a similar source, which would have given bright lines. Thus they came to the conclusion that the sun consists of a very hot mass containing many substances. The light from this *photosphere*, as it is called, passes through a layer of somewhat cooler gases round the sun. The gases present in this *chromosphere*, or *reversing layer*, absorb the light radiated by corresponding elements in the photosphere, and so, in the solar spectrum, dark lines are found instead of the bright ones in the ordinary flame spectra of these elements. The most prominent of these lines in the various colours of the spectrum are lettered and are given below.

Line	Found in	Corresponding element	Wave-length
A	Extreme red	Oxygen } in earth's	7661×10^{-8} cms.
B	Red	Oxygen } atmosphere	6867 ,,
C	Orange	Hydrogen	6563 ,,
D	Yellow (double)	Sodium	5890 and 5896 ,,
E	Green	Iron	5269 ,,
F	Blue green	Hydrogen	4861 ,,
G	Blue	Iron	4308 ,,
H	Violet	Calcium	3968 ,,
K	Extreme violet	Calcium	3953 ,,

In analytical work, there is no limitation of distance in the use of the spectra, and hence heavenly bodies are subjected to spectrum analysis. Fraunhöfer really began this by observing stars and noticing that lines in the stellar spectra were different from those in the solar spectrum.

Absorption Spectra.— Many transparent substances, when placed in the path of the dispersed rays from a prism, give definite dark lines or bands in the spectrum ; these absorption spectra are characteristic of the substances. Solutions, especially organic solutions, and also gases do this. Two interesting absorption spectra are those of potassium permanganate (which shows dark bands in the green) and hæmoglobin (which has two bands in the orange and yellow, with the violet end blotted out). The latter is a valuable blood test.

The Complete Solar Spectrum.—So far we have studied the visible spectrum of the sun—that due to its radiation in the form of light. But this is only a part of the energy radiated by the sun (about 46 per cent.) ; for, as we all know, the sun gives out heat. In 1800 Sir William Herschel discovered that the solar spectrum is not all visible. He held the blackened bulb of a delicate thermometer, in turn, in the different colours, and found that it became heated to different temperatures. Moreover *he found that the heating extended outside the red.* The invisible radiations outside the red were called **infra-red radiations** (L. *infra*, below), and they have been studied with regard to their heating effect (many of the results having been given in Chapter XII). It is estimated that 43 per cent. of the sun's radiated energy is in the form of heat.

But this is not all. The blackening of freshly prepared silver compounds, such as silver chloride, was observed by Scheele in 1777 to take place quicker, in the solar spectrum, in the violet portion. In 1801 J. W. Ritter found that the chemical action extended outside the violet, *i.e.* there were dark radiations, chemically-active, outside the visible violet. These were thus named **ultra-violet** (L. *ultra*, beyond) or **actinic rays.** It was soon shown that the behaviour of substances towards ultra-violet radiations is quite different from their behaviour towards the visible rays ; glass, for example, stops the ultra-violet radiations, but quartz transmits them exceedingly well. Thus for work in connection with ultra-violet rays the lenses in the spectroscope, and the prism used, are made of quartz.

Ultra-violet radiations have been studied mostly with regard to their photographic effect (the darkening of silver salts) and a very beautiful phenomenon known as **fluorescence.** If quinine sulphate, in a test-tube, be moved through the solar spectrum, it glows with a bluish light when held in the violet and beyond it; in sunlight this colour can be seen at the edges of the substance. Paraffin oil in sunlight shows a bluish fluorescence, fluorescein a brilliant green one and eosin a red one. It has been shown that fluorescence is due to the substance absorbing energy of one wave-length and emitting part of it as light of longer wave-length. Thus invisible ultra-violet rays can be absorbed and re-emitted as visible rays, usually violet or possibly green. The effect is, however, confined to surface layers, whilst it only exists as long as the radiations are falling on the fluorescent substance.

When there is a storing up of energy so that the fluorescent effect is seen after light has ceased to be incident upon the substance, the phenomenon is called **phosphorescence.** Often this is due to a chemical action. The term is applied to the glowing of phosphorus seen in the dark, when it is really reacting with the air and so does not actually phosphoresce.

Ultra-violet radiations are present in a bluish light, such as given by a mercury-vapour lamp (with quartz bulb); they are still more common in the light emitted by an electric arc between electrodes of an alloy of cadmium, tin and lead, or better still, iron electrodes, when the arc is more powerful and easier to manipulate.

The recognition of the healing effects of ultra-violet radiations has led to the demand for an efficient ultra-violet producing lamp, and for glass which is transparent to the ultra-violet present in the sun's radiations (to the extent of 11 per cent. approximately). As quartz is costly, experiments have been made with different kinds of glass; it is claimed for "vita-glass" that it is transparent to the ultra-violet to the extreme limit. (Fig. 193 illustrates this.)

The ultra-violet rays have an injurious effect on the eyesight and cause pigments to fade, and metals to corrode. To obviate this a glass has been made which is transparent to visible light rays, and yet totally opaque to ultra-violet radiations. This has come into use for spectacles and for windows in picture galleries, etc. Sir William Crookes experimented with glasses before 1913, and found that, in general, a glass containing cerium cut off the ultra-violet, while a glass containing iron cut off heat rays. Such

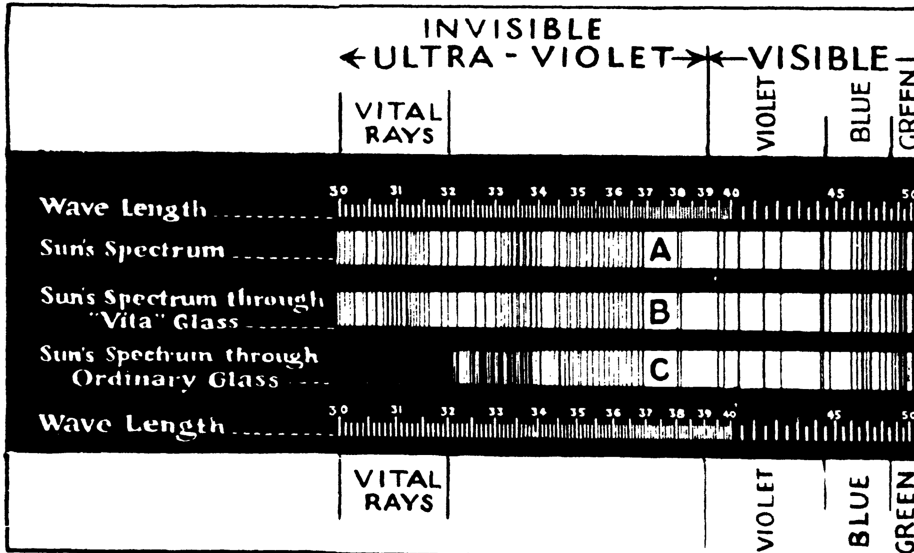


FIG. 193.—Diagram illustrating the composition of light, the yellow, orange and red bands excepted. *A*—shows the sun's spectrum. *B*—shows the sun's spectrum through "Vita" Glass. *C*—shows the sun's spectrum through ordinary glass. *B* illustrates that "Vita" Glass transmits the vital rays to the extreme limit (wave-length $30 \cdot 10^{-6}$ Cms.) of the sun's spectrum. *C* illustrates that the vital rays are completely obstructed by ordinary window glass.—(*By permission of the "Vita" Glass Marketing Board.*)

glasses are called Crookes' glasses, and two important ones are (*a*) one made of 83 parts of fused soda-glass and 17 parts of cerium nitrate, and which is practically opaque to ultra-violet rays, (*b*) one made of 90 parts of raw soda-glass and 10 parts of ferrous oxalate and a little red tartar and wood charcoal, which is sage-green in colour and cuts off 98 per cent. of heat radiations. A 2 per cent. solution of copper chloride absorbs the infra-red, and yet transmits the visible portion of the spectrum.

Colour Effects with Lenses.—Since the refractive index of air to glass is higher for blue than for red light, it follows that the different colours in a parallel beam of white light are focussed differently by a lens. The latter must obviously have a shorter focal length for blue light than for red light. This can be shown by focussing a

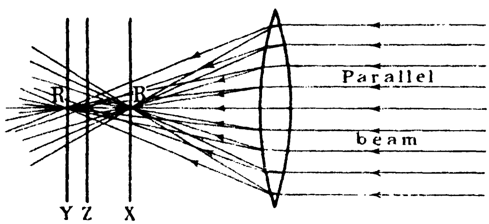


FIG. 194.—Chromatic Aberration due to a Lens.

powerful parallel beam of white light by a large lens. A screen held to receive the light at X (Fig. 194) shows an image with a blue centre and red fringe, at Y one with a red centre and a blue fringe, whilst at Z the image is uniformly white, but blurred, and slightly coloured at the edges. This last effect is known as *chromatic aberration*, and can be observed when looking through inferior opera-glasses.

For a double convex lens, of which the radii of surfaces are 200 cms., the following are values of the focal length of the lens: For violet light 133·2 cms., for dark blue 135·9 cms., for blue-green 138·7 cms., for sodium yellow 142 cms., for orange 143·5 cms. and for red light 144·9 cms.

When lenses came into use for special purposes, described in the next chapter, chromatic aberration proved a great drawback. Newton carried out a few tests with hollow prisms, of different thicknesses, filled with water, and found he could never get a deviation of light by a prism unaccompanied by dispersion. He rather rashly concluded, without full tests of many substances, that there was no remedy for chromatic aberration, and his judgment was accepted for a long time.

However, it has been shown that different substances are not the same in behaviour—flint glass, for example, which contains some lead, disperses light much more than does crown glass, but does not deviate light in the same proportion. It is thus possible to put together two prisms,

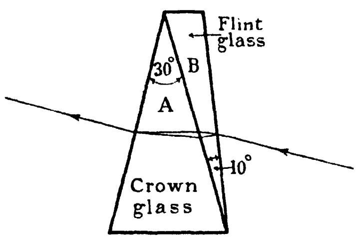


FIG. 195.—Achromatic Prism.

one A of crown glass and the other B of flint glass and of smaller refracting angle, and so obtain a combination which gives no dispersion, but some deviation (Fig. 195). This is called an *achromatic prism*. Newton, of course, combined two equal prisms of similar material to give the effect. A more important use of this idea, applied to prisms, is the making of a combination by means of which no deviation, but some dispersion occurs, *i.e.* a beam of white light passes straight through it giving a spectrum. Such is the principle used in the *direct-vision* spectroscope, conveniently portable as a single tube.

Achromatic Lenses.—To make a convex lens which does not focus the colours of white light differently, an achromatic combination is made, consisting of (*a*) a convex lens of crown glass of shorter focal length ; and

(*b*) a concave lens of flint glass of longer focal length. When suitably chosen such a combination is a convex one, but more dispersive flint glass, even though of longer focal length, *i.e.* less refracting, nullifies the dispersion produced by the more refracting convex lens. With this combination chromatic aberration cannot occur. The lenses are joined together by a suitable colourless substance such as Canada balsam ($\mu = 1 \cdot 53$).

Similarly, to obtain an achromatic concave combination, a shorter-focus concave lens of crown glass is compounded with a longer-focus convex lens of flint glass.

The great advances made in modern astronomy and biology have been, to a great extent, dependent upon the improvement of such instruments as the telescope, microscope, and camera, brought about by the use of achromatic lenses. These instruments are all dealt with in the next chapter.

Visible Spectrum and Colour.—If differently-coloured materials be moved gradually through a clear spectrum, some striking effects are noticed. A piece of white material appears violet when held in the violet, blue when held in the blue, and so on, showing that it reflects any and all of the spectrum colours. But a piece of violet material appears bright violet when held in the violet part of the spectrum, but in other parts it appears black. Obviously, then, the violet material must absorb all coloured rays of light except violet. Similarly a green material appears green only in the green part of the spectrum, a red material appears red only in the red part of the spectrum; in other parts they appear black. This shows the power of *selective reflection*, or *selective absorption*, by materials. The colour of any material is due to the fact that it absorbs most of white light, reflecting a definite portion of coloured light. Somewhat similar is the cause of the colour of transparent solids or solutions when held up to light. Some components are absorbed, the remainder are passed through (usually part are reflected), thus giving the colour as seen by transmitted light. This can be shown by interposing coloured glasses, or coloured filters as supplied for photo-

graphic work, between a dispersed beam and the screen on which the spectrum is observed.

Obviously, then, the light absorbed and the light transmitted or reflected, make up white light. Two colours which together make white light are called *complementary* colours. Some pairs are blue and yellow, red and bluish-green, orange and greenish-blue, green and magenta. If two beams of light, one of which has passed through a glass of one colour and the other through a glass of the complementary colour, are focussed on the same space on a white screen, a white patch of light is obtained. The sensation of white received when perceiving a complementary pair of colours can also be shown by revolving sectors, made up of the two colours, on a Newton's disc turn-table.

Occasionally selective reflection and transmission are not the same for a transparent substance, *e.g.* a glass plate, covered with an alcoholic solution of magenta and then dried, appears green by reflected light and red by transmitted light. Obviously the colours transmitted and reflected must be complementary. Similarly a piece of gold leaf appears yellow by reflection. The complementary blue is absorbed. However, if the gold is in the form of a very thin film, and light is passed through it, it appears bluish, showing that it transmits the blue.

It should be noted that colourless substances, such as ice and soda, in the form of powder appear white, for the simple reason that the minute surfaces scatter all the light received.

The colours of sunset and sunrise are due to the scattering of the shorter wave-length blue light, in passing through the longer layer of the earth's atmosphere when the sun is near the horizon. Thus the sun appears red, for the red light travels through, whilst the scattered light gives the blue, green, and gold colours in the sky and clouds. When high in the heavens the sun appears yellow, instead of pure white, because a little of the blue light from it is scattered; hence the blue sky. The change in the appearance of the sun at sunset can be imitated by passing a powerful beam of light through a

glass tank, containing a solution of sodium hyposulphite, on a screen. Into the tank is poured a solution of hydrochloric acid sufficiently strong to produce a deposit of fine sulphur particles in from two to three minutes. When the acid is poured in and well stirred these very fine particles are gradually formed and remain in suspension, scattering the light. As the number and size of the particles increase, the image of the light on the screen changes from white to yellow, then to orange-red, and finally the whole light is stopped. During the experiment the liquid should also be observed for complementary colours.

Critical Angle and Colours.—In the experiment described on p. 274, when the light is just emerging into the air from glass, or water near the critical position, it is observed to be coloured. This is because the critical angle values are different for the different colours. It was shown that ${}_a\mu_m = \frac{1}{\sin c}$, where c is the critical angle from the medium into the air.

But $\mu_{\text{Blue light}} > \mu_{\text{Red light}}$, and so $\sin c$, and hence c, is smaller for blue light than for red light. Thus when the light, in passing out, is near the critical position, the blue

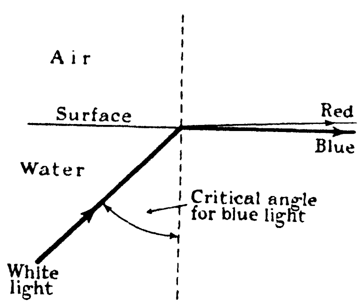


FIG. 196.

portion reaches it first and passes along the surface whilst the red portion is being refracted into the air (see Fig. 196). This effect is observable for all the colours in the usual order violet . . . . . red.

Pigments.—No pigment (or paint) of a single colour reflects only that coloured portion of white light falling on it. For example, blue paint reflects blue and a little green and dark blue (the colours on either side of blue in the spectrum). Yellow paint reflects yellow and a little green and orange. A mixture of blue and yellow paint thus absorbs all the colours in white light except green and so appears green. By virtue, then, of selective absorption and reflection, the colour of a mixture of paints (pigments) is that colour which is not absorbed by either of the constituents.

The Three-Colour Theory of Helmholtz is the most accepted theory in the explanation of colour, which is, of course, a sensation or feeling resulting from the stimulation of nerves at the back of the eye. Since three colours, red, green, and blue, can be combined to give all possible shades of colour, it is considered that these are the *three primary colours*, and that, in the normal eye, there are three nerves, each of which is responsive to one colour only. When the three nerves are equally stimulated a sensation of white is experienced. By a variation of the stimulations given to these three nerves, any colour sensation is possible. There is, however, no physiological evidence in support of this conception of three special nerves. Evidence in favour of the theory is found in the defect of vision known as colour-blindness, which is due to the absence of one or more colour-senses. The commonest form is red colour-blindness (5 per cent. of males and 0·2 per cent. of females are red colour-blind), a form which is hereditary in the second generation, while it can be produced by an excess of smoking. It was discovered by John Dalton, the chemist, who found that a pink geranium appeared blue to him by day, and what he called red by candle-light. Red, orange, yellow, and green were practically all alike to him. The outlook of a red colour-blind person can be visualised by looking at things through a copper sulphate solution or in the light of mercury-vapour lamps. The absence of green colour-sense is the next commonest colour defect, whilst blue colour-blindness is very rare.

EXERCISES ON CHAPTER XVIII

1. How may it be shown that white light can be split up into several parts which can be recombined to form white light again? [*J.M.B.* 1923.]

2. White light passes from a slit, through a prism, and on to a white screen. You are given pieces of red, green, and yellow glass and pieces of paper of the same colour as the glass. Describe a series of experiments you would perform with them. State the results you would expect to obtain and the conclusions you would draw from them. [*J.M.B.* 1922.]

3. What effects are produced when a widely divergent beam of white light falls upon and is transmitted through a glass prism? What apparatus would be needed and how would it be arranged in order to exhibit these effects to the best advantage? [*L.M.* 1919.]

4. Why is it necessary to use a lens in order to produce a well-defined spectrum? Describe, with a suitable diagram, how to project a pure spectrum on to a screen. [*L.G.S.* 1921.]

5. Describe how to set up two convex lenses, a narrow slit, a prism and a source of white light so as to form a pure spectrum. Draw a diagram showing the paths of rays forming the two ends of the spectrum. [*L.M.* 1920.]

6. Explain why one or more lenses are usually used with a prism to form a spectrum. A prism is set in the minimum deviation position for yellow light. How must it be turned to bring it into the minimum deviation position for blue light, and what effect will this turning have on the spectrum? [*L.M.* 1922.]

7. You are required to fit up apparatus to project the solar spectrum on a screen. Describe how you would do this, illustrating your answer by means of a diagram. Write what you know concerning this spectrum. [*L.G.S.* 1925.]

8. What is meant by *dispersion* of light and upon what fact does it depend? Make a diagram of the arrangements usually employed for obtaining on a screen a pure spectrum. Enumerate the conditions which *must* be fulfilled in order that the spectrum shall be pure. [*L.G.S.* 1927.]

9. Describe the experiments you would make to show that white light is composite in character. Mention any common phenomena which lead you to infer that the components of sunlight and of an artificial illuminant such as gaslight are different. [*L.M.* 1926.]

10. Show that, owing to chromatic dispersion, it is impossible to get an absolutely clear image with a single lens; and explain how the difficulty can be overcome by using a composite one. [*J.M.B.* 1926.]

11. Enumerate the apparatus required to obtain a pure spectrum, and show how it should be arranged. In what way do the arrangement of the experiment and the result obtained differ from those in Newton's original experiment? Describe very briefly the spectra obtained when the source of light used is (*a*) an ordinary lamp (gas, electric, or paraffin), (*b*) daylight, (*c*) a bunsen burner with salt in it. [*L.G.S.* 1929.]

12. What kind of prism would you employ to deflect a beam of light (*a*) through 90°, (*b*) through a small angle, say 3°? Would you expect any colour effects in either case? [*L.M.* 1921.]

13. Describe a form of instrument suitable for observing spectra. What are Fraunhöfer lines ?

14. What are absorption spectra ? How do they differ from continuous spectra ?

15. Describe the nature of the general radiations from the sun. State how they are separately studied.

16. What do you understand by ultra-violet radiations and fluorescence ? Describe any uses of ultra-violet radiations which you know.

17. Explain the principle of an achromatic lens, and point out the advantages of using such a lens.

18. Explain why it is that blue and yellow paints when mixed give a green paint, whilst blue and white light, in correct proportions, shone on a screen give a white patch.

19. Explain the formation of the colours seen when a beam of white light is emerging from water, or glass, into air at, or near, the critical point.

CHAPTER XIX

OPTICAL INSTRUMENTS—VELOCITY OF LIGHT

THE commencement of photography was due to the observation that certain substances, such as silver chloride and silver nitrate, white when freshly prepared, become dark when exposed to light. Coupled with it was the invention of the pinhole camera, described on p. 204. We are told that the first photographs, which could not be "fixed," *i.e.* rendered permanent, were produced in 1802. In the next fifty years the art of photography developed and proved an extremely useful supplement to the spectroscope, an important point in the development being the use of lenses. The origin of the lens is very uncertain. Probably Ptolemy (second century A.D.) used convex lenses, whilst spectacles were invented at Florence about A.D. 1285, the glass being ground into the lentil shape at Murano, a place in Italy famous for its glass from early times. The first reference to concave lenses seems to be in a book by Nicolas de Cusa about 1430–1440. After 1600 lenses were used, as will be shown, for many purposes, and in the early part of the nineteenth century the principles of lenses were well understood. A modern camera consists essentially of a dark chamber, painted dull black inside so that no light is reflected from the sides; a lens, usually achromatic; and an arrangement so that a sensitised plate or film may be inserted. The latter carries an emulsion of a silver compound, in gelatine, which is affected immediately wherever light falls on it. The camera has a shutter and a device for opening it for the correct time of exposure to light from the objects to be photographed. Since the distance of the image

from a lens varies with the position of the object, cameras are usually made with the sides in bellows form, so that the distance between the lens and the film or plate (where the image is to be produced) can be varied (Fig. 197).

To reduce spherical aberration, a stop, or diaphragm,

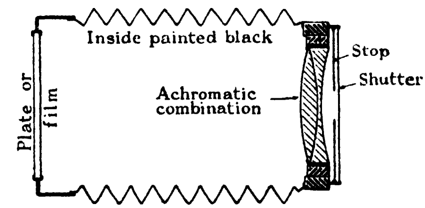


FIG. 197.—Camera.

is used. By means of this a circular gap is left at the centre of the lens, for light to pass, when the shutter is opened.

The diameter of the gap, or the aperture, can be varied, and the size is indicated on the camera, as a fraction of the focal length of the lens—usually $\frac{f}{8}$, $\frac{f}{11}$, $\frac{f}{16}$, $\frac{f}{22}$ and $\frac{f}{32}$, *i.e.* the diameter of the aperture is $\frac{1}{8}$, $\frac{1}{16}$, etc., of the focal length. The amount of light passing through is proportional to the area of the aperture, and this is proportional to the square of the radius (area of circle$=\pi$ radius^2) and thus to the square of the diameter. Thus the illumination of the plate or film is inversely as the square of the stop number, *i.e.*

STOP	$f/8$	$f/11$	$f/16$	$f/22$	$f/32$
Squares of stop numbers	64	121	256	484	1,024
Ratio of squares of stop numbers	1	2 (approx.)	4	8 (approx.)	16
Ratio of Illumination	16	8	4	2	1

This, then, can be employed to deduce the correct exposure for any stop used when one exposure is known, for approximately, the use of a stop of $\frac{f}{16}$, which only gives $\frac{1}{4}$ the illumination of that of stop $\frac{f}{8}$, will necessitate an exposure four times as great as that given when using stop $\frac{f}{8}$.

The plate or film, after exposure, is "developed" chemically, to bring out the varying intensities due to the action of the light, and then fixed. This gives a "negative," so called because it is a reversal picture, the black portions being where most light was received, coming from a white object. A "positive" picture of the original view is obtained from this by passing light, for the correct time, through the negative to a piece of sensitised paper, which is then developed and fixed.

The Optical Lantern is a device to give a large, bright image of a small picture or object placed near to a "projecting" achromatic lens of short focal length, an image being projected on a suitably placed screen. The position of the lens, relative to the object, can be altered so that the lantern can be used in rooms of different sizes. Since a large picture is to be produced, a beam of light from a powerful source must be concentrated on the object or "lantern slide" (the specially mounted picture). This is done by the "condenser," which is usually a pair of mounted plano-convex lenses (see Fig. 198), so forming a very short-focus converging combination. Owing to the concentration of heat, as well as of light, good ventilation of the condenser is necessary, and so the plano-convex lenses are mounted with the curved faces adjacent, in a metal holder with holes in it. In the earlier forms of lantern the source of light was a lime-light—gas flame over a "lime" burning in a supply of oxygen. Later came the use of an electric carbon arc, but now a powerful electric lamp is coming into general use. This has the advantage that a special metal concave mirror can be used behind the lamp to collect more

light and reflect it on to the condenser. Fig. 198 also shows the method of inserting a lantern slide to give an erect image, since the lenses cause a lateral and perpendicular inversion. When the object cannot be inverted, as the slide is shown, an erecting prism is commonly used, between the projecting lens and the screen, its action being described on p. 283.

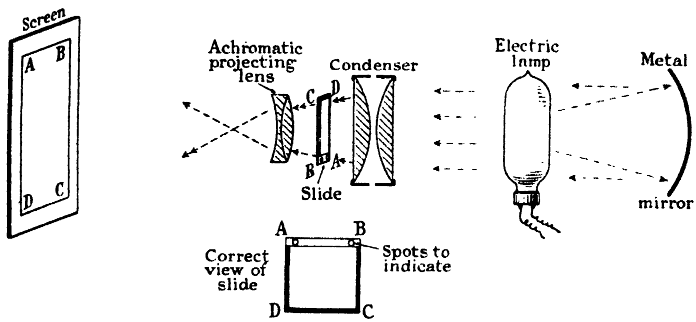


FIG. 198.—Optical Lantern as used for Projecting a Lantern Slide.

The human eye is similar in action to the camera, as can be seen from Fig. 199. Light passes through the cornea (the window of the eye) to the lens, and is refracted by it and through the liquid lenses (refractive index 1·34—1·36) to give an inverted image on a layer of nerve fibres at the back, the retina. Impulses are conveyed from these to the brain, by the optic nerve, and are there translated into sensations. It should be noted that there is a reversal, for an inverted image gives rise to the sensation of erectness. The amount of light entering the eye is regulated by the iris (*cf.* a stop) and the eyelid is, of course, the shutter of the eye. The crystalline lens is built up in concentric layers, increasing in physical and optical density from the outside to the centre. This, together with the different curvatures of the two surfaces (the radius of the front surface being approximately 11 mm. and that of the back surface 8 mm.), considerably diminishes spherical aberration. The lens is suspended

by the suspensory ligaments (which have a shock-absorbing action). The point where the optic nerve enters the eye is not sensitive to light, and is called the " blind spot." If one eye is closed, it is possible to find the position where a small object cannot be seen by the open eye. The retina is most sensitive at its centre—called the " yellow spot."

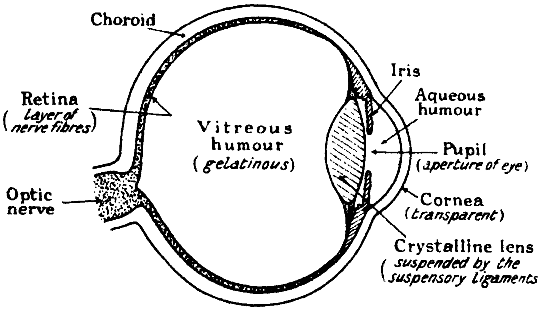


FIG. 199.—The Eye (not to scale).

For the normal eye the principal focus, of the lens combination of the eye, is at the retina, and so distant objects are sharply focussed. Nearer objects should be focussed behind the retina, as is seen from the table on p. 298, but the eye possesses *accommodating power* (or accommodation) to cause images of nearer objects to be produced on the retina. Accommodation is thus a converging action and is brought about by (1) the cornea of the eye being pushed out, so increasing the size and convergency of the watery lens; (2) the suspensory ligaments and muscles slightly increasing the radii of curvature of the faces of the crystalline lens, so increasing its convergency. The control of accommodation becomes automatic, after a period, in most human beings and animals. The normal eye can accommodate for objects up to a distance of 10 ins., or 25 cms., and so this distance is called the *normal least distance of distinct vision* (or the " near point " of the eye).

The Opthalmoscope is an instrument, devised by Helmholtz in 1851, for the examination of eyes, and is a concave mirror pierced at the centre by a small hole for examination. The retina of the eye to be examined is brilliantly illuminated by light reflected into it, by the concave mirror, from a source over the head of the patient. Some modern portable forms have an electric battery and lamp contained in the instrument.

Defects of Vision.—Common faults in eyesight are now discussed.

(1) **Myopia** (Gk. *myeia*, to close + *ops*, eye), or **Short Sight,** is the defect due to over-convergency caused by the distance between the back and front of the eye being greater, or the crystalline lens thicker, than is usual. Light from a distant object is focussed in front of the retina. Thus only objects somewhat nearer (at the "maximum distance of distinct vision" or "far point") are focussed on to the retina without accommodation. A myopic person tries to focus more distant objects by partially closing the eyelids and drawing back the lens Nearer objects are focussed by the use of accommodating power, and objects nearer than the normal distance of distinct vision (10 ins.) are clearly seen ; hence the phrase "short sight." The remedy is the use of spectacles with a divergent (concave) lens or lenses, one pair being suitable for out-of-door use and the other for reading and near work. The former has a lens to enable a myopic eye to see distant objects clearly without accommodation, the latter has a lens by which the eye can see clearly an object held at the normal least distance of distinct vision (10 ins.). Thus the least distance of distinct vision for the myopic eye, called the "near point," must be known.

EXAMPLE.—Suppose a myopic eye has a range 8 ins. to 60 ins.

For out-door use, a lens is required to cause rays from distant objects (*i.e.* parallel rays) to appear to diverge from 60 ins., *i.e.* a concave lens of 60 ins. focal length is required.

If x ins. is the object distance to give an image at 8 ins. from the eye with this lens, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ becomes $\frac{1}{8}-\frac{1}{x}=\frac{1}{60}$.

$$\therefore \frac{1}{x}=\frac{1}{8}-\frac{1}{60}=\frac{15-2}{120}=\frac{13}{120}.$$

$$\therefore x=9 \text{ ins. approx.}$$

The range of the myopic eye is extended—it is from 9 ins. to ∞ with the diverging lens of focal length 60 ins.

For *indoor use*, the lens must be such that an object at the normal distance of distinct vision (10 ins.) appears, to the eye using the lens, to be at the near point (8 ins.). Thus the object distance u must$=+10$ ins. and the distance of the image, caused by the lens, v, must $=+8$ ins.

Now, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, where f=focal length of the lens.

Hence $\frac{1}{f}=\frac{1}{8}-\frac{1}{10}=\frac{5-4}{40}=\frac{1}{40}$, or $\underline{f=+40 \text{ ins.}}$

i.e. the lens must be divergent (concave) and of 40 ins. focal length.

It should be noted that the image of an object presented to a myopic eye by the lens always appears nearer than it really is.

Thus magnification which $=\dfrac{\text{image distance}}{\text{object distance}}$ is always less than 1, and so the retinal images are always diminished; objects appear smaller, and any myopic person will have noticed this when given stronger lenses.

(2) **Hypermetropia** (Gk. *hyper*, beyond, $+$*metron*$+$ *ops*) or **Long Sight,** is the defect due to insufficient convergency caused by the distance between the front and back of the eye being smaller, or the crystalline lens thinner, than is usual. Thus *very* distant objects are clearly focussed on the retina, and the accommodating power is brought into use much more than with the normal eye.

Further, the near point is at a greater distance than 10 ins. (hence the name of the defect), and the eye is

strained in an endeavour to focus nearer things. Unless spectacles (which obviously must have a convergent action) are used for all near work, reading, etc., a long-sighted person may gradually lose the power of accommodation, muscular action becoming imperfect, particularly in old age. This defect, called *presbyopia*, is the cause of many of the road accidents to elderly pedestrians —they fail to recognise the rate of approach of quickly-moving vehicles.

For a hypermetropic eye, as for the myopic eye, it is usual to provide a lens for near work, so that an object at the normal least distance of distinct vision (10 ins., or 25 cms.) gives a virtual image at the near point of the eye, which then sees it clearly.

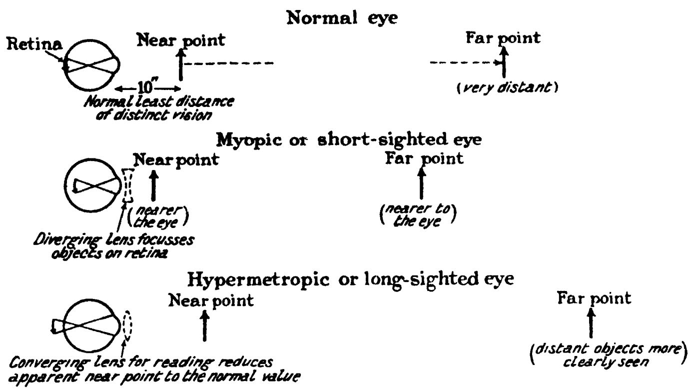


FIG. 200.

EXAMPLE.—Suppose a hypermetropic eye can see clearly to within 30 cms. Then an object at 25 cms. ($u=+25$) must, by means of the lens, appear to be at 30 cms. from the eye and lens (v thus$=+30$).

$$\text{Thus } \frac{1}{v}-\frac{1}{u}=\frac{1}{f} \text{ becomes } \frac{1}{30}-\frac{1}{25}=\frac{1}{f}=\frac{5-6}{150}=-\frac{1}{150},$$

or $f=-150$, and so a *converging* (convex) lens of 150 cms. focal length is required.

Obviously, with such a lens the retinal image in a hypermetropic eye is magnified ($v/u=30/25$), and this is an advantage. (Fig. 200 illustrates the above defects of vision.)

Astigmatism (Gk. *a*, without + *stigmatos*, a point of focus), a very common defect of eyes, is usually due to the lens surface and cornea not being perfectly spherical in shape—generally the curve is more pronounced in the vertical than in the horizontal. Thus lines in different planes are brought to different foci, and so a uniform clear view of anything is not obtained; *e.g.* in looking at a tree, only the twigs in certain planes are clearly seen. In cases of astigmatism a cylindrical lens is used to assist refraction in the plane of the weak refraction of the eye.

The Telescope (Gk. *tele*, at a distance + *scope*) is an instrument which makes distant objects appear much larger and nearer. Its history is uncertain, but it is probable that the earliest was made in 1608 by Hans Lippershey, a spectacle-maker at Middleburg in the Netherlands. His telescope, 18 ins. long, was made of two ordinary spectacle lenses. A convex lens received light from a distant object. The image formed was observed by a concave lens, an erect image being obtained. In 1609 Galileo is said to have heard of it, and then worked out the general method of producing a magnified image. He immediately set up a telescope, called *Galileo's Telescope*, which turned out to be similar in idea to Lippershey's. In any optical instrument the lens receiving the light from the object is called the *object-glass* or *objective;* the lens near the eye is called the *eye-lens*. The principle of Galileo's instrument is shown in Fig. 201. The converging object-glass O would give a real, inverted, diminished image *ab*, but the diverging eye-lens E intercepts the rays before they come to a focus and so produces an upright magnified image A′B′. Galileo gave the details of his telescope, and of his first discoveries made by its use, in the *Starry Messenger*, 1610. With his instrument, also, he first discovered the existence of sun-spots. Drawbacks of this form of telescope are the small field of view and the impossibility of measuring

the final image. It is an advantage if cross-wires (two thin wires at right angles) and a small scale can be inserted in the instrument to be seen with the image. This is done by fixing them at the position of the image produced by the object-glass. Then the eye-lens produces a virtual image of this image of the distant object and of the cross-wires and scale, and so measurements can be made. Obviously, since the object-glass does not produce a real

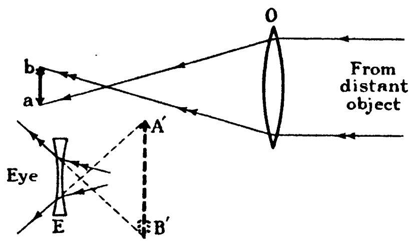


FIG. 201.—Principle of Galileo's Telescope.

image, cross-wires cannot be inserted. Thus this form of telescope is not much used by astronomers. But a combination of two of these telescopes, one for each eye, is used as opera or field glasses, so that the observer may see an erect image of distant objects. In these the eye-lenses should be placed to give a virtual image at infinity ; there is then no eye-strain, for accommodation is not used.

The Astronomical Telescope, using two convex lenses, was suggested by Kepler in 1611, but he never made it up, and the first was probably made by a Jesuit Father, Scheiner, about 1617. It consisted of a converging (convex) object-glass O, which gave a real, inverted image *ab* of a distant object AB (Fig. 202), and an eye-lens E, converging, acting on the principle of the simple microscope (p. 297). The latter was thus at a distance, from the real image, less than its own focal length, and so a magnified image A′B′, still inverted, was obtained. Cross-wires and a scale can thus be inserted at the position of

ab, and for astronomical work the fact that the final image is inverted does not matter.

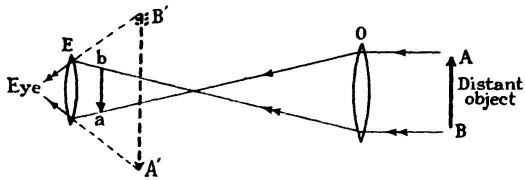


FIG. 202.—Principle of Astronomical Telescope.

Magnification by a Telescope.—The ordinary method of measuring magnification by the ratio of the sizes of the image and object (and thus their distances from the lens) is obviously not applicable to a telescope. The magnification is determined by comparing the angular size of the image produced with that of the object, at the eye. Thus, if in Fig. 203 AB is an object, and A'B' the final image produced,

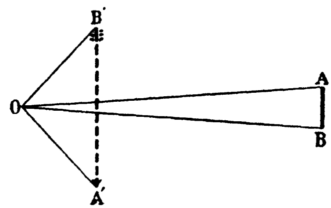


FIG. 203.

the angular size of the image to an observer at O is $A'\hat{O}B'$,
and „ „ object „ „ at O, is $A\hat{O}B$.

The magnification of the telescope is measured as $\dfrac{A'\hat{O}B'}{A\hat{O}B}$.

This can be shown to be equal to

$$\frac{\text{focal length of object-glass } (f_o)}{\text{focal length of eye-lens } (f_e)},$$

and so it is necessary to have an object-glass with a long focal length, that of the eye-lens being small, if a good magnification is required. Thus powerful telescopes have very long tubes; these are made telescopic owing to the variation in focussing of the object-glass with variation in distance of object. If a brighter and

larger image is wanted, it follows that more light must be collected, and so the object-glass must be made of larger aperture, *i.e.* (*a*) for bigger pictures, a longer telescope is required ; (*b*) for brighter pictures, a larger object-glass is required.

As telescopes were made larger and longer, in the early days, distortion troubles increased, and these were attributed solely to spherical aberration. Attempts were made to grind lenses with hyperbolic surfaces to reduce this.

It is interesting to note that the period 1640–1670 marks the rise of the great Scientific Societies in Europe. The Royal Society of England, founded about 1645, was given a Charter of Incorporation in 1662; the French Académie des Sciences of Paris, initiated before 1640, was firmly established in 1666; and the Accademia del Cimento (or Academy of the Experimenters) in Italy, 1657–1667. The growth of these Societies was a great encouragement to inventive genius, stimulated by the needs discussed at the Society meetings. Thus the seventeenth century is the period of instrument invention, and the origin of the barometer, thermometer, pendulum clock, air-pump, telescope, and microscope is attributable to this period.

The Royal Society in its first ten years endeavoured to overcome the telescope difficulty by means of hyperbolic

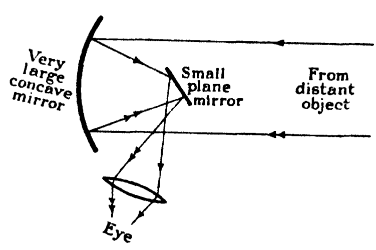


FIG. 204.—Newton Reflecting Telescope.

lenses, but Newton showed that chromatic aberration was the chief source of trouble in these so-called refracting

telescopes—the images were blurred because of the different focussing of the differently-coloured rays of the incident light. As has already been shown on p. 320, Newton thought that it was impossible to remedy this; so he turned his attention to *reflecting telescopes*, in which the light from the distant object is received by a large curved mirror, or reflector, and focussed by reflection. These had already been proposed, and in 1668 Newton made his first one, which is in the possession of the Royal Society. He used a large concave spherical mirror, and intercepted the light reflected from it by a small plane mirror; this reflected the light to the side where it was received by a reading-glass (Fig. 204).

The first form of reflecting telescope, suggested (but not constructed) in 1663 by James Gregory, a Scottish astronomer, was made in 1672 by Robert Hooke. In this *Gregorian Telescope*, parabolic mirrors are used (Hooke probably used spherical ones); a concave one to receive the light, and another small concave one to reflect back the light through a small hole in the large concave reflector (Fig. 205). The image is magnified by a reading-glass.

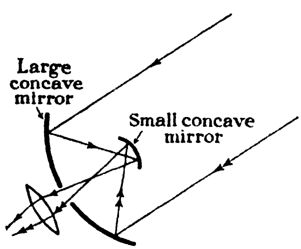


FIG. 205.—Gregorian Telescope.

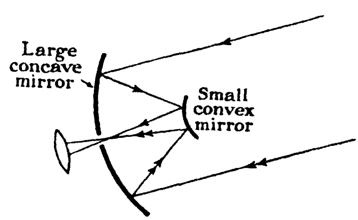


FIG. 206.—Cassegrain Telescope.

Another form, which is coming into more general use, is the *Cassegrain Telescope*. It has great advantages over the other reflecting types with regard to spherical aberration, whilst it is shorter and easier to move. It consists of a large concave reflector, having a small hole at its centre; through this light is reflected back by a small convex mirror (Fig. 206).

Since the seventeenth century the greatest advance in telescopes has been due to the introduction of the achromatic lens by John Dolland in 1758. These have been used in developing larger and longer telescopes to give better definition of detail.

The Yerkes' Telescope at William's Bay, Wisconsin, U.S.A., is the largest refractor in use. Its object-glass is 40 ins. in diameter, with a focal length of 65 ft. The largest reflector is the Hooker Telescope at Mount Wilson Observatory, California, being 100 ins. in diameter. Large telescopes have a mechanical drive on them for use in observing stars, etc. By this means the telescope can be continuously focussed on a particular star or heavenly body, despite the earth's movement. Many, too, of the reflectors can be converted into different types, *e.g.* a 60-in. parabolic reflector at Mount Wilson can be used either as a Newton or Cassegrain reflector (150-ft. focus), a small convex hyperboloidal mirror being used in the latter case.

Prism Glasses.—The difficulty of holding mounted telescopes, which must be long if any degree of magnification is required, is obvious. Thus telescopes were little used, apart from astronomical work, during the last century. In 1898 Zeiss introduced prism glasses, and so portable telescopes (field-glasses) are now very common. Fig. 207 shows clearly the action of the two prisms, which obviates the necessity of a long tube, and explains the total reflection processes which result in the formation of an erect image for the eye-lens to magnify. Each eye sees an image produced by separate systems of lenses and prisms. A great advantage of such glasses is the increase in binocular vision (L. *bini*, two together + *oculus*, eye). Our two eyes are set apart, and with them we get different views of objects at the same time, the brain translating as a whole the different retinal images. This results in (*a*) *relief* (or solidity)—near things stand out from the objects further back, and (*b*) a power of estimating distance. We call it a stereoscopic effect (Gk. *stereos*, solid + *scope*), and its value can be realised if, using only one eye, an attempt be made to thread a needle, or

to touch a spot. As Fig. 207 shows, the object-lenses of prism glasses are set wider apart than the eye-lenses, and so this effect is intensified.

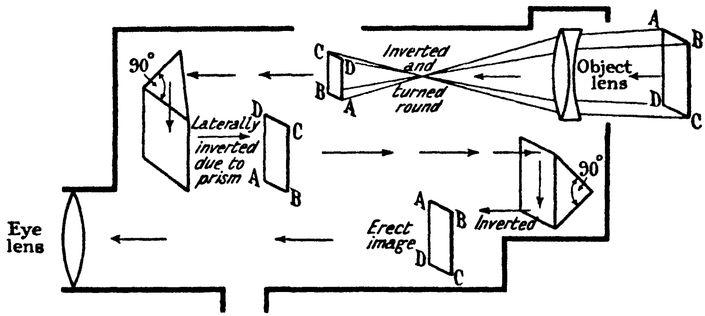


FIG. 207.—Action of Prisms in Prism Glass.

The Microscope.—The principle of the simple microscope, or reading-glass, has already been described on p. 297. The object to be observed is placed nearer to a converging (convex) lens than its focal length. The origin of the simple microscope is lost in antiquity, but its use was very common in the seventeenth century. There is also some uncertainty regarding the discovery of a form of microscope having two lenses, *i.e.* the *compound microscope*. It is said that Galileo made one; several forms appeared between 1650 and 1700, but they were all of so little use, owing to chromatic and spherical aberration, that the naturalists of the time preferred to use the single lens. The compound microscope was not commonly used till after the introduction of achromatic lenses (1758), but it was not till 1835 that the present efficient form was developed. The numerous discoveries in biology and pathology which quickly followed are sufficient evidence of the excellence of the new instrument. The principle involved is similar to that of the telescope, except that the object-glass of the microscope is of much shorter focal length than that of the telescope. Fig. 208 shows that the objective O gives rise to a real,

inverted, magnified image PQ. The eye-lens E acts as a simple microscope, its distance from PQ being less than its own focal length, and so a virtual, still inverted but greatly magnified, image A′B′ is obtained. Lenses O and E are fixed in a tube which can be moved to and fro, whilst cross-wires and a scale are placed in the correct position

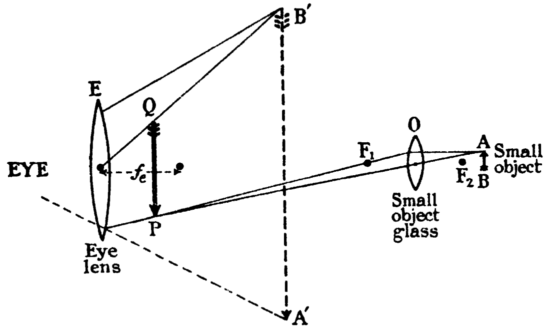


FIG. 208.—Principle of Compound Microscope.

for PQ to give the best magnification. In use, the tube is moved till the distance of O from the object under view is such that its image is in the same plane as the cross-wires, *i.e.* is seen clearly by the use of the lens E. In practice, compound microscopes of a more complicated form are used to overcome spherical and chromatic aberration.

Velocity of Light.—Light travels at such an enormous speed, approximately 186,000 miles per second, that it is not surprising that early thinkers attributed an infinite velocity to light. Galileo was probably the first to attempt to measure its velocity. He tried to measure the interval between the making of a signal, by a lantern, to a person a mile away, and the receiving of a flash signalled by that person directly he saw the first flash. Results, however, were not conclusive—as the interval was so minute.

In 1675 Römer, a Danish astronomer working in Paris, observed the rotation of Jupiter's satellites (moons).

He observed the times of their successive disappearances (*i.e.* eclipses) behind Jupiter, and found that the times varied. The interval between successive eclipses (approximately $42\frac{3}{4}$ hours) became slightly less as Jupiter appeared larger (*i.e.* the earth being nearer Jupiter) and became slightly greater as that planet appeared smaller (*i.e.* the earth further away from Jupiter). Fig. 209 indicates the changes in position, on an exaggerated scale, treating the orbit of the earth E round the sun S, and that of

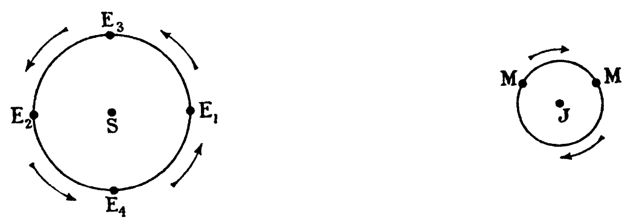


FIG. 209.

Jupiter's first satellite or moon M, as circular. The rotation of heavenly bodies in their orbits is found to take place in regular periods, and so the differences could be explained as due to fluctuations in the rotation of the satellite. Jupiter's first moon is considered because the plane of its orbit is almost the same as that of Jupiter round the sun. The time interval between two successive eclipses of this satellite can be measured accurately when the earth is about the position E_1, for its distance from the satellite is practically constant. Thus the time of future eclipses can be predicted.

Römer concluded that these differences were due to the varying times taken for light, leaving the satellite at the moment of its disappearance behind Jupiter, to travel to the earth, owing to the change in the position of the earth, i.e. *the velocity of light was finite*. Römer's idea was not universally accepted at first.

There are two methods of calculating the velocity of light from the observations, if the radius of the earth's orbit is known. They are as follows :—

(1) When the earth is travelling in its orbit from position

E_2 to E_4 and then to E_1 (Fig. 209) it is approaching Jupiter, and is thus meeting the light coming from the satellite. Thus the observed intervals between successive eclipses of the satellite become gradually shorter than the calculated intervals. On the other hand, when the earth is moving in its orbit from position E_1 to E_3 and then to E_2, it is receding from Jupiter and thus moving away from the light travelling to it from the satellite. Thus the observed intervals between successive eclipess of the satellite become gradually longer than the calculated intervals. The small differences are dependent on the distance moved towards or away from Jupiter, and are obviously a maximum when the earth is moving straight towards or away from it; this is only so in positions E_3 and E_4 respectively. It is found that the maximum difference between two successive eclipses is 15 seconds, and must obviously correspond to either of the two positions. Thus, during the period between two successive eclipses ($42\frac{3}{4}$ hours approx.), the earth moves towards or away from Jupiter, to cause the light to be received 15 seconds earlier or later.

But the speed of the earth is 18 miles per sec. approx.

$\therefore$ In $42\frac{3}{4}$ hours the earth travels $18 \times 42\frac{3}{4} \times 60 \times 60$ miles.

Thus light would travel, in 15 seconds, this distance.

Thus in 1 sec. light would travel $\dfrac{18 \times 171 \times 60 \times 60}{15 \times 4}$ miles

or velocity of light$=$184,680 miles per second approx.

A second method is as follows: It has been shown that as the earth moves round from E_1 to E_3 and then on to E_2, eclipses of Jupiter's satellite occur later than the calculated time. Obviously the time the eclipse occurs when the earth is at E_2 is later than the calculated time of eclipse by a maximum interval. As the earth moves from E_2 to E_4, and then to E_1, the eclipses occur earlier than calculated, and when the earth is at E_1 occur at the calculated time. This maximum lateness of eclipse at E_2 is explained as due to the time taken for the light to travel across the diameter of the earth's orbit, E_1E_2.

Now the average distance of the earth from the sun

$$=92{\cdot}5 \text{ million miles.}$$

$\therefore$ Diameter of earth's orbit$=E_1E_2=2 \times 92{\cdot}5 \times 10^6$ miles.

It was observed by Römer that the maximum lateness of

eclipse at E_2 (as calculated from position E_1) was 16 mins. 28 secs. or 988 secs.

Thus, in 988 secs. light travels from E_1 to E_2, *i.e.*

$$2 \times 92{\cdot}5 \times 10^6 \text{ miles.}$$

$$\therefore \text{ in 1 sec. light travels } \frac{2 \times 92{\cdot}5 \times 10^6}{988} \text{ miles}$$

$$= 187{,}000 \text{ miles approx.}$$

The first terrestrial method of measuring the velocity of light was carried out in 1849 in Paris suburbs by *Fizeau.* Fig. 210 illustrates the method in which light passes from a powerful source S through a space G between two adjacent teeth of a cog-wheel. It then travels a considerable distance (in Fizeau's original experiment, 8,633 metres), is reflected back by a plane

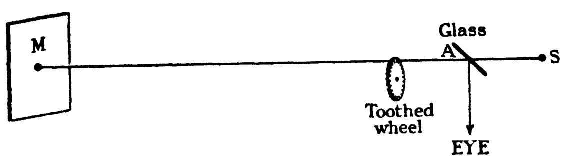


FIG. 210.

mirror M, normal to its path, through the same gap, and is then observed, being reflected to the eye by a thin piece of clean glass A. The cog-wheel is then rotated, its speed being gradually increased until it is such that light which passes through gap G, at any instant, on its return meets the next tooth of the wheel and is eclipsed. In this position of darkness the light travels from G to M and back again ($=2d$) in the same time as it takes for a tooth of the wheel to move to where a gap was previously.

If x=number of teeth in the wheel (=the number of gaps), and T sec.=time of one complete revolution of the wheel, then time between successive arrivals, at a point, of tooth and a gap$=\frac{1}{2}$ of $\frac{T}{x}$ sec.

In $\frac{1}{2}$ of $\frac{T}{x}$ sec. light travelled $2d$.

$\therefore$ in 1 sec. light travelled $\frac{4dx}{T}$ in appropriate units.

The disadvantages of this method are: (1) loss in intensity of the returning light, owing to the distance travelled (light is scattered by the dust particles, etc., in the path); (2) reflection from the teeth of the wheel makes it difficult to pick out light reflected from the mirror; (3) it is difficult to determine when the light is stopped by the middle of a tooth and not by the edge of one. The results obtained were about 0·5 per cent. higher than those estimated by astronomical methods. Fizeau had a wheel with 720 teeth, and revolved it 12·6 times per sec. to get darkness when d=8,633 metres. This gives a value for the velocity as $3{\cdot}13 \times 10^{10}$ cms. per sec.

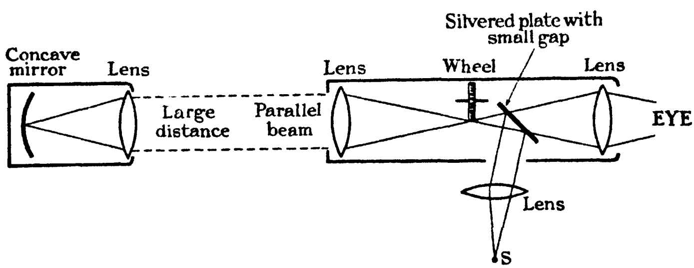


FIG. 211.—Apparatus for Determining the Velocity of Light by Fizeau's Method.

In 1874, Cornu worked at a distance of 23 kilometres, and also doubled the speed of the wheel, so that light entering one space came back through the next, thus giving a continuous light at the eye, instead of darkness. Young and Forbes in England, in 1881, also used this method, bevelling the teeth of the revolving wheel and painting them dull black, to overcome the difficulty caused by scattered light. The modification of the apparatus used is shown in Fig. 211.

Foucault, in 1850, measured the velocity of light by another method, but the importance of his work was that

he also measured the velocity of light in water and showed it to be less than in air. Fig. 212 shows the principle of Foucault's apparatus, in which light from a source S travelled in a narrow beam to a plane mirror M_1, from whence it was reflected at A to meet a concave mirror C normally. It was then reflected back to A and along its original path to S. If, however, the plane mirror was rotated into position M_2 whilst the light was travelling

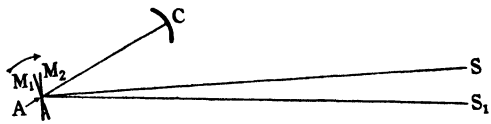


FIG. 212.—Foucault's Rotating Mirror Method for the Velocity of Light.

from A to C and back again, the path of the light was now deviated and became AS_1. Since the angle of rotation of a reflected ray is equal to twice the angle of rotation of the mirror from which the ray is reflected, $S\hat{A}S_1 = 2M_1\hat{A}M_2$.

Since the distance SA was made appreciably large, the angle SAS_1, which is very small, could be taken as equal to its tangent, *i.e.* $\frac{SS_1}{SA}$ radian.

$$\therefore \text{ rotation of mirror } M_1\hat{A}M_2 = \tfrac{1}{2} \text{ of } \frac{SS_1}{SA} \text{ radian}$$
$$= \theta \text{ radian.}$$

Suppose the mirror made a complete revolution (2π radians) in T sec.

$\therefore$ Time for mirror to rotate from position M_1 to M_2 (θ radians) $= \frac{\theta}{2\pi}$ of T sec.

This, then, was the time for the light to travel from A to C and back, *i.e.* 2AC.

$$\therefore \text{ Velocity of light} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{2AC}{\theta/2\pi \text{ of T}}$$
$$= \frac{4\pi \, . \, AC}{T \, . \, \theta}$$

It must be carefully noticed that although light is continuously falling on the plane mirror, it is only reflected to C when mirror is in position M_1.

A difficulty arose here. It was necessary to rotate the mirror at very high speed unless the distance AC was very great. Foucault made AC equivalent to 20 metres by using five fixed mirrors, the light being reflected to and fro five times through 4 metres. He inserted, between A and C, a tube containing a liquid and so measured the velocity of light in a liquid. In this way he showed that

$$\frac{\text{the velocity of light in air}}{\text{the velocity of light in a liquid}} = \text{air } \mu \text{ liquid},$$

thus confirming the relation calculated from the Wave Theory of Light (see p. 268), and so completed the overthrow of the Corpuscular Theory.

In the usual modification of the apparatus, an achromatic lens is used to throw a parallel beam. The chief disadvantage is the diminished brightness of the image as the distance AC is increased. Michelson overcame this in 1878–9, when working with this distance 600 metres, by using the lens between A and C. A little later he worked, with others, making the distance AC various values up to nearly 4,000 metres.

Exercises on Chapter XIX

1. Describe and explain the optical arrangement of a photographic camera. A camera has a lens of focal length 8 ins. What must the range of the bellows be in order that it may photograph objects (*a*) at a great distance, (*b*) so that the picture may be of the same size as the object? [*L.M.* 1927.]

2. A camera is provided with a simple lens of 6 ins. focal length. A sharply defined photograph of a man, standing 5 yds. from the lens, is taken. The height of the man is 70 ins. Find the length of the picture.

3. A photographic camera, with a simple convex lens of 5 ins. focal length, is adjusted to take photographs of distant objects. What alteration will have to be made before it can be used to make a reduced copy of a drawing one-third full size? Prove the magnification formula you employ, and explain the properties of a lens which you assume in your proof. [*L.M.* 1923.]

4. A lantern slide is $3\frac{1}{4}$ ins. square, and an enlarged image of it is to be formed by the aid of a lens of 6 ins. focal length upon a screen 20 ft. distant from the lens. What kind of lens should be used, at what distance from the slide must it be placed, and what will be the size of the image?

5. In what circumstances does a convex lens form a real image? A lens of 12 ins. focus is used to throw an image of a lantern slide 3 ins. square on a screen 50 ft. away. Where must the lantern slide be placed, and what will be the size of the picture? [*L.M.* 1922.]

6. Explain the action and use of a single convex lens when employed (*a*) in a photographic camera, (*b*) as a simple magnifying glass. [*L.G.S.* 1925.]

7. Explain the action of a simple magnifying glass. A person who sees objects most distinctly at a distance of 25 cms. uses a magnifying glass of 5 cms. focal length. Draw a scale diagram, with brief explanation, showing the action of this lens if it is held near the eye. [*L.G.S.* 1923.]

8. What kind of lens would you choose for a magnifying glass? Explain its action, giving a diagram. Why is it sometimes called a "burning glass"? (Give a diagram.) [*J.M.B.* 1927.]

9. Explain the formation of a real image by a single lens. Contrast an ordinary photographic camera with a pinhole one as regards (*a*) the necessity of focussing, and (*b*) the time of exposure. [*J.M.B.* 1926.]

10. Define focal length of a convex lens, and describe some means of finding it experimentally. A convex lens of focal length 5 ins. produces a 3-times magnified real image of an object. Where must the latter be placed? How could this lens be used as a magnifying glass, and what would be its magnifying power, if the nearest distance of distinct vision is 10 ins.? [*C.W.B.* 1927.]

11. Explain, with the aid of diagrams, the use of spectacles in cases of short sight and long sight respectively. Find the focal length of a concave lens which, placed just in front of a convex lens of focal length 0·9 in., brings parallel rays to a focus 1 in. behind the convex lens.

12. What is meant by the *principal focus* of a lens? State what lenses you would need, and describe how you would arrange them, to set up a simple telescope. Draw a diagram showing the image formation.

13. Describe the construction and mode of action of any one form of telescope. Draw a careful diagram illustrating your answer.

14. Describe how to arrange two lenses, one concave and one convex, so that you could use the combination for viewing a distant object. Add a careful diagram to illustrate your answer. [*J.M.B.* 1923.]

15. Draw diagrams showing how a convex lens of focal length 10 cms. produces an image of an object (*a*) 5 cms., (*b*) 15 cms. from it. How would you arrange this lens with another convex lens to form a simple microscope? [*C.W.B.* 1928.]

16. Describe the experimental arrangements you would make in order to obtain (*a*) a real image, (*b*) a virtual image, by means of a convex lens. How would you arrange two lenses to make a simple form of microscope? Make a diagram showing how the images are formed. [*J.M.B.* 1928.]

17. Explain, by the aid of careful diagrams, the use of 90° prisms in the modern prism glasses.

18. How has the velocity of light been determined by observations of the eclipses of Jupiter's first satellite? Assuming that Jupiter's first satellite revolves round the planet in a constant period of 40 hours, that the velocity of the earth in its orbit is 18 miles per sec., and that that of light

is 187,000 miles per sec., find the greatest and the least apparent intervals between successive eclipses.

19. A ray of light AB is incident upon a plane mirror at B and is thence reflected to another plane mirror C 3 miles from B. The mirror C being fixed normally to the direction BC, the ray is reflected back along the path by which it came. If now the mirror B is revolved rapidly, the ray, after leaving B for the second time, instead of following the direction BA, takes another path BD. Explain this and find the angle ABD, assuming that B revolves 100 times per sec. and that the velocity of light is 180,000 miles per sec.

20. Describe Fizeau's method for finding the velocity of light. In an experiment the velocity of light was calculated to be 3×10^{10} cms. per sec. when a wheel with 480 teeth was used and the distance between the wheel and distant plane mirror was 12 kilometres. What was the speed of revolution of the wheel to give the first position of darkness?

21. How was the velocity of light in water first measured? Explain the significance of the result obtained.

22. In an experiment to measure the velocity of light by Foucault's method the distance between the rotating and fixed mirrors was 500 metres, and at 1 metre the beam of light was displaced 1 cm. What was the speed of revolution of the rotating mirror if the velocity of light was found to be $3{\cdot}14 \times 10^{10}$ cms. per sec.?

Miscellaneous Exercises—II. Light

1. Describe the action and use of a simple photometer which you would employ to measure the candle power of an electric lamp stated to be 25 c.p. It may be assumed that a standard 16 c.p. lamp is available for the test. [*L.G.S.* 1925.]

2. A friend has three candles made by different manufacturers and asks you to report on their comparative illuminating powers. Describe the experiments you would carry out, and draw up a form on which to exhibit your observations and the statement of your conclusions. [*L.M.* 1922.]

3. How does the intensity of illumination on a screen depend on (*a*) the distance of the screen from the source, (*b*) the illuminating power of the source? Explain how a photometer utilises these facts. The intensity of illumination of a screen is twice as great when a source A is 50 cms. from it as it is when another source B is 80 cms. from it. Compare the illuminating powers of A and B. [*C.W.B.* 1928.]

4. Define the meaning of the terms reflection and absorption as applied to light. Explain (*a*) how you can use a photometer to show experimentally, that when light is reflected from a plane mirror part of the light falling on the mirror is absorbed; and (*b*) how you can determine the ratio of the part reflected from the mirror to that falling upon it.

5. Describe briefly two different ways of comparing the illuminating powers of two sources of light, and indicate the principle of each. Mention any difficulties which may occur in comparing different kinds of sources. [*J.M.B.* 1928.]

6. State the laws of reflection of light. If you were given a small piece of mirror $\frac{1}{2}$ in. square, how would you determine whether it was plane or concave? In the latter case, how would you find its focal length?

7. Explain the formation of images by multiple reflection. Two parallel mirrors, 12 ft. apart, face each other, and an object is placed 3 ft. from one mirror. Represent to scale the first three images seen in each mirror and draw rays whereby an observer looking into one of the mirrors sees an image formed after five reflections in all. [*L.M.* 1920.]

8. A bright point, P, is 2 ins. from the front surface of a plane mirror consisting of a piece of plate glass ½ in. thick, silvered at the back. Draw a careful diagram showing the positions of the first two images of P formed by the glass. Does the position of either depend upon your point of view? [*L.M.* 1921.]

9. A plate glass mirror, 0·25 in. in thickness, is seen under certain conditions to produce a number of images of a single object. With the aid of a diagram explain their formation, and account for their different brightnesses. [*L.M.* 1926.]

10. An object 2 ins. high stands 5 ins. in front of (*a*) a plane mirror, (*b*) a concave mirror of radius 6 ins. Give full descriptions of the images obtained, and show by diagrams how they are formed. [*C.W.B.* 1927.]

11. What is meant by the *principal focus* of a concave mirror? When a pin is held 20 cms. in front of a concave mirror an erect image is formed at a distance of 60 cms. from the pin. Calculate the focal length of the mirror and verify the calculation by a diagram drawn to the scale of 1 in.=10 cms. [*L.G.S.* 1922.]

12. An object 3 ins. long is placed 4 ins. in front of a convex mirror of 12 ins. radius of curvature. Describe and explain a geometrical construction by which the position and the size of the image may be determined. Find the position and size of the image either by direct measurement applied to your figure or by any other method.

13. Clear water 4 ft. deep seems 3 ft. deep to a person looking straight down into it, but less than this if he is looking obliquely. Explain these facts, and quote the optical laws you have used in your answer. [*L.G.S.* 1926.]

14. A given ray of light falls upon the surface separating two transparent media. Describe and explain a geometrical construction for tracing the refracted ray, and apply it to find the direction of a ray, travelling through glass, that, on passing from the glass into air, grazes the separating surface; the index of refraction of light passing from air into glass being 3/2. Show from the result the direction in which the rising sun would appear as viewed by an eye below the surface of a large lake.

15. Enunciate the laws of refraction of light. Draw three parallel lines, an inch apart in the plane of the paper, to represent rays of light incident upon a glass sphere of radius 2 ins. and with its centre upon the last of the series, and trace, by a geometrical construction, the paths of the refracted rays within and beyond the sphere, taking the refractive index of light passing from air to glass as 3/2. Taking your diagram to be correct for red light, indicate roughly how it would be altered if the incident rays were blue.

16. Construct the path of a ray of light through a right-angled isosceles prism (refractive index 1·5) if the original direction of the ray is parallel to the hypotenuse face. Explain briefly the successive steps in the construction. [*L.G.S.* 1922.]

17. How would you employ a convex lens of 4 ins. focal length (*a*) to throw a beam proceeding from a point-like source to a long distance,

(*b*) to form an image of a distant object upon a photographic plate, (*c*) to enable you to read small print with ease?

18. What are the distinctions between real and virtual images? and how may such images be respectively produced? A lens produces a virtual image of linear dimensions three times those of the object. The distance from the object to the lens is 8 ins. Find the position of the image and the focal length of the lens. [*L.G.S.* 1918.]

19. What do you understand by the term "magnification" as used in an optical sense? Supposing that the image formed by a convex lens is the same size as the object when the latter is 10 ins. from the lens, find the position of the object in order that an erect image, magnified twice, may be formed. [*L.G.S.* 1920.]

20. Explain the use of a convex lens as a magnifying glass. Is a long or short focus lens more suitable for this purpose? [*L.G.S.* 1922.]

21. Explain the meaning of *focal length* as applied to a concave mirror and convex lens respectively. A source of light is 200 cms. from a screen. It is desired to produce a 3-times magnified image on the screen. Show how this could be done with a mirror and a lens respectively, and find the necessary focal lengths. [*C.W.B.* 1929.]

22. Small electric torches are usually made with a concave reflector behind the filament and a convex lens in front of it. Show how either of these separately would produce a parallel beam of light; indicate the best position in which these could be used in conjunction for this purpose. [*J.M.B.* 1927.]

23. Explain the formation of a real image by a convex lens. A convex lens of 20 cms. focus is mounted vertically. Behind it, at a distance of 100 cms., an upright pin is placed. In front of it, with a space of 32·5 cms. between them, is another convex lens of 10 cms. focus. Find the position of the image of the pin when it is viewed through the two lenses. [*J.M.B.* 1927.]

24. A small object, 1 cm. in height, is placed 3 cms. from a convex lens of 4 cms. focal length. Show that an enlarged image of the object would be seen by an eye placed close to the lens on the side remote from the object.

25. If a beam of white light is passed through a prism and then intercepted by a screen we see on the screen a patch of white light with coloured edges. Explain this with the help of a diagram, and show how to modify the experiment so as to obtain a clear spectrum on the screen. [*L.G.S.* 1922.]

26. What information is to be obtained from the well-known experiment of passing white light through a prism? Explain fully how the recombination of white light can be effected. [*L.M.* 1920.]

27. Explain the use of a convex lens as a magnifying glass. Then explain the use of two lenses to obtain an enlarged image of a distant object.

28. Explain the principles involved in the telescope for obtaining (*a*) larger images, (*b*) clearer images.

29. Explain the principle of a compound microscope, showing the difference between it and a telescope.

GENERAL MISCELLANEOUS EXERCISES

Heat

1. Describe how to fit up and use a simple form of barometer. What extra fittings are possessed by a standard laboratory barometer? Explain why the readings on a simple barometer are usually lower in comparison with those of a standard instrument. [*L.G.S.* 1926.]

2. What are the effectsof heat upon solids, liquids, and gases respectively? Mention any scientific or industrial employment of these effects which may be frequently observed. [*J.M.B.* 1922.]

3. Describe how a mercury thermometer is standardised and graduated for the purpose of measuring temperature. How would you use such a thermometer to determine the boiling point of brine? [*L.G.S.* 1918.]

4. If you had one thermometer only, and you doubted the correctness of its fixed points, how would you check them as accurately as possible? [*J.M.B.* 1926.]

5. How would you test the accuracy of the fixed points of a mercury thermometer? If a difference of 2·7 mm. in the barometer pressure causes a difference of a tenth of a degree Centigrade in the boiling point, what should a Fahrenheit thermometer read for the boiling point of water when the barometer reads 750 mm.? [*L.G.S.* 1923.]

6. Describe a thermometer suitable for measuring maximum and minimum temperatures in England. Would it be equally suitable elsewhere?

7. Describe a constant-volume air thermometer, and explain how you would use it to measure the boiling point of a liquid. [*J.M.B.* 1927.]

8. Compare the merits of mercury, alcohol, and air as thermometric substances. [*L.G.S.* 1925.]

9. Distinguish between *temperature* and *quantity of heat*. In what units is each measured? How are these units defined? [*L.M.* 1922.]

10. Describe two simple experiments which indicate that different substances expand differently when heated. Explain the construction and the purpose of any contrivance in which the effect of expansion consequent on change of temperature is eliminated. [*L.G.S.* 1920.]

11. Carefully define the meaning of the term *coefficient of linear expansion*, and show from the definition how to determine the length of a bar of metal at a temperature t° C. from the length at any other temperature T° C. Two bars of iron and copper differ in length by 10 cms. at 0° C. What must be the lengths of the bars in order that they may differ by the same amount at all temperatures? The coefficients of linear expansion of iron and copper are 0·000012 and 0·000018 respectively.

12. How much freedom should be allowed per mile of engine rail to avoid stress in the rails if the variation of temperature between summer and winter is between 25° C. and −5° C. Linear coefficient of expansion of steel is 0·000012 per ° C.

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13. A weight thermometer contains 24 grms. of mercury at 0° C. On being heated to 100° C. it is found to contain 23·622 grms. Calculate the coefficient of linear expansion of the glass, given the coefficient of absolute expansion of mercury is 0·00018.

14. A weight thermometer at 100° C. contains 700 grms. of mercury. What is its internal volume at that temperature? (Density of mercury at 0° C.=13·60, coefficient of expansion=0·000182.)

15. Describe how the coefficient of expansion of a liquid may be determined directly, explaining how the expansion of the vessel is eliminated. If the cold column at 4° C. were 80 cms. high in the experiment and the hot column at 95° C. were 5 mm. higher, what would be the real coefficient of expansion of the liquid?

16. Define the *coefficient of expansion* of a liquid. Distinguish between real and apparent coefficients of expansion. Is such a distinction usually necessary in the case of a gas? Give a reason for your answer. Describe in detail an experimental method of determining the apparent coefficient of expansion of a liquid. [*L.G.S.* 1928.]

17. Draw a graph illustrating the variations of volume of a quantity of water as it passes from the solid condition (ice at −10° C.) to steam at 110° C. The graph need not necessarily be to scale, but the various points of interest with regard to it are to be explained. [*L.G.S.* 1918.]

18. What is Boyle's Law? How would you test it for pressures lower than that of the atmosphere? If 1,000 litres of hydrogen at atmospheric pressure are compressed into a cylinder 100 cms. long and 14 cms. diameter, what is the pressure inside the cylinder?

19. State the laws connecting the volume, pressure, and temperature of a gas. What will be the volume at 300° C. and 770 mm. pressure of a gas which occupies 800 c.c. at 0° C. and 760 mm. pressure?

20. State the laws connecting the pressure, volume, and temperature of a given mass of gas. A quantity of gas is collected in a graduated vessel over mercury. The volume at 15° C. is 45 c.c. and the level of the mercury in the vessel is 12 cms. above the level of the mercury outside. What volume would the gas occupy at 0° C. and 76 cms. pressure if the barometer stands at 75 cms.? [*L.M.* 1927.]

21. Explain the meaning of the expression *coefficient of expansion* of a gas. Supposing that a quantity of gas were to occupy 500 c.c. at 80° C. and 600 c.c. at 120° C., what would be its coefficient of expansion, assuming the gas to expand uniformly?

22. State Boyle's and Charles' Laws with reference to a perfect gas, and describe how you would verify experimentally the latter, especially showing how to tabulate and work out the results of the experiment. Some air is enclosed by a 10-cm. thread of mercury in a capillary tube sealed at one end. The length of the air column when the tube is vertical and the open end at the top is 80 cms. If the pressure of the air is 76 cms. of mercury, what is the length of the air column when the tube is inverted?

23. What do you understand by a degree Centigrade? Describe a constant-volume air thermometer. How would you graduate it? [*L.M.* 1920.]

24. Explain the meaning of *specific heat*. If 1,000 grms. of water at 45° C. are poured into 45 grms. of water at 15° C., standing in an aluminium vessel which weighs 250 grms., what will be the resulting temperature, the specific heat of the metal being 0·2? [*J.M.B.* 1926.]

25. A bottle is filled with water at 100° C. Another bottle exactly similar is filled with a solution of specific gravity 1·1 at 104° C. Both bottles in cooling to 20° C. give out the same amount of heat. Find the specific heat of the solution, neglecting the heat capacity of the bottles. What further particulars would be needed if this latter is taken into consideration? [*L.M.* 1922.]

26. Describe a simple elementary method of determining the latent heat of fusion of ice. Mention the mistakes, in method and manipulation, most likely to lead to an incorrect result. This latent heat is 80 calories per gram. What is it in British Thermal Units per lb.? [*L.M.* 1923.]

27. 45 grms. of water at 20° C. are mixed with 20 grms. of ice at −10° C., and in consequence half of the ice is melted. The latent heat of fusion of ice being 80 calories per gram, find the specific heat of ice. Describe how you would perform an experiment to test the accuracy of the above number, assuming the latent heat of ice to be correct, and point out the most likely sources of possible error in your experiment.

28. A substance has a melting point of 330° C., a specific heat when solid of 0·03, and a latent heat of fusion 5. Explain the meaning of these statements. Supposing heat to be supplied uniformly to a mass of this substance, compare the times of bringing it from 30° C. to its melting point and of melting it. [*L.G.S.* 1925.]

29. The specific heat of ice is 0·5, and the latent heat of fusion is 80. Give the meaning of these numbers, and say how far they are dependent on the system of units employed. How much ice at −10° C. must be put into 10 ozs. of water at 20° C. to lower its temperature to 5° C.?
[*L.M.* 1923.]

30. What is meant by the water equivalent of a calorimeter? A given metal and a given liquid have specific heats of 0·2 and 0·6 respectively. If the boiling point of the liquid is 80° C. and its latent heat of vaporisation 200 calories per grm., what will be the result of mixing 100 grms. of the powdered metal at 220° C. with 50 grms. of the liquid at 20° C.?

31. A copper calorimeter weighs 100 grms. and contains 250 grms. of naphthalene balls. It is originally at 15° C., and dry saturated steam at atmospheric pressure is passed in until the temperature of the whole is 90° C. Using the following data, find the final weight of the calorimeter and its contents. Loss of heat by radiation, etc., and of moisture carried away by uncondensed steam should be neglected. M. Pt. of naphthalene =80° C.; specific heat of solid naphthalene=0·32; L. Ht. of fusion =35; specific heat of liquid naphthalene=0·40; L. Ht. of evaporation of water=540; specific heat of copper 0·1. [*L.M.* 1923.]

32. Explain the statement "one caloric is equivalent to $4{\cdot}2\times10^7$ ergs." The height of a waterfall is 50 metres. What will be the rise in temperature of the water due to falling through this distance? (Acceleration due to gravity=981 cms./secs.^{2}.)

33. Suppose a petrol engine uses each hour 1 lb. of petrol, which produces 22,000 B.Th.U. of heat, and has an efficiency of 30 per cent. What is its H.P.? (1 H.P.=33,000 ft./lbs. per min. and 1 B.Th.U=778 ft./lbs.)

34. From what height would a lump of ice have to fall in order to melt itself?

35. What is meant by a *saturated vapour?* How would you find (*a*) the saturated vapour pressure of water at 30° C., (*b*) the temperature at which the water vapour in a room would become saturated?
[*L.M.* 1920.]

36. An imperfect barometer contains some air above the mercury in the tube and, as a result, the mercury stands at only 720 mm. The length of the air column is 100 mm. If the tube is depressed in the cistern (which is deep enough for the purpose) until the air column measures only 40 mm., what will be the height of the mercury in the tube above that in the cistern, and how far has the tube been depressed? (Neglect any change in level of the mercury in the cistern.) [*J.M.B.* 1927.]

37. Distinguish between evaporation and ebullition. What conditions determine the rapidity of each? The pressure of saturated water vapour at 50° C. is 9·2 cms. of mercury, and it is found that in air, under a pressure of 9·2 cms. of mercury, water boils at 50° C. Explain the relation expressed in this statement.

38. Water has a high thermal capacity, expands irregularly when heated, and is a poor conductor of heat. Explain these statements and give examples where these distinguishing properties are advantageous. [*L.M.* 1928.]

39. In what different ways can heat be transmitted? Illustrate your answer by considering the transmission of heat from the central heating furnace of a building to a person in an upper room. [*J.M.B.* 1928.]

40. In what ways does a hot object lose its heat to its surroundings? Describe a method of hastening the cooling process. [*L.G.S.* 1925.]

41. Define *thermal conductivity*. Describe carefully any experiment by which it can be shown that copper conducts better than iron and by which the ratio of their conductivities can be approximately determined.

42. Under what conditions does dew appear and disappear? A cloud is sometimes observed to cover the top of a mountain and to appear stationary though a strong wind may be blowing. Explain this. [*L.M.* 1922.]

43. Point out very carefully all the various phenomena which come into action when a damp cloth held in front of a fire begins to "steam." [*L.G.S.* 1921.]

LIGHT

51. Describe how to compare the effective candle-power of a lamp covered with a glass shade with its candle-power when the shade is removed. Find the ratio of the distance of the lamp from the photometer if the shade absorbs 9·75 per cent. of the light emitted by the lamp, assuming the distance of the comparison source to be the same in each case. [*L.M.* 1921.]

52. Describe an arrangement by which you could investigate the percentage of light reflected at different angles by a piece of plane glass. [*L.M.* 1921.]

53. Describe and explain the appearance of a grease spot at the centre of a sheet of white paper when viewed (*a*) by reflected light, (*b*) by transmitted light. A 16 c.p. lamp is placed 50 cms. from such a sheet of paper. Where must a 25 c.p. lamp be placed in order that the grease spot may be indistinguishable? [*L.G.S.* 1921.]

54. In an experiment with Rumford's shadow photometer the rod was 20 cms. from the screen. Beyond the rod, 120 and 180 cms. respectively from the screen are two sources of light, which produce on the screen shadows of the rod of equal intensities. Calculate the ratios of the widths of the shadows and of the candle-powers of the lights.

55. Two 16 c.p. lamps are at a distance of 1 and 2 metres respectively from a wall. At what distance from the wall must a 32 c.p. lamp be placed in order to give the same intensity of illumination?

56. State the laws of reflection of light, and describe how you would verify them. The tables in a restaurant are arranged in uniform rows parallel and perpendicular to the wall which is covered by a plane mirror. X, sitting four tables away from the mirror and looking into it at an angle of 45°, sees the image of Y apparently looking at him and sitting three tables from the mirror. Draw a figure approximately to scale showing the relative positions of the mirror, X, Y, and what each is looking at.

57. Prove the formula connecting the distances of object and image from a concave mirror with the radius of curvature of the mirror. Point out the conditions under which this formula is valid.

58. How would you place an object in front of a concave mirror of 12 ins. radius of curvature to give an image three times the size, (*a*) real, (*b*) virtual?

59. Compare the magnifications produced when an object is placed (*a*) 10 cms., (*b*) 20 cms. in front of a mirror of 10 cms. radius of curvature.

60. A letter L, sides each 1 in., is placed with its horizontal part on the principal axis of a concave mirror of 6 ins. radius of curvature and the corner of the L 9 ins. from the pole. What are the dimensions of the image?

61. Draw diagrams to show the formation of the image of an object by the reflection of light (*a*) from a plane, (*b*) from a concave spherical surface. Mention two applications of plane, and two of curved mirrors in practice. [*J.M.B.* 1927.]

62. A luminous object is placed on the principal axis in front of a concave spherical mirror at a distance 20 cms. from the mirror. If the radius of curvature is 30 cms., find (*a*) by geometrical construction, and (*b*) by calculation, the position, nature, and magnification of the image produced. State concisely what optical principles you have employed in your construction. [*L.G.S.* 1925.]

63. What is meant by the *total internal reflection* of light? In what circumstances does it occur? Describe one phenomenon in which the total internal reflection of light plays an important part. [*L.M.* 1920.]

64. Explain the terms *critical angle* and *total reflection*. The sides of a glass prism are each $2\frac{1}{2}$ ins. long and a ray of light is incident normally upon one face of the prism at a point 1 in. from the refracting angle. Draw a diagram showing the path of this ray through the prism, and explain briefly your construction. (Refractive index of glass $=3/2$.) [*L.G.S.* 1923.]

65. ABC is a section of a right-angled glass prism of refractive index 1·5. ABC$=90°$, AB$=$BC$=2$ ins. Rays parallel to AC are incident upon the face AB at points $\frac{1}{2}$ in. and $1\frac{1}{2}$ ins. from A. Trace the subsequent course of the rays. [*L.M.* 1921.]

66. An isosceles right-angled prism may be used (*a*) to turn a beam of light through a right angle, (*b*) to reverse the direction of a beam. Show by means of diagrams how each purpose is accomplished supposing the source of light to be a luminous arrow ↑. Explain clearly why the prism can be used as a mirror, and why it is more efficient than a piece of glass silvered at the back. [*L.G.S.* 1927.]

67. What are the conditions producing total reflection of light when it arrives at the boundary between two transparent media, *e.g.* water and air, or glass and water? Show how the phenomenon is made use of in

one of the following: (*a*) the prismatic compass, (*b*) the optical lantern, (*c*) the prismatic reflector. [*J.M.B.* 1923.]

68. Explain the action of the simple magnifying glass. How must a lens of 5 cms. focal length be placed in front of a small object so as to form an image 25 cms. from the lens on the same side? What is the magnification in this instance? [*L.G.S.* 1924.]

69. A candle flame is 6 ft. from a screen. It is required to throw a real, magnified image of the flame on the screen. Show, with diagrams, how this may be done (*a*) with a lens, (*b*) with a mirror. Find graphically, or otherwise, the position and focal lengths of the lens and of the mirror if the magnification in each case is to be 4. [*L.G.S.* 1926.]

70. What conditions are necessary for a lens to form a real image smaller than the object? Give an explanatory diagram. A camera lens of focal length 1 ft. is in focus for distant objects. What change must be made in the distance between the lens and the film in order that an object 10 ft. away may be in focus? What would the corresponding change be for a camera lens of 6 ins. focal length? [*L.M.* 1923.]

71. A convex lens of 10 ins. focal length forms a virtual image three times the height of the object. Where must the object be placed? Solve the problem first by drawing and then by formula. [*L.M.* 1921.]

72. Define the term *principal focus of a lens*. Rays of light from a luminous object are brought to a focus at a point A. A convex lens of 12 ins. focal length is then placed 12 ins. from A so as to intercept the rays before they meet at A. If they now meet at B, find the distance AB. Where else could the lens be placed so that, after passing through it, the rays might appear to diverge from B? [*L.M.* 1921.]

73. What is meant by a pure spectrum? How is it obtained? For what purpose is it wanted? [*L.G.S.* 1924.]

74. How may it be shown that white light can be split up into several coloured parts which can be recombined to form white light again? [*J.M.B.* 1923.]

75. Explain the terms refraction, deviation, and dispersion of light, and describe how you would arrange apparatus to produce a "pure" spectrum. [*C.W.B.* 1927.]

76. What is meant by *minimum deviation?* White light passes through a vertical slit placed parallel to the refracting edge of a glass prism. Draw a diagram showing the slit and the prism as seen from above and also the paths of the red and blue rays from the slit which suffer minimum deviation in passing through the prism. Has every prism a position of minimum deviation for light of a given colour?

77. A ray of white light is travelling in water. Describe as completely as you can what happens when the ray strikes the free surface of the water. [*L.M.* 1924.]

78. Explain how images are produced on the screen of a pinhole camera and a photographic camera, and contrast the manner of using them. Find the necessary range of the bellows of a camera which is to be used to photograph objects between 10 ft. and 100 ft., the focal length of the lens being 9 ins.

79. Compare the optical principles involved in the use of a photographic camera and the human eye.

80. Explain the principles involved in the projection of lantern slides by an optical lantern, explaining the use of all the parts. A 6-ft. square picture is to be projected on a screen 18 ft. away from a slide 3 ins. square. What must be the focal length of the lens?

81. If the lens of an optical lantern has a focal length of 8 ins. and is 15 ft. from the screen, find the size of the picture if the slide is 3×3 ins. Compare the illuminations of the slide and the lantern.

82. A convex lens P of 20 ins. focal length is placed so that on its principal axis is a luminous point 40 ins. away. The light passes through P and then through a second lens Q of 5 ins. focal length and then emerges as a parallel beam. Where must Q be placed, (*a*) if it is a converging lens, (*b*) if it is a diverging lens?

83. A beam of light is reflected by a rotating mirror to a fixed mirror which sends it back to the rotating mirror from which it is again reflected and then makes an angle of 18° with its original direction. The distance between the two mirrors is 10^6 cms., and the mirror makes 375 revs. per sec. Calculate the velocity of light.

ANSWERS

ANSWERS

HEAT

Exercises 1, p. 20.

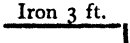
7. 241·54 metres.
9. (*a*) 1,000,620 dynes per sq. cm.
   (*b*) 14·49 lbs. per sq. cm.

Exercises 2, p. 38.

3. 59°, 86°, 122°, 158° and 194° F.
   10°, 25°, 45°, 65° and 75° C.
4. −17·7° C. and 37·7° C.; 89·6° F. and 413·6° F.
5. . . . 45° F. . . . 50° F.
8. 129·2° F.
11. 10° C. and −7·7° C.

Exercises 3, p. 53.

5. 10·0028 ft.; 10·0056 ft.; 10·0084 ft.
6. 0·10464 inch.
7. 202·84 cms.
8. 30·018 ins. approx.
9. (*a*) 35·837 m.; (*b*) 35·829 m.
10. Approx. calc. gives gains nearly 5 secs. a day; exact calc. gives gains 0·4 sec. a day.
11. Iron 3 ft.



Brass 2 ft.

12. 292·7° C.
13. 0·0628 sq. cms.
14. 40° C.; $66\frac{2}{3}$° C.
15. 15·02475 c. ft.; 15·0495 c. ft.
16. 4·983 c. ft.
17. 525·57 c. ft.
18. Steel 412·1 c.c.; space 113·5 c.c.

Exercises 4, p. 69.

5. 0·000005.
6. 0·00045.
7. 1·26 grms.
8. 39·69 grms. approx.
9. 0·000145.
10. 985·22 grms.
11. 12·59 grms. approx.
12. (1) 98·21 grms.; (2) 100·27 grms.; (3) 98·47 grms.
13. 6·25 cms.
14. 22·634 cms.
15. 408 c.mm.
16. 0·000606.

Exercises 5, p. 83.

2. 200 c. ft.
3. $\frac{1}{6}$ goes out; 200 c. ins.
4. $1\frac{2}{45}$ atmospheres.
7. $\frac{1}{272\cdot7}$ or 0·00366.
8. 0·00369.
9. 36·96 atmospheres.
10. −265° C.
11. 689·46 mm.
12. 150 c.c.
13. 2·58 atmospheres.
14. 6·94 %.
15. 958·4 mm.
17. 100·94° F.
18. 19·75 lbs. per sq. in.
19. 44·9° C.

EXERCISES 6, p. 107.

4. 14·54 grms.; 2·857 grms.
5. 17·14° C.
6. 11·995 grms.
7. 0·078 approx.
8. 160·35° C.
9. 0·7 approx.
10. 0·606.
11. 0·511.
12. 11,222·5 cals. per min.
14. 79·5 cals. per grm.
15. 10·31 grms.
16. 47·7 grms.
18. 93·75 kgrms.
19. 13·28 mins.
20. 35·82° C.
21. 1·144 grms.
22. 5425·8 kgrms.
23. 39·85° C.
24. 120 grms.
25. $6\frac{1}{4}$ mins.; $40\frac{1}{2}$ mins.
26. 1·6 pence.
27. 11·55 grms.
28. 8° C.
29. 14·28 mins.: 7·28 mins.
30. 31·03° C.

EXERCISES 7, p. 118.

4. 11·57° F. approx.
5. 0·0234° C.
6. 4·27° F.
7. 3·41 lbs.
8. 798·5 ft.-lbs. per B.Th.U.
9. 0·071.
10. 42·11 H.P.; 23·74 %.
11. $\frac{25}{78}$ or 0·32 lb.
12. $\frac{3}{280}$ or 0·0107.

EXERCISES 9, p. 146.

5. 27·73 %.

EXERCISES 11, p. 174.

4. 5 cms.; 30 cms. per sec.; $\frac{1}{6}$ sec.

EXERCISES 12, p. 189.

11. 1680 cals. per min.
14. 37 : 27.

MISCELLANEOUS EXERCISES—I. HEAT, p. 190.

2. −459·4° F.
3. 288° absolute.
4. 40° C.; 72° F.
8. 29·7285 ins.
11. 17·27 c.c.
13. 313° C.
14. 1169·6 grms. per sq. cm. (or 136 grms. per sq. cm. above atmospheric).
16. 100 grms.
17. 950 cals.
18. 25,240 cals.
19. $12\frac{1}{2}$ grms.; 30·77 C. (resultant temp.).
20. 195,700 cals.
21. 540 cals. per grm.
22. 162·93 grms.
23. 12·58° C.
24. 0·1329.
27. 0·05 lb. approx.
33. 76 cms. of Hg.

LIGHT

EXERCISES 13, p. 217

7. 27 : 64 or 1 : 2·37.
8. 6·12 ft.
9. 3 : 2; 2·24 ft. from weaker lamp.
11. $\sqrt{2}$ or 1·414 ft.
12. 25 c.p.
13. 125 : 64.
14. 125 : 27.
15. 2 foot-candles.
16. 152 : 125 or 1·216 : 1.
20. 2 : 1.

EXERCISES 14, p. 239.

16. 0·0125 radian or 0·716° approx.
17. 2° 52′.

EXERCISES 15, p. 260.

2. Real, inverted image, 22·5 cms. from mirror; magnification $\frac{1}{2}$.
3. Real, inverted image, 3 ins. from mirror; magnification $\frac{1}{2}$.
4. $1\frac{2}{3}$ ft.

5. Approx. 1·1 ft. from candle and 11·1 ft. from wall.
6. Real, inverted image, $4\frac{2}{7}$ ins. from mirror; magnification $\frac{3}{7}$.
8. 12 ins. behind mirror, virtual, erect, magnified 3 times; $8\frac{4}{7}$ ins. in front of mirror, real, inverted, magnification $\frac{3}{7}$.
11. (*a*) $26\frac{2}{3}$ cms.; (*b*) $13\frac{1}{3}$ cms.
12. $-6\frac{2}{3}$ cms.; $3\frac{1}{18}$ cms.
13. (*a*) 30 cms.; (*b*) 10 cms.
14. 12 ins.
15. 3 times as large.
17. Image 6 ins. behind mirror.
18. 4 ft. away.
21. (*a*) 8 ins. behind mirror, magnification $\frac{1}{6}$; (*b*) $6\frac{2}{3}$ ins. behind mirror, magnification $\frac{1}{3}$; (*c*) $3\frac{1}{3}$ ins. behind mirror, magnification $\frac{2}{3}$.

EXERCISES 16, p 284.

25. $41\frac{3}{4}°$ approx.
27. 39° approx.
28. 37° approx.

EXERCISES 17, p. 306.

1. (*a*) Real, inverted image, magnification 2; 30 cms. on other side of lens. (*b*) Virtual image at ∞. (*c*) Virtual erect image, magnification 2; 10 cms. from lens and 5 cms. from candle.
9. $1\frac{1}{4}$ ft. from object.
10. $-42\frac{6}{7}$ cms.
12. $4\frac{1}{6}$ cms.
13. 10·144 ins.; move lens 27·712 ins. nearer screen
14. Converging lens, $f=-\frac{9}{16}$ ft., placed $2\frac{1}{4}$ ft. from screen and $\frac{3}{4}$ ft. from object.
15. -24 cms.
16. 2 ins. from the object.
17. $\frac{12}{13}$ in. from lens.
18. $7\frac{1}{2}$ cms. from lens.
19. $f=-18\frac{3}{4}$ cms.; 50 cms. nearer.
20. Real, inverted image, magnified twice; $f=-1\frac{1}{3}$ ft.; when lens 2 ft. from wall, real inverted image, magnification $\frac{1}{2}$.
22. AQ=12 cms.; PQ=18 cms.

EXERCISES 19, p. 348.

1. 8 to 16 ins. from lens.
2. 2·41 ins.
3. Increase distance between lens and plate from 5 ins. to $6\frac{2}{3}$ ins.
4. Converging lens placed $6\frac{2}{13}$ ins. from slide; image 10 ft. $6\frac{3}{4}$ ins. square.
5. $1\frac{1}{49}$ ft. from lens and $51\frac{1}{49}$ ft. from screen; $12\frac{1}{4}$ ft. square.
10. $6\frac{2}{3}$ ins. from lens; 3.
11. 9 ins.
18. 40 hrs. $\pm$ 13·86 secs.
19. 2·4°.
20. 13 revs. per sec. approx.
22. 250 revs. per sec.

MISCELLANEOUS EXERCISES—II. LIGHT, p. 350.

3. 25 : 32.
10. (*a*) Virtual, erect image, same size as object, 5 ins. behind mirror. (*b*) Real, inverted image, size 3 ins., $7\frac{1}{2}$ ins. from mirror.
11. 40 cms.
12. Virtual, erect image, 2·4 ins. behind mirror, 1·8 ins. high.
18. Image 24 ins. from lens and 16 ins. from object; focal length of lens -12 ins.
19. $2\frac{1}{2}$ ins. from lens.
21. Mirror focal length $+75$ cms. placed 100 cms. from lamp and 300 cms. from screen.
Lens focal length $-37·5$ cms. placed 50 cms. from lamp and 150 cms. from screen.
23. 30 cms. from lens $f-10$ cms., and 2·5 cms. from lens $f-20$ cms.

GENERAL MISCELLANEOUS EXERCISES

HEAT, p. 353.

5. 211·3.
11. Copper 20 cms.; iron 30 cms.
12. 1·9008 ft.
13. 0·000006.
14. 52·4 c.c.
15. 0·0000687.
18. 64·96 atmospheres, or 4936·76 cms. of Hg.
19. 1657·31 c.c.
20. 35·36 c.c.
21. $\frac{1}{120}$.
22. $104{\cdot}\dot{2}\dot{4}$ cms.
24. 42·4° C. approx.
25. 0·8658.
26. 144 B.Th.Units.
27. 0·5.
28. 9 : 5.
29. $1\frac{2}{3}$ ozs.
30. 5 grms. of liquid vaporised and rest at 80° C.
31. $378{\cdot}\dot{5}\dot{4}$ grms.
32. 0·1168° C.
33. 2·593 H.P.
34. $3{\cdot}42 \times 10^6$ cms. or 112,033 ft.
36. Height of Hg 660 mm.; tube depressed 120 mm.

LIGHT, p. 356.

51. With shade : without shade = 0·95 : 1.
53. 62·5 cms. from paper.
54. Powers 9 : 4; widths 15 : 16.
55. 1·265 metres.
58. 8 ins. in front; 4 ins. in front.
59. (*a*) 1; (*b*) $\frac{1}{8}$; ratio 3 : 1.
60. Upright portion $\frac{1}{2}$ in., horizontal portion $\frac{3}{14}$ in.
62. Real, inverted image, magnified 3 times, 60 cms. in front of mirror.
68. $4\frac{1}{6}$ cms. in front of object; 6.
69. Lens $f = -\frac{24}{25}$ ft.; $4\frac{4}{5}$ ft. from wall. Mirror $f = 1\frac{3}{5}$ ft.; 8 ft. from wall.
70. Move film back $\frac{1}{3}$ ft.; move film back $\frac{1}{38}$ ft.
71. $6\frac{2}{3}$ ins. from lens.
72. 6 ins.; 6 ins. from A and 12 ins. from B.
78. 9·068 ins. to 9·73 ins.
80. −8·294 ins.
81. $64\frac{1}{2}$ ins. square; 462·25 : 1.
82. 45 ins. from P; 35 ins. from P.
83. 3×10^{10} cms. per sec.

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SOUND

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SOUND

CHAPTER XX

SOUND—LONGITUDINAL WAVES IN AIR—CONDUCTION OF SOUND

IN the early days of Broadcasting, people were frequently remarking how wonderful it was that sounds made could be heard at a distant place. To-day Broadcasting is accepted without remark, just as from our early days we accept the process of speech and hearing as being quite natural. Yet *Sound*, or *Acoustics*, is Nature's telephone for man and other creatures, and its instruments are very remarkable. Think of the marvellous qualities of the ear, enabling us to listen to a band and pick out the sounds of different instruments!

Our study of Sound will include the external causes of sound, its transmission and reception. But the sensations produced, and how they are produced inside our bodies, are outside the scope of this book.

The first important point to notice is that **a material medium is essential to the transference of sound,** and so sound does not pass through a vacuum. (The ether of space transmits light and heat but not sound energy.) This is shown when sounds are made in a vacuum and cannot be detected outside. A suitable apparatus is a glass receiver, for an air-pump, having a rubber bung tightly fitting in the top. Leading out of this are two wires, sealed in the bung and fixed to a small electric bell inside. The receiver is placed on the metal plate of an air-pump and the wires joined externally to sufficient dry cells to ring the bell. The air is gradually removed

from the receiver by working the pump, and the sound of the bell gradually becomes weaker until, when most of the air is removed, it cannot be heard at all.

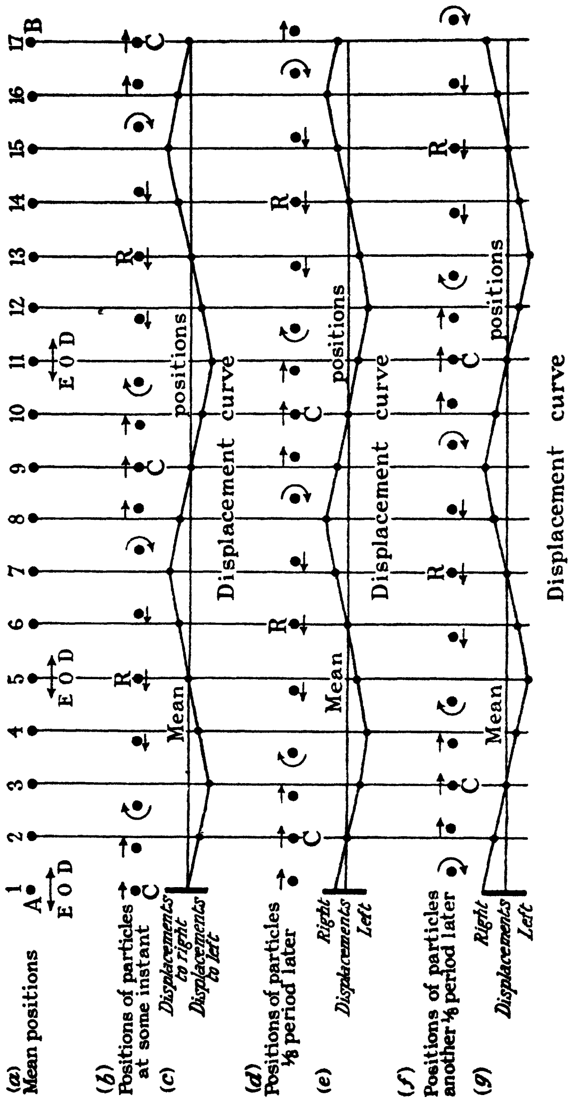
Ordinarily, then, sound is carried by the air, but other materials transmit sounds as well At some time you have probably put your ear at one end of a fallen tree and listened to the sound of some one lightly tapping the other end of the tree. Most of you know, too, that the sound of galloping horses or the rumbling of a distant train can be heard a mile or so away if an ear is pressed against the ground to receive the sounds transmitted by the earth.

The next important point to realise is that **sound is produced by some body in vibration.** The vibrating body disturbs the material medium and *the disturbance is passed on through the medium*, *i.e.* transmitted to a distant point, where it is there able to affect special instruments, *e.g.* the ear. That sound *is* connected with vibration is evidenced by (*a*) the shaking of window-panes on the beating of drums in the street; (*b*) the trembling of the earth on a report of a cannon; (*c*) the trembling of the lips of an oboe or clarionet player when playing his instrument; (*d*) the visible vibrations of a piano-wire, violin-string, etc., when the instrument is being played.

Further, *sound is produced by a repetition*, at very short intervals, *of mechanical action* in such a device as a rattle, or a flexible card pressing against a rotating cog-wheel. The discovery of the latter fact is attributed to Galileo, who is said to have been found, when quite a small boy, making sounds by pulling a knife-edge over the milled edge of a coin. The *siren* is a mechanical device for producing a repetition of vibrations in the air (see later), whilst the *reed* is a kind of elastic tongue which will vibrate to and fro and is inserted in many musical instruments.

The Transmission of Sound.—Air is the usual vehicle of the energy which gives to us the sensation of sound. But *there is* obviously *no movement of the air, as a whole, when sounds are transmitted.* There is, for example, no blast of air from a violin or trumpet when played, nor even from a cannon when it is fired.

The prevalent theory is that sound energy is transferred in the form of a wave-motion. Wave-motion in general, and transverse waves in particular, have been studied in Chapter XI. But we consider that **sound waves are not transverse,** *i.e.* the air particles transmitting sound energy are not considered to move in a line perpendicular to the direction of the wave-motion. There is, however, another form of wave-motion. You have probably stood on the edge of a cornfield and watched the wind-waves across the field, how the stalks of corn bend, in succession, to a gentle breeze. Each pushes the adjacent stalks in turn and so the ears of corn move slowly, to and fro, in the direction in which the waves appear to travel. This kind of wave-motion, in which **the particles vibrate along the line in which the waves are propagated,** is called **longitudinal wave-motion.** It can be illustrated by means of a specially-wound spiral coil of wire. According to Professor Barton's standard work on Sound, if the coil is to be effective for a slow longitudinal wave-motion along it, the wire should be about 0·15 cm. diameter (copper wire is suitable), and the coil should consist of at least 200 turns, about 1 cm. apart and 10 cms. in diameter. Each turn should be supported by a fine silk thread in the form of a V, the point of the V being at the top of a turn of wire, the sides being 1 metre long and ½ metre apart at the ends which are fixed to a wooden framework. If the end is moved to and fro, in the direction of the length of the coil, successive turns are seen to move to and fro in the direction of the length of the coil whilst waves travel up and down the coil. But the appearance is not one of wave-motion as compared with water waves. There are no crests or troughs, for example. Let us then analyse the motion, by considering a line of small particles as we did in the case of transverse wave-motion (on p. 168). Suppose the particles 1, 2, 3, etc., etc., are spaced at equal distances, when at rest, along the line AB (Fig. 213*a*), and that the first particle is caused to move to and fro about its mean position and along the direction of AB. Suppose that in so doing it jogs the next particle and sets that


(a) Mean positions
(b) Positions of particles at some instant
(c)
Displacements to right
Displacements to left
(d) Positions of particles ⅛ period later
(e)
Right
Displacements
Left
(f) Positions of particles another ⅛ period later
(g)
Right
Displacements
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E O D
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Mean positions
Displacement curve
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FIG. 213.

moving similarly, but a little later, and that this process is continued along the row. Then the positions of the particles at any instant will be as shown in Fig. 213*b*.

Suppose at the instant considered, No. 1 particle is in its mean position and moving towards B, and that the extremes of vibration motion of the particles are E and D as indicated in Fig. 213*a*. Then in $\frac{1}{8}$th of a period any particle moves $\frac{1}{8}$th of the distance E to D and back again, *i.e.* $\frac{1}{2}$ of the amplitude OE or OD. Thus if we suppose each of the numbered particles in turn attains a similar position to that occupied by the next lower-numbered particle $\frac{1}{8}$th of a period before, the difference in position between successive particles at any instant will always be $\frac{1}{8}$th of a period, or $\frac{1}{2}$ of the amplitude OE or OD.

Thus when :

No. 1	particle is	in its mean position and moving to right,
No. 2	,,	$\frac{1}{2}$ of OE to left of its mean position and moving to right,
No. 3	,,	at extreme left of its mean position and just reversing direction,
No. 4	,,	$\frac{1}{2}$ of OE to left of its mean position and moving to left,
No. 5	,,	at its mean position and moving to left,
No. 6	,,	$\frac{1}{2}$ of OD (or OE) to right of its mean position and moving to left,
No. 7	,,	at extreme right of its mean position and just reversing direction,
No. 8	,,	$\frac{1}{2}$ of OD (or OE) to right of its mean position and moving to right,
No. 9	,,	at its mean position and moving to right ; and so on.

Fig. 213*b* shows how the particles are spaced out, and it is seen that Nos. 1 and 9 particles, which are at the mean positions and moving to the right, *i.e.* in the direction in which energy is transmitted, have the particles on either side closer to them than in the normal spacing ; they are said to be centres of compression or condensation (the particles around being squeezed in towards them) and the density of particles around them is greater than normal.

Nos. 5 and 13 particles, which are at their mean positions and moving to the left, *i.e.* in the opposite direction to which energy is transmitted, have the particles on either side farther away from them than in the normal

spacing; they are said to be centres of rarefaction, the density of particles around them being less than normal.

Now, it is possible for us to realise why we call this method of transmission of energy wave-motion. Fig. 213*c* is a curve showing the displacement of the particles, at the instant indicated in Fig. 213*b*, plotted against their mean position, the displacements to the right of the mean position being plotted above the line and displacements to the left of the mean position below the line. This displacement curve is similar in form to the curve in Fig. 80 for transverse wave-motion. There are other features of similarity between the two motions, which cannot be treated here, all making clear that the motion of the particles is a form of wave-motion in which the particles move to and fro in the direction of the waves, *i.e.* it is longitudinal wave-motion.

It should be noticed that particles at the extremes of their motions (*e.g.* Nos. 3, 7, 11, etc.) have no velocity; they are also centres of normal density, and have an area of condensation on one side and an area of rarefaction on the other side.

Figs. 213, *d*, *f*, *e*, *g*, show the positions of the particles and the curves of displacement after two successive intervals of $\frac{1}{8}$th of a period, and it can be seen from the set of figures how the crest (indicated X) progresses, and also how the centres of condensation and rarefaction move. It should then be clear that during one period, any particle, in its vibration to and fro, goes through a gradual cycle of normal density, centre of condensation, normal density, and centre of rarefaction. Further, the arrangement shown repeats itself at regular distances. These are essential features of *progressive longitudinal wave-motion.*

Further, the wave progresses a distance equal to a wave-length (distance between two nearest particles in the same phase, *e.g.* 1 and 9, 2 and 10, etc.) in a time equal to the period, and so the equations deduced on p. 171 hold; *i.e.* $V=\frac{\lambda}{T}$ or $V=n\lambda$.

Crova's Disc is a device for illustrating longitudinal waves. In the middle of a large sheet of paper a small circle (0·7 cm. radius) is drawn (Fig. 214) and its circumference divided and numbered as is the face of a clock.

With centre No. 12 and radius 1·5 cms., describe a circle ;
" " No. 1 " " 1·5+·5 cms., " " ;
" " No. 2 " " 2+·5 " " " ;

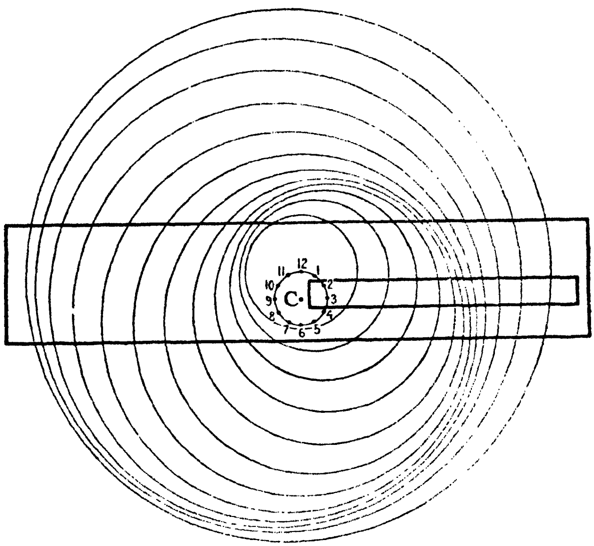


FIG. 214.—Crova's Disc.

proceed round the numbers in order, drawing circles with the same increase in radius each time. A piece of cardboard, or metal, with a rectangular slot as shown, is held over the disc, which is rotated about its centre C. Longitudinal waves (compression and rarefaction alternating) appear to travel to and fro in the space seen through the slot.

Progressive Longitudinal Waves in Air.—Air and, in fact, all gases offer no permanent resistance to a change of shape. They do offer a slight resistance (in the case of speedy changes, *e.g.* in the movement of an aeroplane or motor-car, the resistance may be very

appreciable), but the force opposing motion disappears when the motion stops and so has no restoring effect; there is no tendency for a gas to return to its original shape. **But gases do offer a resistance to a change in volume.** If a gas is compressed by a pressure it exerts an opposing pressure, and if the compressing pressure is removed this opposing one acts as a force of restitution, and so the gas expands to its original volume. This is well realised when a finger is held over the end of a bicycle pump and the handle forced in a little and then released. Thus, *a gas is elastic in the sense that it can be compressed and will recover from the compression.* (Refer to pp. 6 and 167.)

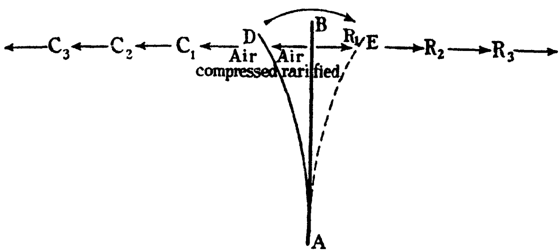


FIG. 215.

Thus waves involving compression and rarefaction are possible, in air, *i.e.* longitudinal waves of the form just discussed. It must be realised, of course, that these waves (which will produce sounds) are superimposed upon the molecular motions of the air particles (the motions which, according to the Kinetic Theory treated in the Heat Section, are revealed by their temperature).

Suppose a flexible, flat, steel rod AB is quickly displaced to position D as shown in Fig. 215. Instantaneously then the air to the immediate left of the rod is compressed, whilst to the immediate right is a space of no air, or, as we say, the air is rarefied. Immediately, the compressed air to the left of the rod expands to regain its normal condition, by virtue of its elasticity, so compressing the air to its left. This action continues outwards

—through C_1, C_2, C_3, etc., or, as we say, a wave of compression travels outward to the left of the rod.

On the immediate right of rod AB, when in position AD, a rarefaction, or space of low pressure, was produced. The air surrounding is at atmospheric pressure, and so moves in to equalise the pressures. Thus a space of rarefied air is left on the right, say at R_1, and by the repetition of the process travels out through R_2, R_3, etc., or, as we say, a wave of rarefaction travels outwards to the right.

Very quickly the elastic, flexible rod recovers its position, and its inertia causes it to go through its mean position, reaching position E. During this movement the air to the right is this time compressed, and in the space between D and E there is momentarily little air, *i.e.* rarefaction. Thus, by actions similar to those just described a wave of compression travels out to the right and a wave of rarefaction to the left. Now as the rod recovers its position and goes over nearly to D again, a compression wave travels out to the left and a wave of rarefaction to the right. So, as the flexible rod moves to and fro compressional and rarefactional waves travel out alternately in both directions, and these are progressive longitudinal waves. It is found that if such a flexible rod makes more than 20 complete to-and-fro motions per second, a sound is heard. The changes in pressure travelling out in the air set up similar changes in the air in the external ear of a person intercepting them (Fig. 216). These changes affect the *drum-skin* (or drum, as we call it), forcing it into corresponding vibrations, for it is of such a nature that it readily responds to these " audible " vibrations. These motions of the drum are transmitted to three bones, called the hammer (in contact with the drum), anvil and stirrup, which form a kind of bridge across the space at the back, this space being the " drum " (or tympanum) of the ear. The drum is frequently open to the air in the throat *via* the Eustachian tube, and so the drum-skin is not strained, for the pressure is the same (atmospheric) on both sides of it The stirrup is against, and transmits the vibrations to a membrane which covers the *inner ear* or *cochlea*, the seat of hearing. This is like

the spiral shell of a snail, and contains a liquid in which is another thin flexible tube filled with another liquid. The

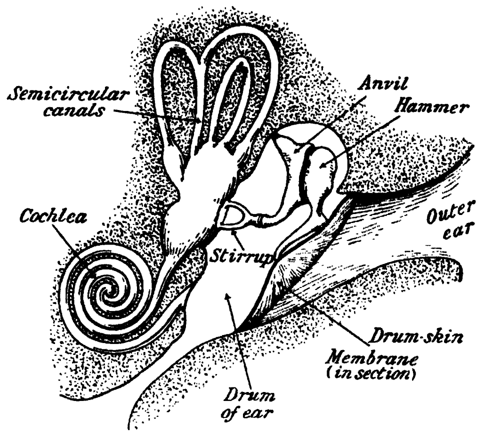


FIG. 216.—Human Ear.

vibrations produced in the membrane are considered to be analysed by the cochlea and converted into nerve impulses which are interpreted by the brain. Thus, although the outer ear or drum may be damaged, a person can be made to hear if a suitable connection is made between the outer air and the membrane of the inner ear, provided the cochlea, etc., is undamaged. For example, in such cases people can often hear *via* their teeth. An apparently deaf person can hear the sounds of a piano when gripping one end of a stick by the teeth, the other end being held by the top cover of the piano. Such persons can "listen-in" by using a tight pair of telephones pressing against the bony part of the head, along which the vibrations are carried to the inner ear. The following experiment illustrates that. Close your ears by cotton-wool pads so that you cannot hear the sound of ticking of a watch. Grip the watch by the handle between your teeth, and you will hear the watch very distinctly.

The above are examples of the *conduction of sounds by solids*, and the earliest form of *stethoscope* made by

Hænnic in 1819 is an application of this. It is a hollow, wooden cylinder applied by a doctor to the chest, etc., of a patient in order to listen to the action of the heart or lungs. The sound is carried partly by the wood and partly by air in the cylinder. In later forms a diaphragm was introduced in the cylinder to better set the air in vibration.

Of course, all solids do not conduct sounds well; substances such as hemp, straw, wadding, carpet, are all bad conductors (as they are of heat, too), and so are used to stifle sounds.

The subject of noise and its restriction is receiving much attention—in this age of wireless loud-speakers, jazz bands, motor-car horns, etc., it is clear that a means of reducing noise is essential, particularly for houses and public buildings. The materials which make for warmth in a house are not necessarily the best to deaden sounds. Hollow walls and porous plasters are found to increase reverberation (or reflection, which will shortly be discussed); absorbent plasters are being studied. Experiments are being made, with some success, in filling air spaces in walls with sawdust or hair-felt.

Liquids also conduct sounds. Persons swimming under water can hear sounds made in the water, and, to a certain extent, sounds made outside it, whilst anglers will testify that fish in water hear sounds made around. Many of you have probably noticed, when at the Aquarium at the Zoo, that sounds made in water by some of the larger fish are heard outside.

As regards gases, we have already seen that air transmits sounds, but it is found that sounds become more faint, and the voice is more difficult to hear, in mountainous regions and high altitudes, whilst in polar regions sounds are more clear and intense. A simple way of illustrating this is as follows: Fix solid glass rods in rubber bungs which fit the necks of two round-bottomed flasks. Attach a small bell to each rod, at the end which goes inside the flask (the toy bells used on baby-rattles will do). Into one flask place a little water and boil it vigorously for 10 minutes or so, to drive out the air;

whilst still boiling, and when hardly any water is left, push in the rubber bung so that the bell is inside the flask. Allow it to cool. After a time most of the water vapour in the flask will condense, leaving a fairly good vacuum. Shake the flask and notice that the bell is scarcely audible, whereas a similar bell in the other flask, which contains air (or coal-gas, if filled by leading coal-gas in for some time), is heard quite clearly. We have seen on p. 381 that sounds do not travel through a vacuum.

Exercises on Chapter XX

1. Describe an experiment to illustrate the fact that a material medium is necessary for the transmission of sound. Explain how the air waves travel through it. [*L.G.S.* 1921.]

2. Illustrate by means of diagrams the nature of the wave-motion which occurs when sound is propagated in air. Explain clearly the meaning of the terms amplitude, wave-length and frequency as applied to wave-motion. [*L.G.S.* 1919.]

3. Write a short account of the production and propagation of sound in air. [*L.G.S.* 1925.]

4. Explain the method of representing longitudinal waves by a curved line. [*L.G.S.* 1924.]

5. How is sound (*a*) produced, (*b*) transmitted from place to place? Describe briefly two experiments to illustrate (*a*), and one to illustrate (*b*). [*J.M.B.* 1928.]

CHAPTER XXI

VELOCITY OF SOUND—REFLECTION AND REFRACTION OF SOUND

SOUND is not propagated instantaneously, *i.e.* it takes time to travel. You have, no doubt, all realised this, especially since you learned that the velocity of light is so great that it travels several miles in a negligible time. There are many common examples which illustrate the difference in speed between light and sound. A cricketer is seen, from the boundary of a cricket-field, to hit the ball an appreciable time before the sound is heard, the interval between the sight and sound getting longer with increasing distance from the batsman. If you are standing on a hillside watching the approach of a train in the valley below, you may see the steam as a white cloud as it commences to issue from the engine-whistle, but the sound follows much later. Possibly you have seen a cinematograph picture of a column of soldiers, headed by a band, and noticed how the soldiers are out of step, since the rear ranks hear the marching music of the band, and march to it, later than the front ranks. You may even have noticed the small ripple or wave which appears to travel along the column as the heads move up and down to the movement of the feet. Another example is that of thunder and lightning. When they occur, the electrical discharges between clouds are so intense as to give light, *i.e.* lightning, whilst the changes in pressure in the clouds and air around produce sounds, *i.e.* thunder. The lightning is seen much earlier than the thunder is heard; in fact, if the velocity of sound is known, it is a simple matter to time the interval between the flash and

the sound and thus to calculate the distance at which the thunderstorm is taking place.

The first experiments to measure the velocity of sound were based on the difference in time of travel between light and sound propagated when a cannon was fired. Mersenne, a French scientist who lived in the middle of the seventeenth century and who acted as a correspondent or intermediary between scientists of the time, recorded that observations were made by an officer at the Siege of Rochelle (1628). The results were inconsistent, and the velocity of sound was thought to depend on local atmospheric conditions. Mersenne himself made observations, using firearms, and concluded that the velocity was 1,380 ft. per sec. Some time later, the Florentine Academy made experiments, getting a value of 1,175 f.s., but the *first exact determination was made in* 1738 *by a Commission appointed by the French Academy*, near Paris, over a distance of 17–18 miles. A gun was fired at the first station, observations of the interval between the arrival of the flash and the sound of the explosion being made at the second station. Later, the second station fired a gun and a similar observation was made at the first station, and this was continued. In each case pendulums were used to measure time intervals, and the idea of this so-called "*reciprocal firing*," and the averaging of the results obtained, was to eliminate any wind effect which, it was realised, had an appreciable effect on sound travel. The results obtained gave a value at 8° C. of 337 metres per sec., which gives, as we shall see later, a velocity of 332 metres per sec. approximately at 0° C.

In 1822, further experiments were carried out by the French Academy at the request of Laplace, a great French mathematical scientist who was interested in the movements of the heavenly bodies, and so had fully studied the classic work, the *Principia*, of the most famous English scientist, Sir Isaac Newton. In the experiments, reciprocal firing was carried out at 5-minute intervals over a distance of 11 miles, chronometers, accurate to one-tenth of a second, being used. The mean result at 15·9° C. was 340·9 metres per sec., which, as will be shown

later, gives a velocity at 0° C. of 331 metres per sec. This result confirmed Laplace's expectations, for he had been studying Newton's work, in the second volume of the *Principia*, on the Velocity of Sound. There Newton showed mathematically that the

$$\left.\begin{array}{r}\text{Vel. of sound in}\\ \text{a medium must}\end{array}\right\}=\sqrt{\frac{\text{the elasticity of the medium (E)}}{\text{the density of the medium (D)}}}.$$

For a gas, Newton showed that if the temperature were constant, *i.e.* any changes were isothermal ones, the elasticity is equal to the pressure; so he considered that for a gas medium,

$$\text{Velocity of sound}=\sqrt{\frac{\text{Pressure P}}{\text{Density D}}}.$$

He calculated the value for air; *e.g.* when the pressure is atmospheric it $=1{\cdot}01396\times10^6$ dynes per sq. cm. (see p. 17), and the density of air at 0° C. $=0{\cdot}001293$ grm. per c.c.

$$\text{Thus Vel. of sound at } 0°\text{C.}=\sqrt{\frac{1{\cdot}01396\times10^6}{0{\cdot}001293}}\ \text{cms. per sec.}$$

$$=28{,}000 \text{ cms. per sec. or } 920 \text{ f.s.}$$

Newton, of course, realised the difference between this calculated value and the experimental results. He endeavoured to explain it by saying that $\frac{1}{8}$ of the space through which sounds travel consisted of air molecules and the sound travelled through them instantaneously, whilst the same thing happened when water-vapour was in the air. So the real value of the velocity was bound to be greater than the calculated value.

Laplace in 1816 came to a conclusion as to why the two sets of results did not agree—that Newton's idea regarding the elasticity of a gas, in which sound was travelling, was a mistaken one. Hence the experiments of 1822, which justified Laplace's explanation, now applied, and often called "Laplace's correction."

The value of the elasticity of a gas, as calculated by Newton, involved Boyle's Law, which states a relationship between the pressure and volume of a given mass of gas

at a constant temperature. Newton assumed that the changes in the air taking place in wave-motion had no effect on the temperature, *i.e.* they were isothermal. Laplace pointed out the error in this assumption. He said the vibrations were so rapid that the compressions and rarefactions were too rapid for the heat produced to be dissipated, or that lost to be replaced. Any change, involving pressure and volume, in which heat is not allowed to enter or leave, is called an *adiabatic change;* Laplace said the value for the elasticity under adiabatic conditions ought to be substituted in Newton's formula for the Velocity of Sound. It can be shown that the elasticity of a gas, under adiabatic conditions $=\gamma$ pressure, where γ is a constant for the gas, and

$$=\frac{\text{the specific heat of the gas at constant pressure}}{\text{the specific heat of the gas at constant volume}}.$$

$$\left.\begin{array}{c}\text{Thus the velocity of}\\ \text{sound in a gas}\end{array}\right\}=\sqrt{\frac{\gamma\,.\,\text{Pressure}}{\text{Density}}}=\sqrt{\frac{\gamma P}{D}}.$$

There are various methods for measuring γ, and it has been shown that for air $\gamma=1{\cdot}41$.

Introducing this value into Newton's formula and calculation,

$$\text{Vel.}=28{,}000\times\sqrt{1{\cdot}41}\text{ cms. per sec.}$$
$$=332{\cdot}5\text{ metres per sec.,}$$

a result near enough to the experimental results to justify Laplace's argument that the transmission of sound in gases consisted of adiabatic pressure changes; this has been confirmed in many ways.

Further experiments have been made at different times since 1822 to measure the velocity of sound from gun-firing, the latest methods introducing electrical methods of recording to eliminate the small errors always occurring when observations are made. Other means of measuring the velocity of sound will be dealt with later in the book.

Effects of Pressure Change on the Velocity of Sound in a Gas.—We have seen that the density D of a substance $=\dfrac{\text{its mass M}}{\text{its volume V}}$.

Thus velocity of sound in a gas $=\sqrt{\frac{\gamma P}{D}}=\sqrt{\frac{\gamma PV}{M}}$, all of which are constants, PV being a constant, if the temperature is constant.

Thus the **velocity of sound in a gas does not vary with pressure.** This is rather an obvious result, for as the pressure of a gas is increased so is its density.

This result was confirmed experimentally, in 1823, by determinations made in the Tyrol between two stations differing in height by 1,360 metres, and in 1844 in the Alps between two stations differing in height by 2,080 metres, the result in both cases for the velocity at 0° C. being 332·4 metres per sec.

Effect of Temperature Change on the Velocity of Sound in a Gas.—Since the velocity of sound (Vel.) in a gas $=\sqrt{\frac{\gamma P}{D}}$, using P_t, D_t, $\mathrm{Vol.}_t$ to represent the pressure, density and vol. of a gas at $t°$ C., and P_0, D_0, $\mathrm{Vol.}_0$ to represent the pressure, density and vol. of a gas at 0° C.,

$$\frac{\text{Vel. at } t° \text{ C.}}{\text{Vel. at } 0° \text{ C.}}=\sqrt{\frac{\gamma P_t}{D_t}}\div\sqrt{\frac{\gamma P_0}{D_0}}=\sqrt{\frac{P_t D_0}{P_0 D_t}}.$$

But on p. 57 we saw that the density of a given mass of a fluid is inversely proportional to the volume, *i.e.*

$$\frac{D_0}{D_t}=\frac{\mathrm{Vol.}_t}{\mathrm{Vol.}_0},$$

$$\therefore \frac{\text{Vel. at } t° \text{ C.}}{\text{Vel. at } 0° \text{ C.}}=\sqrt{\frac{P_t \mathrm{Vol.}_t}{P_0 \mathrm{Vol.}_0}}.$$

On p. 80 we saw that for a gas, Pressure × Volume ∝ absolute temp. or $\frac{P_t \mathrm{Vol.}_t}{P_0 \mathrm{Vol.}_0}=\frac{T_t}{T_0}$ where T_t and T_0 are absolute temp. values corresponding to $t°$ C. and 0° C.

$$\text{Thus} \frac{\text{Vel. at } t°\text{C.}}{\text{Vel. at } 0°\text{C.}}=\sqrt{\frac{T_t}{T_0}}=\sqrt{\frac{273+t}{273}} \text{ (or Vel. at } 0°\text{C } \sqrt{T}),$$

i.e. **the velocity of sound in a gas is proportional to the square root of its absolute temperature.**

$$\text{Thus velocity at } 1^\circ \text{ C.} = \text{Vel. at } 0^\circ \text{ C.} \sqrt{\frac{274}{273}}$$

$$= 332\sqrt{\frac{274}{273}} \text{ metres per sec.}$$

$$= 332{\cdot}6 \text{ metres per sec. approx.}$$

i.e. *the velocity of sound increases by* 0·6 *metre or* 60 *cms.* (*or* 2 *ft.*) *per sec. for each degree C. rise in temperature.*

This has been freely tested and confirmed in Arctic regions between temperatures 0° C. and −45° C.

Velocity of Sound in Liquids.—It was thought for a long period that sounds could not travel in liquids, on the ground that liquids were incompressible and so pressure changes were not possible. But in 1826, Savart showed that sound waves were propagated in water, and two Geneva scientists, *Colladon and Sturm*, *determined the velocity of sound in water*, in Lake Geneva, over a distance of 13½ kilometres. At one end they had a large bell under water. A hammer was caused to strike the bell, and, at the same time, to ignite gunpowder above water, the experiment being carried out in the dark. At the other end an observer watched the flash and timed the arrival of the sound travelling in water, intercepting it by means of a long tube, carrying a membrane, held below the water surface. Their results gave a velocity for sound in water of 1,435 metres (or 4,700 ft. approx.) per sec. at 8° C.

The higher value of the velocity of sound in liquids, and many solids (as we shall see later), as compared with the velocity in air and gases, is because of the very much greater values for their elasticities as compared with that of a gas, despite the fact that gases have much lower densities.

Using Newton's equation, velocity of sound

$$= \sqrt{\frac{\text{Adiabatic Elasticity}}{\text{Density}}},$$

and taking the values for water, density $=1$ grm. per c.c., adiabatic elasticity $=2{\cdot}1 \times 10^{11}$ dynes per sq. cm.,

the velocity of sound in water should

$$=\sqrt{\frac{2{\cdot}1 \times 10^{10}}{1}} = 10^5\sqrt{2{\cdot}1} \text{ cms. per sec.}$$

$$=1{,}449 \text{ metres per sec.}$$

(which agrees fairly well with the experimental result).

Somewhere the writer came across an interesting way of expressing the difference in velocity of sound in different media. Supposing a Channel Tunnel were built between Cap Grisnez and Eastware Point, a distance of 21 miles, and a loud sound were made at one end of the tunnel, the sound would be heard at the other end,

travelling through air, in 97 secs.,
,, ,, sea-water, in 23 secs.,
,, ,, iron rails, in 6 secs.,
,, ,, copper telegraph wires, a little quicker,
and ,, ,, laths of fir wood in 5 secs.

Reflection of Sound.—Sound is reflected from large obstacles such as cliffs and walls, and the reflected sound is called an **echo**. The production of echoes is

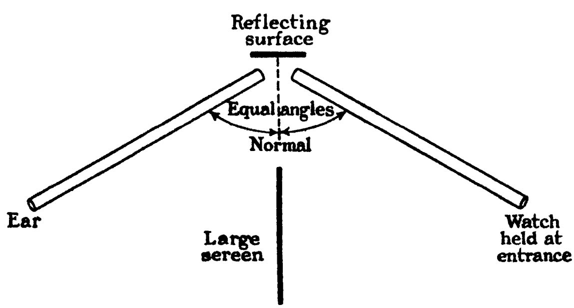


FIG. 217.—Showing Regular Reflection of Sound.

well known to all of us ; it is one of the unpleasant features of many public buildings, school halls, etc., which makes listening to speeches very difficult. Sound and light are

reflected from surfaces according to similar laws. The regular reflection of sound can be shown quite easily by using two long tubes inclined to a surface in approximately the same plane (see Fig. 217). Hold a watch to one tube and listen at the end of the other tube, distant from the reflecting surface. It will be found that the sound is best heard when the tubes are equally inclined to the surface—*i.e.* using the same definitions as for light, the angle of incidence=the angle of reflection. During the experiment a screen should be placed as shown, so that sound from the ticking watch does not travel directly to the ear.

Sounds can also be detected in such experiments by a "*sensitive*" *flame*. This is a special type of flame so arranged that, although it is steady as regards shape and size under ordinary circumstances, when sounds reach it the flame shortens and roars, immediately regaining its quiet form when the sound has passed. Tyndall set up a very sensitive high-pressure gas flame and showed that the sensitiveness is in the issuing jet where not ignited, and not really in the flame, the nature of the flame indicating changes in the jet. High-pressure jets are not suitable for ordinary laboratories, but there are several low-pressure forms which can be used on the ordinary gas supply, *e.g.* the Rayleigh sensitive flame (made by Messrs. Gallenkamp), containing a tissue-paper membrane as the front of a gas-chamber.

Ordinary bunsen burners with a single side hole as an air entrance can be used if the hole is closed and the gas pressure reduced till the flame is just quiet. Suitable jets can also be made by drawing out ordinary 6 mm. glass delivery tubing, after softening in a bunsen flame. The end, from which the gas issuing is to be a sensitive jet, should have a diameter of from 1 to 2 mm.

Sound is also reflected from curved mirrors according to the same laws as for light. If two large spherical or parabolic mirrors be arranged facing each other, and a watch or metronome be placed at the focus F_1 of one mirror, the spherical sound waves leaving F_1 are reflected from the mirror as plane waves which are then reflected

from the other mirror as waves which converge to its focus F_2. Here they can be intercepted by a small funnel joined by rubber to another small funnel to which the ear is placed.

An interesting case of sound reflection is that which occurs in large round buildings such as the Whispering Gallery of St. Paul's, and Gloucester Cathedral. A sound made close to, and along the direction of, the round wall is reflected close to the wall and around it as shown in Fig. 218, and the echo is heard clearly by the speaker. The sound can quite easily be shut off by an obstacle.

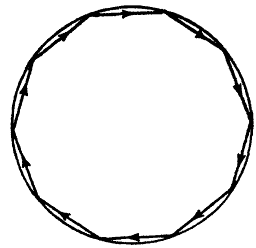


FIG. 218.

In churches there is often a board above the pulpit to reflect the sounds made by the preacher down to the congregation.

The subject of echoes is of great importance in the design of public buildings. In 1850, John Tyndall presented a report to a Committee set up to inquire into the faults causing the bad acoustic properties of the House of Commons. Wallace Sabine, an American Professor, studied the subject and soon recognised that curved walls had disadvantages. He realised the necessity of the walls absorbing the sound waves reaching them to avoid the occurrence of echoes. For the details of the principles of design and equipment (such as the use of wall coverings to absorb the sound) the student should read the March issue of *Discovery*, 1928, and also the *School Science Review*, Vol. 4, No. 16, p. 191.

Sound is also reflected at the surface separating two media, *e.g.* in water it is not easy to hear sounds made outside, for they are reflected from the surface. Again, sound is reflected from the surface of a fog-bank; if the sound source and the observer are both inside, the sounds are better heard than if one is inside and the other outside. On board a ship just outside a fog-bank it is difficult to hear the siren of one inside it, and this fact has often nearly caused disaster.

It is probable that the suggestion of timing the interval between a sound and its echo as reflected from a wall, at a known distance, came from Mersenne. He found that shouting rapidly he could say " one, two, three, . . . seven," in 1 sec., and he measured the shortest distance he had to stand from a wall so that the echo of similar figures came back to him. The velocity of sound was thus twice that distance per second. A modern method of measuring the velocity of sound is to have two electrically controlled clickers (or metronomes) actuated by a pendulum. These make sounds together and at regular intervals, and one is removed to such a distance that it sounds in between, or else at the same time as the sounds of the clicker close to the observer. A modification of this replaces one clicker by a wall, making use of its echo. In either case, the calculation is obvious.

The timing of echoes is of great value in sounding for the depth of the sea, or in detecting icebergs (most of the iceberg being under water), wrecks, submerged rocks, etc. A ship generates sounds, *e.g.* by exploding a depth charge, or by a mechanical device, under water and the reflected waves are received by a *hydrophone*. If the time interval between the making of the sound and the arrival of its echo is measured, knowing the velocity of sound, the depth, or distance away, of the reflecting surface can be calculated.

Fig. 219 shows a hydrophone, with a solid plate, the

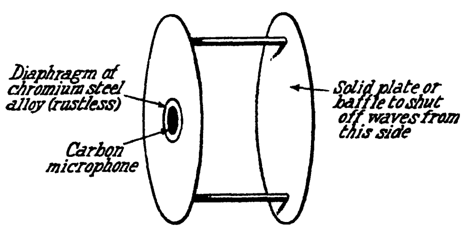


FIG. 219.—Hydrophone

baffle, to cut off sounds from one direction. Such a shielded hydrophone can be rotated to get a maximum loudness of reflected sound and the direction of the

reflecting obstacle accurately found. This was used in the Great War for the detection of enemy submarines. The essential part of the hydrophone is the granular carbon microphone, which varies electrically with variation in pressure on it. It is a water-tight capsule operated by a chromium-steel (rustless) diaphragm, which is sensitive to sound waves, thus causing changes in the microphone. The latter is joined by wires to an electric circuit and indicator on the ship.

Echelon Echo.—Huyghens pointed out a very interesting kind of echo, called an *echelon echo*, which was due to multiple reflection of a sound from a series of reflecting surfaces such as a wooden fence (particularly if clinker-planked, *i.e.* planks overlapping) or iron railings. Huyghens observed a case which he found was due to reflection from a series of steps, and which did not occur when snow was on the ground. The observer receives the reflected waves in quick succession, and at periodic intervals, due to the different distances travelled by them, and so hears a high-pitched musical note. Fig. 220 illustrates the common case occurring when a man is walking by a series of planks in echelon formation (*e.g.* a fence) or railings and has metal tips on his heels, particularly if he stamps with his heels.

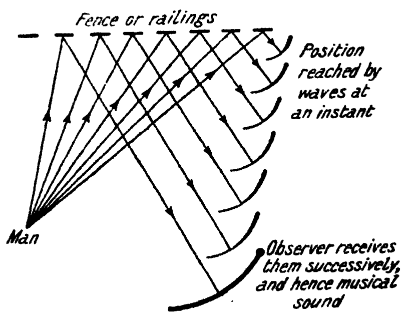


FIG. 220.—Echelon Echo Formation.

Refraction of Sound.—Sound, like light, is deviated, or refracted, when entering one medium from another. Tyndall showed this by causing sound, made by a high-pitched whistle, to pass through a soap-bubble filled with carbon dioxide. He showed, by means of a sensitive flame, that the sound was focussed to a point. This can also be shown by using a toy balloon which has

been filled with carbon dioxide or some other gas which is denser than air.

Very interesting effects of sound refraction are produced by differences in (*a*) temperature, (*b*) wind velocity, at different layers of the atmosphere.

(*a*) *Temperature Effects.*—During the daytime the air near to the ground becomes heated by radiation from the ground, and so is relatively warmer than the air above it. Sound waves travel more quickly in warmer air and so tend to become deviated upwards (the direction of the wave-front changing) as indicated in Fig. 221, with the result that no sound is received a little distance away.

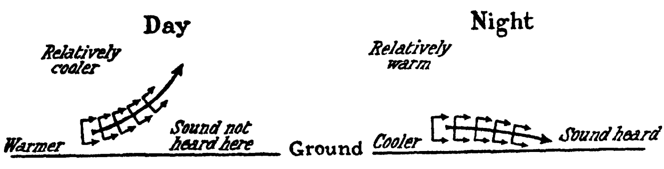


FIG. 221.—Refraction of Sound Waves (Temperature Effect).

At night the effect is reversed, the air near the ground becomes cooled down and the air above being a little warmer owing to radiation from the earth. Thus sound waves tend to be refracted into the ground surface, and, of course, are reflected from it, with the consequence that *sounds are heard much better at a distance by night than by day.* For a similar reason sounds are heard better across water than across land in the daytime.

(*b*) *Wind-Gradient Effect.*—The velocity of the wind is usually greater above the ground than at the surface, and so the velocities of the wind added to the velocity of the air waves cause the sounds to travel faster a little above the ground than at the surface. Thus the wave-front of the sound waves tends to become bent into the ground (Fig. 222), so that sounds carried by the wind are heard at ground level for a considerable way. But sound waves made against the wind are bent, or refracted, as shown in the figure, with the result that sounds are not carried well against the wind. You will have noticed

that while listening from a distance to a band playing in a park on a gusty day.

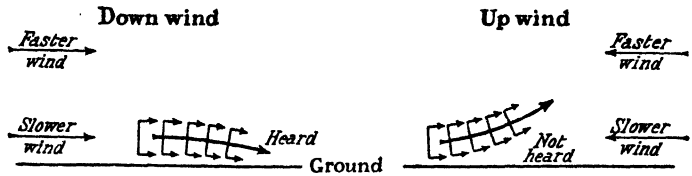


FIG. 222.—Refraction of Sound Waves (Wind Effect).

Variations in density of the air at different heights, and local variations in wind, etc., often cause peculiar results When a powerful sound is made, it may be heard at a place 20 miles away and not heard at a place between which is only 10 miles away. The latter is in a silent area, as shown in Fig. 223. A staff-officer serving in the American War of Independence first recorded this—how he watched a fight but could not hear the sounds of it. This was observed continuously during the Great War. The writer remembers many times leaving the Ypres sector when heavy gunfire was going on

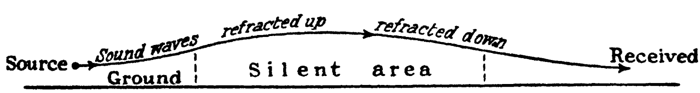


FIG. 223.

and being unable to hear it 10 miles away. At the same time the sounds were very clear at the coast near Boulogne.

Another example is that of the well-known Silvertown explosion at London in January, 1917. The noise of this was inaudible at Cambridge, yet it was heard distinctly in Lincolnshire and the North. In some cases two or more explosions were apparently heard—owing to the different paths, and consequently different times of arrival, taken by the sound waves generated. This duplication is an interesting one, and some experiments to study it have been made in recent years involving the generation of loud explosions and the observation of the sound in various directions.

The Huyghens Construction.—Huyghens' idea of the process by which energy was actually transmitted by wave-motion was as follows. Consider a spherical wave to be travelling from S, and at any instant part of the wave-front is WF (Fig. 224). Then *every particle on this wave-front becomes the centre of a little spherical wave*, e.g. W becomes the centre of a wave which travels out so that, after a small interval of time *t*,

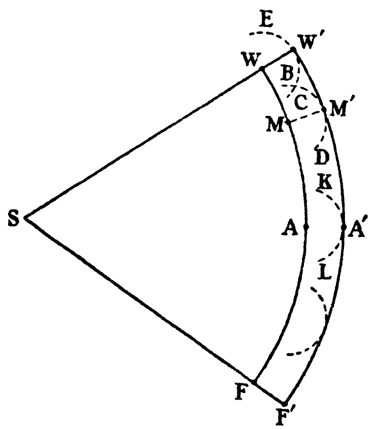


FIG. 224.—Huyghens' Construction.

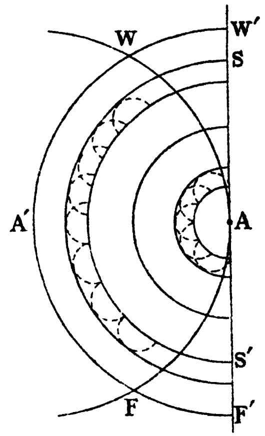


FIG. 225.—Huyghens' Construction for Reflection of a Wave.

its wave-front is EW'B. Similarly, particle M becomes the centre of a spherical wave, and its wave-front, after the same time *t*, is CM'D. The new wave-front as a whole, after that time *t*, is contributed to by all these small wavelets, or *secondary waves* as they are called, and is thus the surface touching all of them, *i.e.* W'M'A'F'. A simple example of the use of this construction is the case of reflection of a spherical wave meeting a large obstacle at right angles to its path. Huyghens considered that when the wave met such a surface SS' (Fig. 225) the first part A of the wave-front, which met the surface, became the centre of a secondary wave in the medium it was travelling in, and so travelled backwards. Each point of the wave-front containing A followed this procedure as it met the surface SS'. Thus the original wave-front WAF was reversed, A moving forward to A' whilst W and F advanced to the surface SS',

i.e. to W′ and F′ respectively, and so the reflected wave-front was W′A′F′.

The principle of Huyghens can also be used to explain why sounds made outside houses can be heard clearly inside. When the sound waves are incident upon any hole which is small compared with the wave-length of the sound, it becomes the centre of spherical waves which travel inwards, and so sounds spread in all directions in the house. This holds for open-window spaces, etc.

Exercises on Chapter XXI

1. Describe a method for measuring the velocity of sound in the open air. Point out the sources of error and explain how these may be allowed for.

2. Name some instances demonstrating the fact that the transmission of sound through air is not instantaneous. Describe a method of finding the velocity of sound in air based upon one of these examples. Work out a numerical example in illustration of your answers. [*L.G.S.* 1925.]

3. How does sound travel through gases? Indicate briefly how, if at all, the velocity of a sound depends (*a*) on its wave-length, (*b*) on the density of the gas, (*c*) on the temperature of the gas. [*L.M.* 1921.]

4. How does the velocity of sound in air depend upon (1) temperature, (2) pressure? A boy standing in a disused quarry claps his hands sharply once every second and hears an echo from the face of the opposite cutting. He moves until the echo is heard midway between the claps. How far is he then from the reflecting surface if the velocity of sound at the time was 1,120 ft. per sec.? [*L.G.S.* 1923.]

5. Explain how echoes are produced. How may the phenomenon be used to measure the velocity of sound in air? [*L.M.* 1921.]

6. Explain how echoes are formed. Determine the intervals between successive echoes formed by two cliffs, 1,500 ft. apart, if the observer who makes the initial sound stands 500 ft. from one of them. Assume the velocity of sound through the air to be 1,100 ft. per sec. [*L.G.S.* 1920.]

7. Two bells are struck regularly and simultaneously 40 times per minute. At first they are equidistant from an observer, but the distance of one of them is gradually increased. Assuming the velocity of sound to be 1,120 ft. per sec., calculate what the difference between the distances of the bells will be before he again hears them simultaneously. Why would it be difficult to verify the calculation by direct experiment?

8. A gun is fired from a distant fort, and an observer at a house 5 miles away times the interval between flash and report as 24 secs. On another day he makes the time $23\frac{3}{8}$ secs. What changes in conditions may have caused the difference? [*L.M.* 1922.]

9. Do sounds of all kinds travel with the same velocity? What conditions in the air itself may affect the velocity? Give experimental evidence for your answer. [*L.G.S.* 1926.]

10. Discuss the statement that the velocity of sound in air increases by 2 f.s. for each 1° C. rise in temperature, and find what it will be between 0° and 10° C. and also between 100° and 110° C., given that the velocity of sound in air at 0° C.=1,090 f.s.

11. Describe any experiments to illustrate the reflection of sound from a plane surface. What difference in results would you expect when the surface is one of (*a*) wood, (*b*) wool? What effect has the surface of the walls in a room on speaking? Explain why it is easier to listen to a man speaking in a large well-filled room than it is when he is speaking in the same room nearly empty.

12. How could you show that sound waves suffer reflection? Describe an experiment by which the reflection of sound waves is made the means of measuring the velocity of sound in a medium. Describe an apparatus depending for its efficiency on the reflection of sound waves. [*L.M.* 1297.]

13. Describe a laboratory experiment to illustrate the reflection of sound at a concave surface. Explain the action of any form of whispering gallery. [*L.M.* 1920.]

14. Explain how a single sound such as a pistol shot is transmitted by the air. What kind of surface will give a musical echo to such a sound? [*L.M.* 1923.]

15. During bomb practice in the neighbourhood of a corrugated fence an observer noticed that the explosion of each bomb was followed by a shrill echo. Explain this with a rough diagram of the relative positions of the bomb, the fence, and the observer. [*L.M.* 1920.]

16. The interval between the flash of lightning and the sound of thunder is 3 secs. when the temperature is 12° C. How far away is the storm? (V. of sound in air at 0° C. = 1,090 f.s.).

REFERENCE BOOKS

"Acoustics of Buildings." F. R. Wilson. (Chapman & Hall.)
"Acoustics of Buildings." A. H. Davis and G. W. C. Kaye. (Bell & Sons)

CHAPTER XXII

CHARACTERISTICS OF SOUNDS—INTENSITY, PITCH AND QUALITY

SOUNDS are commonly classified as (1) noises, which are usually irregular and often suddenly made and very brief in duration; (2) notes, which some people regard as being musical. It is perhaps better to distinguish notes as (*a*) definitely musical ones, or (*b*) simple tones such as those of tuning-forks. These are quite distinct, for whereas musical sounds are pleasing and bear repetition, simple notes like those made by tuning-forks seem dull and monotonous, and unbearable on continuous repetition. Try the effect of continuously knocking and sounding a single tuning-fork!

The task of distinguishing between musical notes and noises is not always simple, and it also depends to a great extent on the sensitiveness of the ear of the observer. To some people modern jazz music gives great pleasure and classical music is merely boring; to others the reverse is the case. However, musical sounds are regular, being produced by regular or periodic vibrations, whereas noises are produced by irregular vibrations. Yet these noises, when continuous, may be musical. Think of the sound of falling or running water, the "soft murmur" of the running brook, the sounds of wind in a chimney, and the rustling of leaves. It would seem as if the impression must be prolonged—that musical feeling takes time to grow and so to be distinctly realised. What are by themselves noises appear musical in relation to other similar noises. Hence tunes can be played by dropping or knocking differently sized pieces of wood or by pulling

corks from bottles. So cymbals, castanets, triangles, etc., are used in orchestral and band music.

In our work we shall be mainly concerned with **musical sounds** which, it will be readily realised, **have three distinct characteristics.** They are :

(*a*) **intensity ;**
(*b*) **pitch,** which indicates whether sounds are high or low (or, as we call them, treble or bass) ;
(*c*) **quality,** or timbre, which enables us to distinguish easily the same note played with the same intensity by two different instruments. This latter characteristic is extremely interesting, for it enables us to identify the sounds of the different instruments in an orchestra and to recognise the voice of some one we cannot see.

Intensity of Sounds.—The intensity of a sound is independent of the ear receiving it, and depends mainly on the energy of the movement producing the sound. We ought, therefore, to differentiate between intensity and loudness, or sensation produced by a sound, for the latter depends on the sensitiveness of the ear of the receiver. It can be shown that the intensity of sound depends on the amplitude of the wave (and is $\propto$ amplitude^2) —the greater the energy of the vibration displacing the medium, the greater is the amplitude of the wave produced. At present there is no instrument to measure the intensity of a sound and the subject is not easy to study. One difficulty can easily be realised by observing the variations in the notes or sounds of a siren (see later) worked continuously by the same mechanical power. Deep musical notes are first produced and do not seem very loud ; the shrill notes produced later are much more piercing, the ear being more sensitive to notes of higher pitch. Many of you will have noticed this when listening to wireless reproduction, particularly from Morse stations. The intensity (and apparent loudness) of sounds is affected by the nature of the medium conducting them, by virtue of changes in the density, as previously shown. Temperature also has an effect, since it influences the density (p. 391).

Inverse Square Law.—On p. 173 we saw that when a source is radiating energy in all directions, the intensity of the effect of the energy at a distant point varies inversely as the square of its distance from the source of energy. This applies to the case of sound freely travelling out from a source in all directions. De la Roche and Dumal verified this about 1820 with bells identical in tone and intensity. Four of the bells were made to sound at once at a distance from a single bell also being struck. The observer placed himself between so that the one bell seemed to make as loud a sound as the four bells sounded together. It was found that the observer was then twice as far from the four bells as from the one bell, thus verifying the inverse square law.

The method of radiating sound in all directions is usually wasteful, for it is generally required to send out sound in one definite direction only. If, then, the energy generating sound waves can be confined to straight lines (*i.e.* so that the wave-front is plane and not spherical), there is little waste of energy, and in addition the sound travels much farther. We are all aware of this principle as applied in the megaphone, or speaking trumpet; and when we wish to call any one at a distance we automatically shout with our hands cupped around the mouth. The famous "Horn of Alexander" is said to have been used to call his soldiers at a distance of ten to twelve miles. The principle is similar to that of the searchlight and to the "Beam system" in wireless—the energy sent out being confined to a beam in one chosen direction. Under these circumstances the amplitude of the wave-motion remains nearly the same and does not become appreciably smaller as it does in the case of spreading waves, *e.g.* water ripples, which are seen to have less movement at greater distances from the generating source.

Conversely, to aid reception, receiving trumpets are used to collect as much energy as possible, just in the same way as large-apertured object-glasses are used in telescopes to collect as much light energy as possible from a distant luminous object and so give a clearer image.

Intensity and Velocity of Sound.—The relation-

ship for the velocity of sound (p. 396) is true as long as the amplitude (*i.e.* intensity) of the sound is not too large. When sounds are intense, *i.e.* the amplitudes of the waves produced are large (as in the case of cannon sounds), they travel somewhat faster.

Pitch.—Most people have some sort of a musical ear and so are able to recognise pitch in a note—*i.e.* to distinguish a high shrill note from a low deep one. It is said that Galileo, when a boy, was the first to realise that *pitch depends on the frequency of the vibration,*

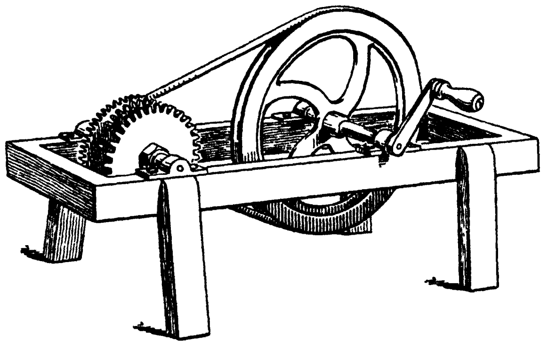


FIG. 226.—Savart's Toothed Wheels.

discovering it by drawing the edge of a knife-blade across the milled edge of a coin. The quicker the knife is moved across, the more frequent are the impulses sent out and the higher the pitch of the note received. In 1830 *Savart*, a Frenchman, showed this by means of his *toothed wheels*. He made wheels, with different numbers of teeth on them, and rotated them, holding a flexible metal plate against the teeth (Fig. 226). As each tooth moved on, it lifted the plate, which then fell back upon the next tooth, and this procedure was continued. Thus the flexible plate moved to and fro, so generating air waves. A registering apparatus (known as a "revolution counter") gave the number of complete revolutions of the wheel in a time which could be measured. Thus the number of vibrations per second of the vibrating

plate, and so the frequency of the air waves = the number of revolutions per second × number of teeth on the wheel. It is easily shown, if these wheels are adjusted to revolve at the same rate, that the highest note is obtained from the wheel with the greatest number of teeth, and the lowest from the wheel with the smallest number of teeth.

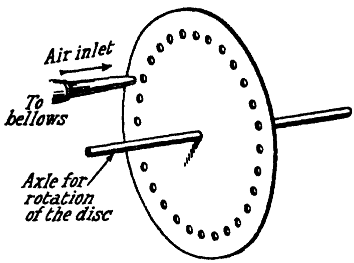


FIG. 227.—Seebeck's Disc Siren.

The Siren.—A form of siren commonly used to-day is that of Cagniard de la Tour, who is said to have called his instrument a siren because with it he was able to make sounds in water (cf. Gk. mythological siren who attracted seafarers by sweet music). An early form was *Seebeck's Disc Siren* (Fig. 227), which consisted of a metal disc with a series of holes, near its edge, through which an air blast could be blown. The disc was rotated and so there was a succession of puffs of air through the holes, *i.e.* waves travelled out. As the speed of rotation of the disc was increased, so the pitch of the note emitted was raised. Cagniard de la Tour's improvement is shown in Figs. 228 and 229. Air was made to enter from a bellows, or wind-bag, into a metal cylinder closed at the top by a disc like Seebeck's disc, *i.e.* it was perforated near the edge by a series of holes. On this disc rested another similarly perforated disc, turning on a vertical axis. When the holes coincided the air passed through and so, during the rotation of the top disc, there were periodic puffs of air

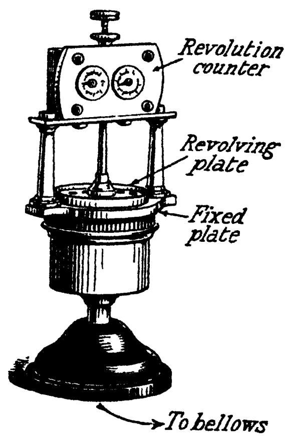
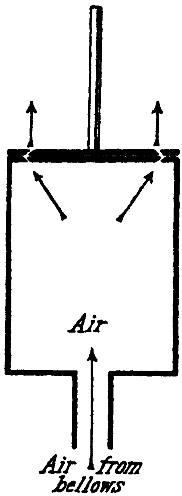


FIG. 228.—Cagniard de la Tour's Siren.

passing out, *i.e.* alternate compressions and rarefactions travelled out. Cagniard de la Tour made the perforations obliquely through the two discs (Fig. 229) in such a way that when the holes met the channels through them were nearly at right angles. Thus the current of air forced from the bellows not only passed through the holes but also gave an impulse to the top rotatable disc sufficient to turn it. If the air current was kept up the velocity of rotation increased continuously (to a limit) and the note emitted rose in pitch. Attached to such a siren is a revolution counter (actuated by the rotating disc or its axle), and so the frequency of the note emitted at any time can be calculated, the holes in the disc being equivalent to the teeth of Savart's wheel.



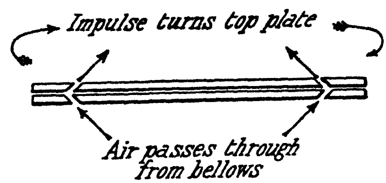


FIG. 229.—Principle of Cagniard de la Tour's Siren.

Thus, to measure the *absolute pitch*, or *frequency* (*i.e.* the number of vibrations per second), of any note, we can do it by adjusting the speed of rotation of a Savart's wheel or a siren to give the same note, and then time a certain number of revolutions of the instrument used, calculating the pitch as previously shown. It is difficult to keep them emitting a steady note, and so better methods for determining pitch are given below and later.

It should be noted that a siren has been found to be the most effective instrument for giving warnings at sea. Air or

steam, at from 10 to 40 lbs. pressure, is used. Kelvin found that the mouth of the horn must be oval, or elliptical, with the longer axis vertical and the horizontal axis ¼ of the vertical axis. The siren at Trevose Head, N. Cornwall, has a horn 21 ft. long and a mouthpiece about 7½ ft. by 2½ ft. and has been heard by a Cunard liner 31 miles away at sea. A lightship siren has been heard as far as 16 miles away through the water.

A method of measuring the absolute pitch of a tuning-fork is to fix on the fork a very light style, or pen, which can be inked. The fork is made to vibrate so that its indicator moves against a rotating cylindrical drum, or falling plate, carrying paper, and so traces a wavy line on it. In the former case the number of complete vibrations marked on the paper, in one revolution, is counted; the number of revolutions of the drum per second being known, the frequency of the fork is obtained. In the case of the falling plate the distance the plate moves, when a counted number of vibrations are indicated on it, is measured, and the acceleration of the falling plate due to gravity being known, the corresponding time can be calculated and so the frequency of the fork found. Other methods of measuring the frequency of a note (or of a tuning-fork) will be given later.

Sounds and Pitch.—It is found that audible sounds have a frequency, or pitch, of between 30,000–40,000 (depending on the ear of the listener) and 20 vibrations per second. It is interesting to note the lengths of waves corresponding to these audible limits and other more common notes, and observe how they differ from the wave-lengths of visible light (see p. 317). Remember velocity of waves = frequency × wave-length.

Sound	Frequency	Wave-length
Highest audible note	40,000	⅓ in. (approx.)
Highest musical note used (on piccolo)	4,750 (approx.)	2·4 ins. „
Highest note on piano	3,500	3·2 ins. „
Woman's ordinary voice	280	4 ft. „
Man's ordinary voice	140	8 ft. „
Lowest note on piano	27	40·8 ft. „
Lowest audible note	20 (approx.)	55 ft. „

It should, of course, be noted that the velocity of sound is the same for all notes (except *very* loud sounds), no matter what the pitch is.

The Musical Scale.—Not only do we express pitch absolutely, by the number of vibrations per second, but we also express it relatively, by what is known as the musical method. In this certain sounds constitute what we call a musical scale. The musical scale used by European nations for centuries is called the *True* or *Major Diatonic Scale*, and its origin is uncertain. The notes can be obtained, as we shall see later, by simple subdivision of a string which is made to vibrate, and so the scale was possibly built up in that way. Most of us from our early days have become familiar with the eight notes of the octave (Gk. *okto*, eight) in this scale, if only from church bells, or from learning the tonic sol-fa notes d, r, m, f, s, l, t, d′. Helmholtz's notation, which we now use, is C, D, E, F, G, A, B, C′.

The development of the Major Diatonic Scale is bound up with the subject of *harmony*, or *consonance* (when different notes played together give a pleasing impression), and *discord*, or *dissonance* (when different notes played together give an unpleasant impression). Pythagoras, the Early Greek philosopher, showed that strings, whose lengths bore a simple relation, harmonised when vibrated together. It has been realised that the frequency ratio of two notes which harmonise is simple in character. If not, then the combination of the two notes is discord. **The frequency ratio of two notes expresses the musical interval between them. The common ones which give harmony are :—**

ratio 1 : 1 called unison
„ 2 : 1 „ octave
„ 3 : 2 „ fifth
„ 4 : 3 „ fourth

ratio 6 : 5 called minor 3rd, and ratio 5 : 4 called major 3rd
„ 8 : 5 „ „ 6th, „ 5 : 3 „ „ 6th

and the naming of these is realised when the frequency ratios of the notes in the diatonic scale are considered :

Number of Note	1	2	3	4	5	6	7	8
Notation	C	D	E	F	G	A	B	C′
Tonic Sol-Fa	d	r	m	f	s	l	t	d′
Relative Frequency	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2
Relative Freq. as whole nos. . . .	24	27	30	32	36	40	45	48
Interval ratios	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{16}{15}$	

Note that C to C′, or No. 1 to No 8 note, is the octave, freq. ratio = 2 : 1
„ C „ E, „ 3 „ „ major third, „ = 5 : 4
„ C „ F, „ 4 „ „ fourth, „ = 4 : 3
„ C „ G, „ 5 „ „ fifth, „ = 3 : 2
„ C „ A, „ 6 „ „ major sixth, „ = 5 : 3

The interval ratios $\frac{9}{8}$ and $\frac{10}{9}$ are called *major* and *minor tones* respectively : those of $\frac{16}{15}$ are called *semitones*.

The Major Diatonic Scale, however, has ceased to be sufficient, or exact enough, for modern music, in which more notes are required. A device used to obtain variety in music is that of *modulation*, or change of key. Suppose some music is based on the simple scale C, D, E, ... C′, the interval the third from C, *i.e.* C to E, has an interval ratio of 30 : 24 or 5 : 4. Suppose now E becomes the basis of the new scale or key of the music when modulation takes place. Then the corresponding third interval is from E to G, which has a frequency ratio of 36 : 30 or 6 : 5, *i.e.* the minor instead of the major third. Such differences would happen for most of the intervals. This would not matter for unaccompanied singing for the voice can adjust itself to such small differences, or for stringed instruments where the length of the string is adjusted by the player ; but it would be impossible for many instruments such as the piano and organ. Hence a compromise has been adopted and is called the *Scale of Equal Temperament*. This uses twelve equal semitone intervals in the octave and aims to give a good representation of all the different musical intervals possible. Thus if x = the interval ratio of two successive notes, and since there are twelve of them in the octave (frequency ratio 2 : 1),

$$x^{12}=2 \quad \text{or} \quad x=2^{\frac{1}{12}}=1{\cdot}059.$$

The equal temperament scale is as follows :

Tonic Sol-Fa notes	d	**de**	r	**re**	m	f	**fe**	s	**se**	l	**ta**	t	d
Frequency ratios	1	$2^{\frac{1}{12}}$	$2^{\frac{2}{12}}$	$2^{\frac{3}{12}}$	. . . .						$2^{\frac{10}{12}}$	$2^{\frac{11}{12}}$	$2^{\frac{12}{12}}$
or	1	1·059	1·122	1·189	. . . .						1·782	1·888	2

The black type corresponds to black notes of piano : rest to the white.

For purposes of scientific standard the middle C of the piano, and the tuning-fork corresponding, is a frequency of 256 (probably because $256=2^8$), but there is no common international standard for the musical pitch of instruments.

Our concert pitch is A=439 at 68° F.

The continental pitch is A=435 at 15° C.

The American standard is A=440 at 68° F. (for at least the next ten years).

The English Army "Kneller Hall" standard has been A=452·4 at 68° F., and this was connected with the fact that our Army spends much time in India. But it has been decided to lower the pitch and equip all Army bands with instruments at the low pitch, our concert pitch, of 439 vibrations for A at 68° F.

A Harmonic Series is a series of notes with vibration numbers in the ratio 1 : 2 : 3 : 4 : 5, etc., etc. Commencing with d (or C) as the first of the series, the *fundamental* or the first harmonic, the second harmonic is the note whose frequency is twice as great, *i.e.* the octave note, *i.e.* d′ or C′.

The third harmonic is the note whose frequency is $3\times$that of the fundamental or $\frac{3}{2}\times(2\times$the frequency of the fundamental), *i.e.* is a fifth above C′ and is thus G′ (or s′). Similarly the fourth harmonic has a frequency four times that of the fundamental, and is thus the second octave note, or C″ (or d″). The fifth harmonic is of frequency

$5\times$that of the fundamental

or $\frac{5}{4}\times4$ " " "

and is thus the major third (interval ratio $\frac{5}{4}$) above C″, thus being E″ or m″.

Similarly the sixth harmonic can be shown to be G″ or s″. The harmonics above a fundamental are often called *overtones* of the fundamental, the second harmonic being the first overtone, and so on.

Quality of Sound.—Quality, or timbre, is the characteristic which enables us to distinguish a note on one instrument from a note of the same pitch played on another instrument. Much of the charm of orchestral music lies in the contrasts of the varying qualities of sounds

from different instruments. Note also the power of the ear to analyse sounds when several are received at the same time, and contrast it with the failure of the eye to detect two or more colours at once—in this case the eye experiences a combination effect.

Our exact knowledge of the subject of sound-quality is due to *Helmholtz* who, in his earlier days, was a teacher of Physiology and Anatomy and, in 1871, was made Professor of Physics at Berlin University. *Helmholtz* made a very detailed analysis of sounds, using the principle of **resonance.** Most elastic bodies have their own peculiar note which they emit when struck. You have no doubt noticed a salesman hold a vase or dish at the bottom and flick it, causing it to sound what is called its fundamental note—emitted when it is vibrating freely in its own natural manner. But most vibrating systems have several possible modes of vibration, each with its own period, and when the system is disturbed and left to itself a number of these may coexist, but the sound heard is of the pitch of the fundamental. A method of demonstrating the various modes of vibration is due to Chladni, an Italian scientist, with his *Chladni's Plate*. A common form of this is a metal plate 12 ins. square with a hole in the centre by which it is held horizontally and screwed down on a narrow vertical stand, leather washers being used to allow vibration of the plate all over. The degree of freedom can be varied. Fine sand is sprinkled over the plate, which is then set in vibration by a violin bow drawn across an edge. The sand is thrown off vibrating portions and collects on non-moving portions, and various patterns corresponding to varying modes of vibration can be obtained by varying the pressure of the bow, slightly touching the plate at one spot, etc. If the plate be left free (*free vibration*) it always gives the same resultant pattern for the same tension.

Now, an external agency can affect a vibrating body, *e.g.* a person touching a swing can alter its vibration. If displaced and left to itself, the free vibrations of a swing are a definite number per minute. But, by exerting a force periodically, one can make the swing vibrate to the same

period as that force—*it is then said to be in forced vibration.* However, the most interesting case is seen in the method by which we usually endeavour to get a swing moving well. First of all we displace the swing and then, just as it has come back and is about to move away again, we give it a push, or impulse. In this manner *we build up the movement of the swing by exerting a force to act on it in period with its own natural free vibration*, and we call this **resonant forced** (or **reinforced**) **vibration,** or simply **resonance.** Lord Kelvin said that with a pea-shooter and an unlimited supply of peas it should be possible to aim them at, and in period with, London Bridge and ultimately set it swinging so violently that it would collapse. It is a military rule that, when a body of soldiers is marching over a bridge, the order "Break step" must be given, so as to prevent a great force being exerted periodically on the bridge. An appreciable movement was once observed by the writer when several members of a company conspired to keep step when marching over Chelsea Suspension Bridge.

An enclosed volume of air has its own particular period of vibration (and note emitted), and so when external pressure changes are in the same period as the natural period of the volume of air, the latter is set vibrating and is reinforced, the air resounding loudly or resonating. This can be shown very simply by means of a number of similar bottles with a different volume of water in each, so that the volume of air above each is different. Set different tuning-forks vibrating, and test them above the bottles. Each bottle will be found to resound best to one certain fork. No doubt, too, you have noticed the variation in the sound emitted when water is running from a tap into a jug or kettle. As less air is left in the jug or kettle the note emitted gets higher in pitch, for the smaller the volume of air the smaller its periodic time of vibration and so the greater is its frequency.

Helmholtz utilised this principle of resonance to analyse sounds by making a large range of **Helmholtz resonators,** or sonorous globes, which are of the form shown in Fig 230. They have openings which are small

in comparison with the enclosed space and respond very sensitively to their own particular note. Helmholtz used a very large series of these with gradually varying periods of vibration, finding the natural note of each one by testing with tuning-forks. He then used them to listen

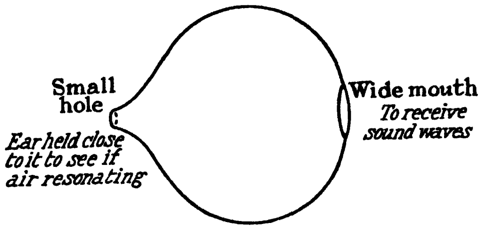


FIG. 230.—Helmholtz Resonator.

to any sound, so ascertaining which of the resonators resounded to it. **By this means he found that when a note was sounded not only was the fundamental vibration present, but the vibrations of harmonics of the fundamental could be detected.** A close investigation of the quality of the note emitted led Helmholtz to the conclusion that **the all-important point in determining the quality of a note is the number and relative proportions of the tones which build up the sound.** The presence of harmonics up to the sixth give character and brilliancy to the note, whilst musical sounds which are pure, *i.e.* simple and devoid of harmonics, are dull and wanting in brilliancy, *e.g.* the notes of tuning-forks. If the first, third, fifth, etc., harmonics only are present, the sound is nasal in character, whilst if higher harmonics than the sixth are present the sounds are piercing, *e.g.* the high notes of a violin. These conclusions Helmholtz confirmed by imitating sounds by making use of suitable combinations of tuning-forks, particularly for vowel sounds, showing that they depend on the relative intensities of the component harmonics.

Some interesting experiments can be performed with a piano to illustrate this. (1) Strike the note middle C loudly and then damp it quickly by touching the corresponding wire inside (*via* the open top) and listen carefully. You will hear

the upper C (or C′) wire in vibration. When the middle C vibrated there were also some upper C vibrations, and these set the wire of the upper C note vibrating. (2) Put down the open pedal and so raise all the dampers of all notes. Sing sharply the vowel sound *ah* against the sounding-board and listen for a response. Each wire selects the particular constituent of the sound corresponding to its own fundamental vibration (if present), with the result that several wires are set in vibration and a compound sound, very much like *ah*, is given back. After damping the whole and releasing, this should be repeated with such sounds as *oo*, *ee*, etc.

Interference between Sound Waves.—Beats.— If a vibrating tuning-fork be held close to the ear and rotated about its own axis it is found that in four directions the sound of the tuning-fork appears to be a maximum, and at four other directions, in the one revolution, little sound is heard. Suppose A and B (Fig. 231) are the two prongs of the tuning-fork. These move outwards together giving compression waves as shown, whilst rarefactional waves travel out towards C and D at the same time. As the prongs of the fork move to and fro these pressure changes alternate, and loud sounds are heard in these four directions. In the directions at 45° (along the dotted lines) the waves from the outer surface of one prong, say A, are almost exactly balanced by the opposite waves from the inner surface of the prong, say those towards C, and so at points along the dotted line little sound should be heard. Since the ear occupies some space it receives waves also on either side of the dotted line, and so faint sounds are heard.

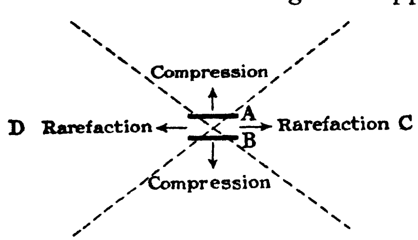


FIG. 231.

This phenomenon of two sets of waves neutralising one another is an example of *interference* in sound. Interference is not necessarily a neutralising effect—when a second system of waves is superposed on a system

of waves, at some points the amplitudes are greatly increased, at others they are reduced to zero in the resultant wave. The waves are said to interfere. It is quite easy to show this graphically. Fig. 232 shows the compounding of two waves with the same period and

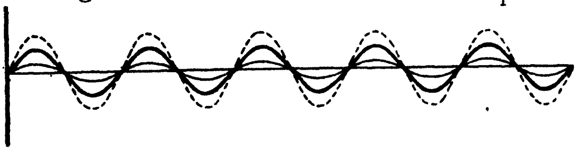


FIG. 232.—Compounding Two Waves in the Same Phase.

which are in the same phase, or differ in phase by a whole number of wave-lengths, and so the corresponding particles in each are at the same stage of their vibrations. Their amplitudes are shown to be different and one wave system is shown in a thick black line, and the other in a

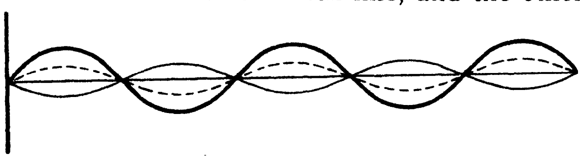


FIG. 233.—Compounding Two Waves in Opposite Phase.

thin black line. The resultant wave is shown in a dotted line, and is obviously obtained by finding the algebraic sums of the amplitudes at different points. Fig. 233 shows the result of compounding two waves in opposite phase (or phase difference may be $\frac{1}{2}\lambda$, or $\frac{3}{2}\lambda$, or $\frac{5}{2}\lambda$, etc.

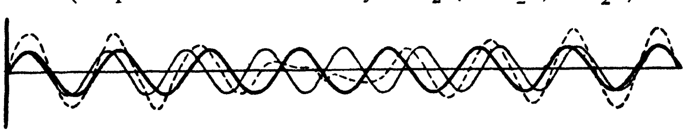


FIG. 234.—Compounding Two Waves, the Periods of which are 8 : 7.

Obviously then, if the amplitudes of the two waves were equal, in the first case the resultant would be 2 × each wave, in the second case the resultant would be 0, *i.e.* no displacements.

Fig. 234 shows the compounding of waves with different

periods, the example taken being where the periods are in the ratio 8 : 7. The bends in the resultant wave (dotted line) are not regular in form ; the maximum displacements are greater in some places than in others, so that in some regions the condensations and rarefactions are intense, in others only slight. Thus, the sound heard by an ear, at which such a wave system arrives, alternates —the intensity being sometimes increased and sometimes diminished. These alternations are called *beats*, and they are most noticeable when two sources of nearly equal period (and thus frequency) are sounding together. This can be shown by obtaining two exactly similar tuning-forks, then slightly altering one by putting a little wax on one prong, or by fastening on a specially made small screw clamp, and then sounding together.

It is found that **the number of beats per second is the difference between the two frequencies.**

Thus, if the frequencies of two wave systems, which are combined, are n_1 and n_2 there will be $n_1 - n_2$ beats per second produced.

This is used to find the frequency of one source of sound if another source of known frequency is provided, and if there is little difference in frequency between the two sources, so that the ear can detect, and it is possible to count, the beats per second.

E.g. suppose a known tuning-fork vibrates 200 times per second, and an unknown fork gives, with the above, 4 beats per second. Then the frequency of the latter must be 200 ± 4, *i.e.* 204 or 196. Add a little wax to a prong of that fork and so give it a greater period, *i.e.* a lower frequency, and then observe whether there are fewer or more beats, when the two forks sound together. If there are now more beats, then the frequency of the fork must be 196 ; if there are now less beats, then the frequency of the fork must be 204.

This method of counting beats is a common method of tuning one note to another, and is used by piano-tuners to tune a central note on the piano to a standard fork (there are no beats when the notes are in unison). We shall have occasion to use the method later.

Helmholtz also found that discord between two notes is due to beats of 30–40 per sec. taking place between them (or their harmonics), giving rise to unpleasantness of sound. He considered that the rapid recurrence of the intense sound was distressing to the sensitive ear (between certain frequency limits and particularly about 33 per sec.), just as the continuous recurrence of bright light (*i.e.* flickering) is trying when one is walking along by a hedge or tall railings and the sun is shining through the gaps.

Schleibler's Tonometer is a set of standard tuning-forks (all of known frequencies) used for measuring the absolute pitch of any note. The standard forks, carefully tuned, cover an octave and the forks differ by a frequency of 4, *i.e.* their frequencies are N, N+4, N+8, N+16, . . . 2N.

Thus the frequency of any note within this range can be found by finding either a fork with which no beats are given (*i.e.* unison) or two adjacent forks with which it gives 1, 2, or 3 beats per sec. *E.g.*:

suppose with fork,		frequency	N+64,	note gives	3 beats,	
and	,,	,,	,, N+68,	,,	1 beat,	

the frequency of the note=N+67. (N, of course, is known.)

This is a convenient instrument because it is portable. Schleibler's own tonometer had a range of 256–512 vibrations, and so consisted of

$$\frac{512-256}{4}+1=\underline{65\text{ forks}}.$$

Doppler's Principle.—In 1842, Doppler pointed out a very interesting change in the apparent frequency of wave-motion if the source of the waves were moving relatively to the observer. You may have noticed this effect while standing on a railway platform, when an express train has dashed through with its engine whistling. It can also be distinctly observed if the horn of a motor-car is being operated while the car is passing quickly by you, or better still, if you are in a train or car passing another

moving train or car from which the sound is being made. In the first case the observer is at rest and a source generating sound waves moves towards him. Then each successive wave generated has a shorter distance to travel to reach the observer, thus doing so more quickly. To him accordingly the frequency of the waves gets gradually greater and greater, *i.e.* the pitch of the note heard seems gradually to rise. Obviously as the source goes away from the observer the effect is the reverse, *i.e.* the pitch seems to fall. Thus, as the source of sound passes by the observer there is a distinct drop in pitch from the apparently higher to the apparently lower than the real pitch of the note sounded. This effect, very appreciable owing to the great speed of the seaplanes concerned, was very clear at the recent Schneider Trophy Race in the Solent, the sound sources being the rotating propellers. It can be well shown in a room to a class by swinging on the end of a stout cord, about 3 ft. long, a sounding tuning-fork with a sound-box, in a circle towards and away from the class.

EXAMPLE.—Suppose a train, moving at 30 mls. per hr. towards a man standing on a station, is whistling the note C′ (frequency $2\times256=512$ per sec.).

The train moves $\frac{88}{2}=44$ f.s. (since 60 m.p.h. $=88$ f.s.). If the whistle were at rest there would be 512 waves in the distance travelled by the sound in 1 sec., *i.e.* approx. 1,100 ft. But since source travels 44 ft. towards the observer in 1 sec., the 512 waves are apparently contained in $1100-44=1{,}056$ ft.

$$\therefore \text{ Apparent wave-length of waves} = \frac{1{,}056}{512} \text{ ft.}$$

$\therefore$ Frequency of waves to observer

$$=\frac{\text{Velocity}}{\text{wave-length}}=\frac{1{,}100 \text{ f.s.}}{\frac{1{,}056}{512 \text{ ft.}}}=512\times\frac{1{,}100}{1{,}056} \text{ per sec.}=533\tfrac{1}{3} \text{ per sec.}$$

Similarly it can be shown that the apparent frequency of the waves when source is travelling away from observer

$$=512\times\frac{1{,}100}{1{,}100+44} \text{ per sec.}=492{\cdot}3 \text{ per sec.}$$

Thus drop in frequency on passing$=533\frac{1}{3}-492{\cdot}3$ per sec. $=41$ per sec. which is more than a semitone at the pitch quoted.

This can also be illustrated by two boys on bicycles ringing their bells while passing one another.

It has been suggested that a musical policeman could estimate the speed of a motor-car by the drop in pitch of the horn conveniently sounded as the car passed him.

A moving observer passing a source of sound at rest experiences a similar occurrence, and it is left as an exercise for the student to work out the changes in pitch.

Exercises on Chapter XXII

1. How do you explain the formation of musical notes, and the fact that they may differ in pitch, quality, and loudness? [*J.M.B.* 1927.]

2. Describe an experiment to show that the difference in pitch between two notes depends only on the ratio of the frequencies of the vibrating sources. If you are given two notes whose vibration frequencies are 288 and 450, what is the vibration number of the note whose pitch is midway between? [*L.M.* 1928.]

3. Explain the construction and action of the siren. What is it used for, and what are some of its drawbacks? [*L.M.* 1920.]

4. If there are thirty-two holes in the disc of a siren which makes 1,050 revolutions per minute, what is the frequency of the note emitted by the siren?

5. Distinguish between pitch and frequency. How are they related to one another? Describe experiments to support your statements. [*L.M.* 1923.]

6. Describe a form of siren or other suitable apparatus for carrying out experiments to show that (*a*) different sources of the same frequency give out notes of the same pitch, (*b*) the pitch interval between two notes is governed by the frequency ratio of the sources. Indicate briefly how you would carry out the experiments. [*L.M.* 1925.]

7. What relation exists between the vibration frequencies of two notes, one of which is the octave of the other? How would you prove the relation?

8. On what does the difference in pitch between two notes depend? A note of frequency 384 is said to be a "fifth" higher in pitch than one of 256. What is the frequency of the note a "fifth" higher than the 384 note? What is the difference in pitch between it and the 256 note? [*L.M.* 1928.]

9. What do you understand by (*a*) the frequency, (*b*) the amplitude of a vibration? How does a noise differ from a musical note? Describe a method of finding the frequency of a note given by a tuning-fork. [*J.M.B.* 1928.]

10. Under what circumstances is a body a source of sound? What determines whether the sound produced is a musical note or a mere noise? Describe experiments to illustrate your answer, including one to show that even the noise produced by dropping a piece of wood has a detectable pitch. [*L.M.* 1926.]

11. Describe the determination of the frequency of a tuning-fork by means of a siren. What assumption is made in this method of determining frequency? If the frequency of the middle C on a piano is 260 what is the frequency of the next higher C and of the note midway between them? [*L.G.S.* 1929.]

12. An observer standing by the side of a railway line watches the approach of a train as it is leaving a tunnel. He hears the whistle (apparent frequency=576). Immediately after he hears the echo from the hill through which the train has passed. The note seems to be considerably lower. Explain this and calculate the true frequency of the note, and also that of the echo, assuming the train moved at 60 m.p.h. and the velocity of sound is 1,100 f.s.

13. The pitch of the note emitted (real frequency=120) by the horn of a motor-car, speed 30 m.p.h., appears to fall as it passes an observer at rest. What will be the apparent drop in the note if the velocity of sound is 1,100 f.s.? Discuss the effect observed when one is moving in a car at 30 m.p.h. and passing the same horn sounded whilst at rest.

14. Much attention is being given to the desirability of exact reproduction of sounds in developing "wireless" loud speakers. What factors in regard to the sounds need be considered? Suggest methods of studying them.

CHAPTER XXIII

VIBRATION OF STRINGS—TRANSVERSE WAVES

STRINGED musical instruments have been used for many centuries, and it is found that very primitive races have instruments which consist essentially of one or more strings which are vibrated. We have already seen that Pythagoras had some knowledge regarding vibrating strings. They were certainly studied by Galileo and Mersenne, but the greatest development in knowledge concerning them was made by a French scientist, Joseph Sauveur (1653–1716), who, despite the fact that he had a poor ear for music, spent much time on the study of sound.

A simple idea of a way in which a string or cord can vibrate can be obtained by fastening a long string, or rope (or about 6 yds. or so of rubber bunsen tubing) to a wall and then jerking the other end which is held reasonably firmly by the hand. The displacement, or hump, made at one end is seen to be carried along the string, and each part of it is seen to repeat the motion of the part which moved just before. This movement, or wave, is reflected from the fixed end, and, if the other end be held firmly by the hand, is seen to be reflected backwards and forwards till the imperfect elasticity of the string causes it to stop. It is noticed that the particles of string move perpendicularly to the direction of its length, *i.e.* to the wave-motion, and so we say *progressive transverse waves* are set up in the string—progressive because they travel up and down the string.

In using a catgut string for producing musical sounds it is held firmly at both ends, and is set in vibration at one end by plucking, or often by a bow which is drawn across it. The latter pulls the strings a little way till the opposing force it offers causes it to slip back. It is then caught by the next piece of the bow and carried forward again

till it slips back. This process is continued as the bow is drawn across, with the result that the string is set into rapid vibration and a sound is emitted.

It can be shown mathematically that the velocity of wave-motion in a stretched string depends only on its mass per unit length and the tension of the string, and the velocity $V=\sqrt{\frac{T}{m}}$, where T=the tension, and m=the mass per unit length.

V is in cms. per sec. when T is in dynes and m in grms. per cm.

V is in ft. per sec. when T is in poundals and m in lbs. per ft.

We must consider in greater detail this case of vibration in a stretched string, for a wave set travelling along a string, and reflected back from a taut end (with the same velocity) must interfere with the further waves set travelling along the string. Consider a train of waves which travel to a clamped end. The string gives periodic impulses to the clamp and these in turn give periodic

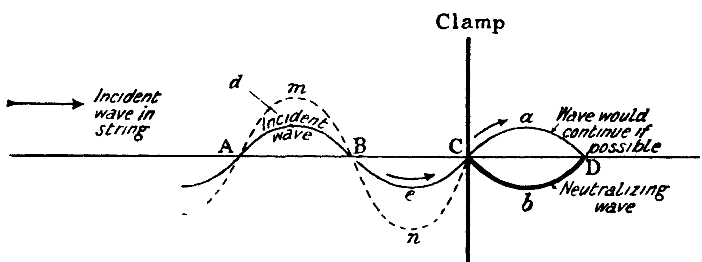


FIG. 235.

impulses back to the string, *i.e.* the clamp acts as a reflector. Fig. 235 shows the effect of a wave A*d*B*e*C at the clamp. The wave would continue, if possible, as shown (C*a*D). But obviously this is neutralised by the clamp, *i.e.* we can consider that a neutralising wave C*b*D is set up. But, at the same time as this neutralising wave is produced, a similar (opposite in direction) wave must be set up in the string in contact with the clamp,

i.e. a wave C*e*B*d*A, and this must be the reflected wave. The resultant of the original wave and this reflected wave is then obviously the wave C*n*B*m*A shown in dotted line.

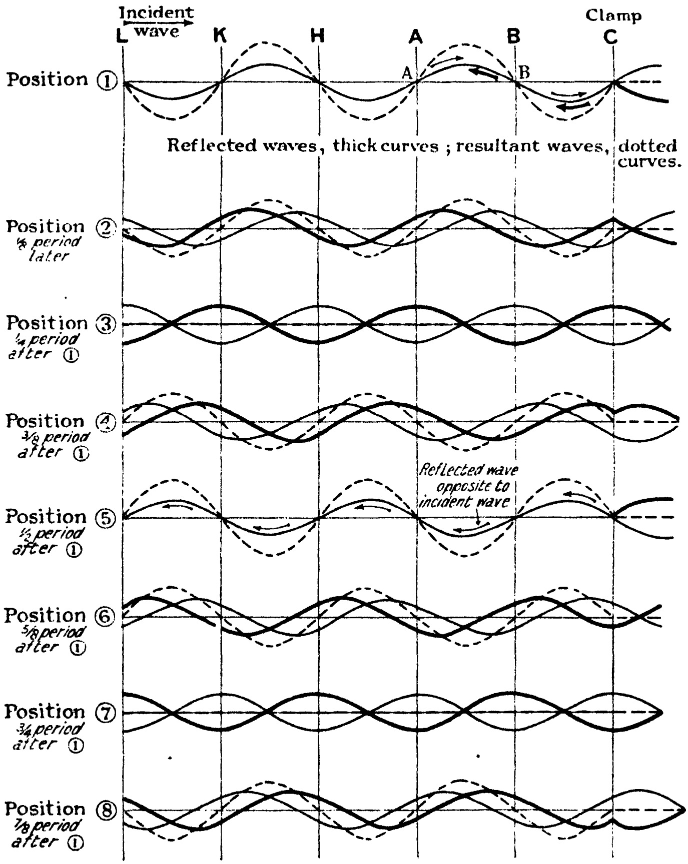


FIG. 236.

We can now consider the action similar to this for different positions in the wave-motion (or period), and Fig. 236 shows these results. From them it is seen that

there must be a kind of wave formation, or periodic displacements, in the string and *certain equally-spaced points*, the clamp C, B, A, H, K, L, etc., *have no displacement.*

Thus, considering the behaviour of the string between two of these points which are adjacent, *e.g.* A and B (and see Fig. 237), the particles between undergo periodic vibrations between the heads of the arrows as indicated, *e.g.* particle 1 at A is always at rest, particle 2 vibrates between a and a_1, particle 4, half-way between A and B, has the maximum amplitude and vibrates between x and x_1, etc. Hence when you look at such a vibrating string, if the maximum amplitudes are sufficiently large, it is possible to see the string divided up into a number of loops or segments (Fig. 238), each end of a loop signifying a position of no displacement, or

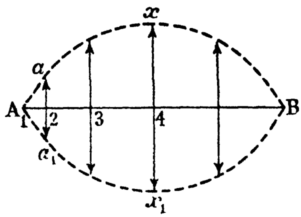


FIG. 237.

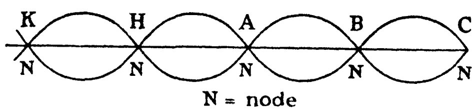


FIG. 238.

a *node*. The distance between two successive nodes (and called a loop or segment) is obviously half a wave-length. A point, half-way between two nodes, which undergoes maximum displacements is called an *antinode*.

If the string is tightly clamped at both ends, the waves being reflected backwards and forwards, then obviously the two ends must be nodes, and so it is theoretically possible for the string to vibrate so that it is divided up into any whole number of loops. This kind of wave-motion where uniformly-spaced parts are at rest is called *stationary vibration*, and the waves are called *stationary transverse waves*, since the other particles move perpendicularly to the wave propagation (along the length of the string).

Then, as before, and in the stated units, the velocity $V=\sqrt{\frac{T}{m}}$, where T is the tension and m the mass per unit length.

But $V=n\lambda$, where n=frequency and λ=wave-length, ∴ frequency of the wave-motion

$$=n=\frac{1}{\lambda}\sqrt{\frac{T}{m}}=\text{frequency of the note emitted,}$$

and n=the number per sec., and λ is in cms. when T is in dynes and m in grms. per cm.

The *fundamental* or normal vibration is when the whole string is acting as one loop, *i.e.* the only nodes are the two ends of the string. Then the

wave-length = 2 loops = $2l$, where l = length of string in cms.

and **frequency** $n=\frac{1}{2l}\sqrt{\frac{T}{m}}$.

This is the ordinary case of a violin, harp, etc.

(In English units n=no. per sec. when l is in ft., T is tension in poundals, and m is mass per unit length in lbs. per ft.)

From this mathematical relationship we thus see the **laws of vibrating strings: the frequency of the fundamental note emitted is**

(1) **inversely proportional to the length** ($n \propto \frac{1}{l}$);

(2) **directly proportional to the square root of the tension** ($n \propto \sqrt{T}$);

(3) **inversely proportional to the square root of the mass per unit length** ($n \propto \frac{1}{\sqrt{m}}$).

These laws can be verified by means of a *sonometer* (or monochord) (Fig. 239). This consists of one or two steel wires fixed to one end of a sounding-box by a fixed screw or clamp. The other end of one of the wires passes over a pulley fastened to the sounding-box, and a rod or pan for carrying weights is attached to it. Thus the

tension of the wire can be altered by known weights. The second wire (if attached) is often fixed at its other end to a thumbscrew, moving through a fixed nut, and so its tension can be altered. Two movable bridges are provided to vary the length of the string used (often 100 cms. downwards). They are made of wood and come

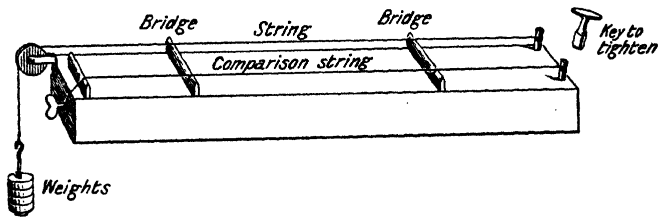


FIG. 239.—Sonometer.

to an edge at the top where they meet the wire (or triangular glass prisms can be used). With this instrument you can quickly show by plucking the wire that—

(1) by moving the bridges closer together, the length of wire between emits a higher note;

(2) by tightening up the wire, a higher note is emitted.

An exact experimental verification of the laws is described below.

(1) *To verify* $n \propto \frac{1}{l}$. For this a set of standard tuning-forks covering at least an octave should be used. Add sufficient weights to the string over the pulley for its maximum length between the bridges, when bowed by a violin bow, to emit a note equal to, or slightly higher than, that of the lowest tuning-fork. Then, if necessary, adjust the bridges till the wire and the fork give the same note. If your ear is not musical enough to detect it, test by the beats method: bow the wire between the bridges, quickly strike the tuning-fork and hold it on the sounding-box of the sonometer. You will then hear both wire and fork sounding at the same time, and when they are in unison there will be no beats. Measure exactly the length of the string between the two bridges. Now repeat with the different forks, each time adjusting the length of the string till its note is the same as that of the fork. Enter your results in

a table as below (and also plot a graph of $\frac{1}{n}$ against l, which should give a straight line).

Fork and Frequency (n)	Length of string in cms. (l)	nl
C 256 D 288 (see p. 445) C′ 512		This column should give a constant result to verify the law.

(2) *To verify* $n \propto \sqrt{T}$.—For this the length of the string (*i.e.* the distance between the two bridges) is kept constant and the tension is varied, by means of weights, until the note emitted by the fixed length of string when bowed is the same as that of one of the tuning-forks. This is carried out for the series of forks and the results tabulated as follows (a graph of n and $\sqrt{T}$ should give a straight line):

Fork and Frequency (n)	Tension of string in kilograms wt. = T	$\sqrt{T}$	$\frac{\sqrt{T}}{n}$
C 256 D 288 (see p. 445) C′ 512			This should be constant to verify the law.

Owing to the difficulty of exactly tuning the string to a vibrating fork by adding weights, it is a more common practice to verify the 2nd law on assuming the 1st law previously verified. A series of experiments, similar to the one described for verifying the 1st law, are carried out with different known tensions. It is then shown from the results that $\frac{nl}{\sqrt{T}}$ is a constant, the mean value for nl being obtained from each set of experiments.

(3) *To verify* $n \propto \frac{1}{\sqrt{m}}$ (not ordinarily done in the laboratory).—For this, different gauge wires (and of different metals) are required; a known length of each must be weighed and the mass per unit length of each determined. In turn the same

length of each wire is made to undergo the same tension, and the frequency of the note it emits when plucked is determined. This can be done by means of a siren or a Savart's wheel (or by using another sonometer wire). It is then shown that in each case $n\sqrt{m}$ is a constant quantity.

Overtones or Harmonics with Strings.—We saw on p. 432 that a string might be set in motion so that it was subdivided into any number of loops. We have just considered the usual, or fundamental case of one loop. Now, having a sonometer wire AB (Fig. 240) tuned to the ordinary middle C of the piano, and while lightly touching it at its middle point D by a feather or match stick, bow it and so set it in vibration. You should immediately recognise that it now emits the note an octave above its fundamental (*i.e.* the note sounded when not held lightly at its centre). A node is created at the middle point D, and so the vibration is as shown in Fig. 240. Obviously, then, the wave-length $\lambda = l$, the length of the string.

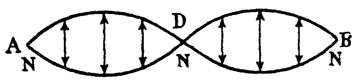


FIG. 240.

$$\text{Thus} \quad n = \frac{1}{\lambda}\sqrt{\frac{T}{m}} = \frac{1}{l}\sqrt{\frac{T}{m}} = 2\left(\frac{1}{2l}\sqrt{\frac{T}{m}}\right),$$

i.e. its frequency is twice that of the fundamental.

The note (C′) is the overtone, or second harmonic of the fundamental note (which was C).

If now the string be lightly held as before at E, a distance one-third its length from its end, and bowed, its vibrations are as shown in Fig. 241.

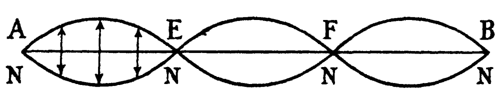


FIG. 241.

The wave-length in this case is obviously $= \frac{2}{3}l$, and so the frequency $n = \frac{1}{\lambda}\sqrt{\frac{T}{m}} = \frac{3}{2l}\sqrt{\frac{T}{m}}$ and so is three

times that of the fundamental, *i.e.* it is the third harmonic (or second overtone) of the fundamental note (C) and is G′.

There is a very neat way of showing this. Make little paper riders and place them on the string, one at F ($\frac{1}{3}$ of the length of string from the end B) and others at other places along the string (see Fig. 242). Hold the string lightly at E as before, and bow near the end B. You will see all the riders jump off the wire (showing the string vibrated where they were) *except the one which is at the node* F.

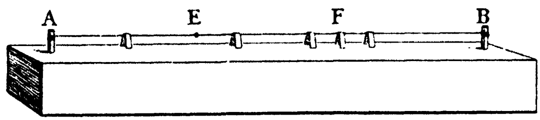


FIG. 242.—Using Sonometer Wire and Paper Riders to show Transverse Vibrations.

Higher harmonics can be produced by lightly holding the wire, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, etc., its length from its end, and the existence of nodes can be demonstrated in each case, as above, by paper riders.

Pianoforte Strings.—In a piano the steel wires, the strings, are stretched on wooden or metal frames to which a thin wooden soundboard is attached. The vibrations of this board are forced, not resonant, and so are small, but owing to its large area it imparts movement to a large volume of air and so gives an adequate volume of sound. For the low notes single strings, often several feet in length, are used and they are usually weighted by wire twisted round them. The wires of high notes are often made up of two or three wires, tuned in unison, to give better quality. Those for the highest notes are only 2–3 ins. long. The wires are struck by hard narrow hammers, the ends of which are covered with felt, when the piano keys are depressed. Since at the point where the string is struck there is no corresponding harmonic, a wire should be struck at a point $\frac{1}{7}$ of its length from one end, since the seventh harmonic is dissonant. However,

most pianos are so constructed that the wires are struck $\frac{1}{8}$ or $\frac{1}{9}$ the length from one end, so giving a more metallic note, whilst the shorter wires are often struck even nearer the ends in order to improve the quality of their tone, their rigid nature making it more difficult for the higher harmonics (which give quality) to be produced.

The construction of a *harp* is somewhat similar in principle to that of a piano; the soundboard is smaller and the sounds weaker. Again, the wires are plucked near the centre, so that the higher harmonics are not produced, and thus the sounds are soft and also deficient in brilliancy.

A *violin* (and viola) produces the sixth to the tenth harmonics, as Helmholtz showed, and these give the notes a piercing character. The shape of a violin is not a matter of theory, but of experience handed down for over 200 years. In *banjos*, etc., frets are used to indicate where the fingers must be placed to give the higher notes.

Problems on Vibrations of Strings.—(1) A string 100 cms. long, stretched by a weight of 16 kilograms, makes 256 vibrations per sec. (middle C). How could this be altered to 320 vibrations (note E) by altering (*a*) the length, (*b*) the tension of the string?

(*a*) Since frequency $\propto \frac{1}{\text{wave-length}}$, $\frac{\text{first frequency}}{\text{second frequency}} = \frac{\text{second length}}{\text{first length}}$.

$$\therefore \frac{256}{320} = \frac{\text{second length in cms.}}{100},$$

or second length $= \frac{256 \times 100}{320} = \underline{80 \text{ cms.}}$

(*b*) Since frequency $\propto \sqrt{\text{Tension}}$, $\frac{\text{first frequency}}{\text{second frequency}} = \sqrt{\frac{\text{first tension}}{\text{second tension}}}$

$$\therefore \frac{256}{320} = \sqrt{\frac{16}{\text{second tension in kgrs.}}} = \frac{4}{5}.$$

$$\therefore \text{Squaring, } \frac{16}{\text{second tension in kgrs.}} = \frac{16}{25},$$

$\therefore$ $\underline{\text{Second tension} = 25 \text{ kgrs.}}$

(2) A stretched string emits a certain note, and when the

load is increased by 4 kilograms wt. the note rises an interval of a fifth (*i.e.* in ratio 3 : 2). What was the original stretching force?

Let T kilograms weight=original stretching force so that
T+4 „ „ =second „ force.

Now since frequency $\propto \sqrt{\text{Tension}}$, $\dfrac{\text{new frequency}}{\text{old frequency}} = \sqrt{\dfrac{\text{new tension}}{\text{old tension}}}$

$$\therefore \frac{3}{2} = \sqrt{\frac{T+4}{T}}, \text{ or squaring, } \frac{9}{4} = \frac{T+4}{T},$$

$$\therefore 9T - 4T = 16 \quad \text{or} \quad \underline{T = 3{\cdot}2 \text{ kgrs. wt.}}$$

(3) Two wires of the same length, and nature, have diameters in the ratio 10 : 7 and give the same note when plucked. Compare the tensions.

Now, masses *per unit length* $\propto$ volumes,
and so are $\propto$ radii^2 or diameters^2,
and so are in ratio $10^2 : 7^2$ or 100 : 49.

But since $n = \dfrac{1}{2l}\sqrt{\dfrac{T}{m}}$, it follows that, when l and n are the same, the tensions $\propto$ masses per unit length.

$\therefore$ Tensions must be in the ratio 100 : 49, the wire with the greater diameter having the greater tension.

Exercises on Chapter XXIII

1. State the laws governing the vibration of stretched strings, and describe how these laws may be verified experimentally. [*L.M.* 1920.]

2. Prove the relation connecting the velocity of propagation of a wave with its frequency and wave-length. Give an account of the various modes of transverse vibration of a stretched string and explain how you would show that the frequency is inversely proportional to the wave-length, the tension being constant. [*L.M.* 1919.]

3. Regular series of transverse waves are being sent along a rope. Draw figures showing (1) two wave-trains of the same frequency but different amplitude, (2) two wave-trains of the same amplitude but different frequency. If all waves travel with the same velocity, what connection exists between (3) frequency and wave-length, (4) amplitude and wave-length? [*L.M.* 1922.]

4. State the laws governing the transverse vibrations of stretched strings. Describe how you would use a stretched string to compare the frequencies of the notes emitted by two tuning-forks. [*L.G.S.* 1924.]

5. State what you know of the way in which stretched strings vibrate when bowed. Being supplied with two tuning-forks, the frequency of one of which is known, describe *in detail* the experiment you would carry out with a sonometer to determine the frequency of the second fork. [*L.G.S.* 1928.]

6. Describe simple methods of altering (1) the tension, (2) the length of a sounding wire. Two such wires are in unison. The tension of one is doubled. How must the lengths of the other be altered so as again to be in unison with the first? [*L.G.S.* 1924.]

7. Enumerate the factors which determine the frequency of a vibrating string. A string, stretched by an unknown weight, emits a certain note when plucked and released. On increasing the load by 5 kilograms the note rises to the octave. Calculate the original load on the string. [*L.G.S.* 1920.]

8. Describe, giving possible numerical results, how you would prove that the frequency of vibration of a string vibrating transversely is proportional to the $\sqrt{\text{stretching force}}$. On increasing the weight stretching a given string by 2·5 kilograms, the frequency is altered in the ratio 3 : 2. Find the original stretching weight.

9. If you were provided with a series of tuning-forks of known frequencies, how would you proceed to verify the relation between the frequency of a stretched string and its length? Two tuning-forks A and B are in unison with lengths 50 cms. and 75 cms. respectively of a stretched wire. How would you alter the tension on the wire so that A vibrates in unison with 75 cms. of it? [*L.G.S.* 1923.]

10. How does the frequency of a stretched string vary with the length? Describe a way of showing this. A string, 39 ins. long, is divided by two bridges into three vibrating segments, having frequencies in the ratio 2 : 3 : 4. Find the lengths of the segments. [*L.M.* 1924.]

11. If you were provided with a tuning-fork, describe an experiment whereby you could show the relation between the length of a vibrating wire and its tension while its frequency remains constant. A piano wire is replaced by another whose diameter is 0·9 of the former. How will the tension of the new wire compare with the former if the same note is obtained? The length of wire employed is the same in each case. [*L.M.* 1927.]

12. How would you demonstrate the different modes of vibration of a stretched string? In what ratio are the frequencies of these modes? [*L.G.S.* 1924.]

13. A string 50 cms. long, stretched by a weight of 10 kilograms, makes 256 transverse vibrations per sec. How could the frequency of the note emitted be raised to 384, (1) by altering the length of the string, (1) by altering the stretching weight? Could the string be made to suit a harmonic note of frequency 384 without altering either the length or stretching weight? Explain fully.

14. Explain how you would demonstrate the existence of harmonics in the vibrating wire of a sonometer. A copper wire 1 metre long is vibrating in two segments when stretched by a weight of $\frac{1}{4}$ kilogram. Find the frequency of the note if the mass per cm. is 0·01 grm. (g=980 cms. per sec. per sec.)

15. Explain the term *overtone*. Describe how to compare the frequencies of the notes given by two wires of different materials when their lengths and

tensions are the same, using a two-string sonometer fitted with a reference wire. [*L.G.S.* 1921.]

16. A strained sonometer wire gives 2 beats per second with a tuning-fork when its length is 143 cms. long and also when its length is 145 cms. long. What is the frequency of the tuning-fork and of the wire in each case?

17. Two wires A and B of the same material and having equal lengths are in unison when stretched by forces of 3·5 and 10·5 kilograms weight respectively. What is the ratio of their diameters? If the force on A is increased to 5 kilograms, what is the new ratio of their frequencies? [*L.M.* 1922.]

18. You are provided with a sonometer with two strings, adjustable bridges, and weights. How would you investigate the relation between the tension and the frequency of a stretched string? What assumption would you have to make? Three strings of the same length are subjected to equal tensions, but are found to give notes with a frequency ratio 2 : 3 : 4. The tensions of the first two are altered equally until the first is in tune with the third. How must the length of the second now be altered to make it in tune with the third? [*L.M.* 1923.]

19. On what does the frequency of the note emitted by a stretched string depend? Compare the frequencies of the notes emitted by two sonometer wires, made of the same wire, stretched by forces of 5·76 kilograms weight and 4 kilograms weight respectively. What would be the effect on the frequencies of these wires if they were wrapped with a layer of very thin wire? [*J.M.B.* 1929.]

20. How would you investigate the relation between the frequency of the note obtained by plucking a stretched wire and the stretching force on the wire? The relation between frequency and length may be assumed. A wire 1 metre long stretched by a force of 50 lbs. wt. gives a note of frequency 256. A bridge is placed $33\frac{1}{3}$ cms. from one end. What must the stretching force be made so that the longer segment gives a note of 256? [*L.G.S.* 1929.]

21. A rather unmusical person wishes to tune a wire to the note of a tuning-fork. How can this be done by him? [*L.G.S.* 1925.]

22. State the laws governing the frequency of the note emitted by a stretched string when vibrating transversely. A steel wire and a catgut string of the same length and stretched with the same force give the same note when plucked. If the density of steel is eight times that of catgut, what is the ratio of their diameters? [*L.M.* 1929.]

23. State how the frequency of the note given by a stretched wire when vibrating transversely depends upon its length and its tension. If you were supplied with a monochord and a tuning-fork of known frequency, describe an experiment you would perform to compare two unknown weights. [*L.M.* 1930.]

24. How would you investigate experimentally the relation between the tension and the vibration frequency of a stretched wire? A stretched wire 3 feet long is divided by two bridges so that the segments give notes of vibration frequencies in the ratio 3 : 4 : 5. Where must the bridges be placed? [*L.G.S.* 1930.]

CHAPTER XXIV

VIBRATION OF AIR IN PIPES — ORGAN PIPES — WIND INSTRUMENTS — LONGITUDINAL VIBRATIONS IN RODS

THE fact that an enclosed volume of air can vibrate and give a musical note has already been mentioned or p. 420 when dealing with resonance. This resounding of the air, or, as we often say, of a pipe or chamber containing the air, has been used in musical instruments from early days. In the " Pipes of Pan " musical sounds were produced by blowing across the tops of hollow tubes of varying lengths, and Aristotle (384–322 B.C.) hac some correct ideas on the motion of air in a pipe.

In all the instruments in which air vibrates, the laws of vibration are the same, and it is convenient to consider first the simple case of a column of air set into vibration by a tuning-fork. Let us take the case of a tube containing air and closed at one end. Use a cylindrical glass tube, about 50 cms. long and 3–7 cms. in diameter supported by a clamp so that it can be moved up and dow in a tall glass jar which is nearly filled with water. I a C tuning-fork ($n=256$) be sounded and held just abov the open end of the tube which is then moved up an down, its other end being in the water in the jar, it wil be found that in one position (approx. 30 cms. of tub outside the water) the sound of the tuning-fork is reinforce by the vibrations of the column of air in the tube, and th sound is loud—the tube " sings out " in this one positior

Consider what happens : As the prongs of the tuning fork move outwards (above the open end of the tube) wave of compression is sent down the air in the tube This must be reflected back from the closed to the ope

end. Here its emergence results in the formation of a region of low pressure just inside the end, and this travels down the pipe as a wave of rarefaction carrying the energy of the previous wave down the tube. *It is found that very little energy is lost from the tube.* If the length of the tube is of a certain value, this rarefactional wave starts to travel down exactly at the same moment as the rarefaction wave, due to the prong of the tuning-fork returning inwards, starts to travel down, *i.e.* there is reinforcement (greater amplitude produced). The combined wave of rarefaction travels down the tube, is reflected at the closed end, and returns to the open end where, on emergence, it gives rise to another compressional wave which travels down the tube. The length of the tube being correct (as already shown), the prong of the tuning-fork above its open end at this very moment moves outwards again, so setting up a further compression wave, which amplifies the compression wave just starting to travel down the tube. In this manner there is a rapid building up of the vibration of the air in the tube and so "resonance" is obtained, energy then being radiated as sound waves of appreciable amplitude.

From this it is seen that in one period of vibration, in one complete to-and-fro motion of the tuning-fork, the wave travelled up and down the tube (length l) twice, *i.e.* a distance of $4l$.

Thus the wave-length, λ, of the vibration $=4l$,

$\therefore$ Velocity of wave-motion $=V=$ frequency $n \times \lambda = 4ln$.

It is found in practice, however, that l is *not* the exact length of the air column which is in vibration, but a little of the air outside the tube is also vibrating. Thus the real length of the column vibrating $=l+c$ and the wave-length $=4(l+c)$.

Thus $V=4n(l+c)$, where c is a quantity called the **end-correction** for the tube. It equals 0·6 of the radius of the open end of the tube, approximately, and we shall shortly see how to get its value exactly.

In the case of a tube *open at both ends* a compression wave at one end gives rise to a rarefactional wave at the

other end as it emerges. This, in turn, gives rise to a compression wave at the first end on emergence. Thus, for resonance, the length (l) of the tube must be such that the period of vibration of the tuning-fork (or outside source) must be the time during which the wave travels up and down the tube, *i.e.* $\lambda=2l$ approx.

In the case of the open tube, there is an end-correction for each end, and so the effective length of tube$=l+2c$.

Thus $\lambda=2(l+2c)$, and so $\underline{V=n\lambda=2n(l+2c)}$.

It should be noticed that for resonance with the same tuning-fork the length of the open tube must be approximately twice that of a closed pipe, *i.e.* a tube closed at one end and nearly 30 cms. long and an open tube of nearly 60 cms. will both resound to fork C ($n=256$). This principle of simple resonance of an air column is the basis of a laboratory method of measuring the **velocity of sound,** known as the **resonance method.** It is usually carried out with a tube closed at one end, chiefly because of the shorter length required. It is also a little more convenient to alter quickly the end of a "closed" or "stopped" tube, as we call it. The relation obtained above, $V=4n(l+c)$, is used and so standard forks of known frequencies must be provided, and also C must be determined. The most precise method is to find a value for l for a series of tuning-forks, *e.g.* for the octave series C, D, E, ... C′. For this experiment, the tube described, moved up and down in water, can be used, or a special apparatus, something like a modern Boyle's Law tube, is suitable. As shown in Fig. 243, both ends are open and one tube can be moved up and down to vary the position

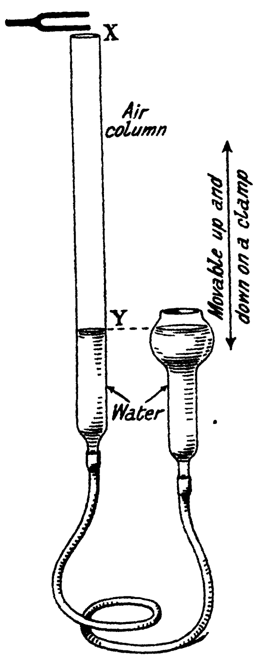


FIG. 243.—Resonance Apparatus.

of the level of the water, and hence the length of air column in the tube, above which a tuning-fork is held The results should be tabulated as follows :

Fork Frequency n		$\frac{1}{n}$ (Ratio)	Length of air column XY for resonance (l cm.)	End-correction, obtained from graph $=c=$ cms.	$l+c$	$V=4n(l+c)$
C	256	180				
D	288	160				
E	320	144				
F	341·3	135				
G	384	120				
A	426·7	108				
B	480	96				
C′	512	90				

Mean value for V= ________

This value can be corrected for temperature (read in the laboratory) to 0° C. (see p. 398).

<u>*To obtain c*</u>, a graph of l against $1/n$ should be plotted (Fig. 244) on a large scale.

Now since, for the temperature, V is constant and $=4n(l+c)$,

$$\frac{1}{n} \propto (l+c)$$

Thus $1/n$ plotted against l should give a straight line, cutting the $1/n$ axis above zero (at A).

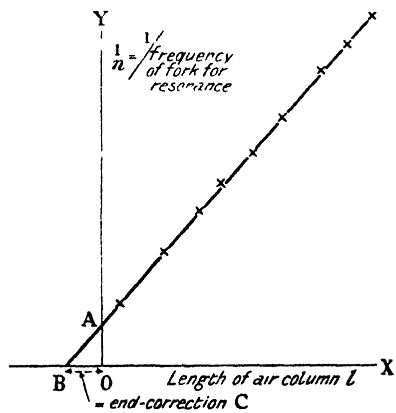


FIG. 244.—Graph of Results from Resonance Tube Experiments, to Measure the End-correction, c.

The best straight line possible is drawn through the mean positions of the points plotted. Suppose it cuts OY at A and is produced to meet OX (the l axis) produced at B.

Then, since $1/n$ must be 0 when $l+c=0$, B must be the true origin for the graph of $1/n$ against $l+c$, and so OB *must equal the end-correction.* The effective length of air column can be measured from B along BX, instead of from O.

This value of c can be introduced in the table above, and the velocity of sound calculated.

A second method of obtaining the end-correction is from the octave positions.

Suppose with fork frequency 256, length of air column for resonance $=l_1$,
and with fork frequency 512, length of air column for resonance $=l_2$.

Then, in case 1, $V=4n(l_1+c)=4\times256(l_1+c)$,
and, in case 2, $V=4n(l_2+c)=4\times512(l_2+c)$,

$$\therefore\ l_1+c=2(l_2+c)$$

$$\text{or}\quad c=l_1-2l_2.$$

The mode of vibration described, both for the closed and the open pipe, corresponds to the lowest, or fundamental note of the length of air column set in vibration Tuning-forks are often fixed into "resonance boxes" which have the necessary volume of air to resound to, or reinforce, the pitch of the fork. Thus the boxes of higher frequency tuning-forks are smaller than those required for forks of lower frequency.

But other modes of vibration are possible, exactly analogous to the stationary vibrations in strings, except that in the latter they are transverse, whereas in air they are longitudinal. The case of a tube of air closed at one end is analogous with that of a string clamped at one end. The principle of obtaining the reflected and resultant waves is illustrated in Fig. 245 (and is similar in principle to that described on pp. 430, 431).

Considering the whole series of positions for an air column as analogous to the series given for a string on p 431, it is seen that in a vibrating tube there will be

a series of points, which must be half a wave-length apart, where the displacement is always zero, and these points are alternately centres of condensation and rarefaction ; these points are *nodes*. The particles elsewhere

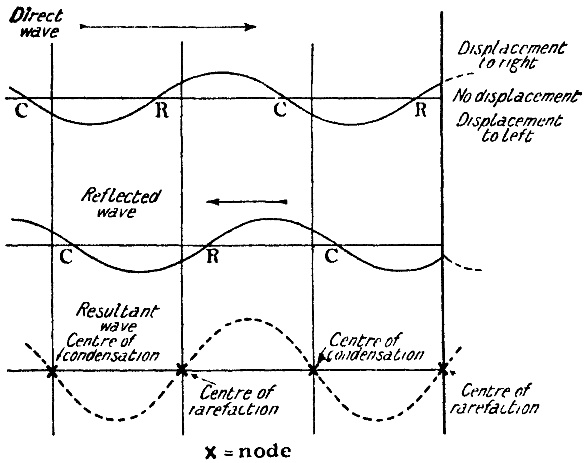


FIG. 245.

vibrate to and fro longitudinally, and there is a train of *stationary longitudinal waves* which consist of vibrating loops or segments separated from each other by the equidistant nodes. The points half-way between nodes are points at which the particles of air undergo maximum displacement, and are called *antinodes*.

Thus we see that a column of air may be set into vibration so that it encloses any number of loops, or segments :

(*a*) *Open tubes* (*i.e.* open at both ends) must have positions of maximum displacement, *i.e.* antinodes, at both ends ;

(*b*) *Closed tubes* (*i.e.* open at one end only) must have a position of maximum displacement, *i.e.* an antinode, at the open end, and a position of no displacement, *i.e* a node, at the closed end.

We represent diagrammatically the fundamental

modes of vibrations in the two cases of resonance as follows (Nodes N, Antinodes A):

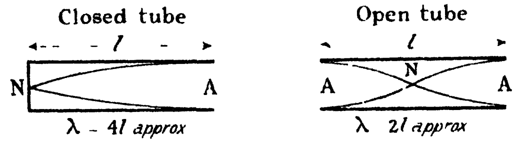


FIG. 246.

Some of the other modes of vibration are seen from the following, using the same method of indicating loops, etc.:

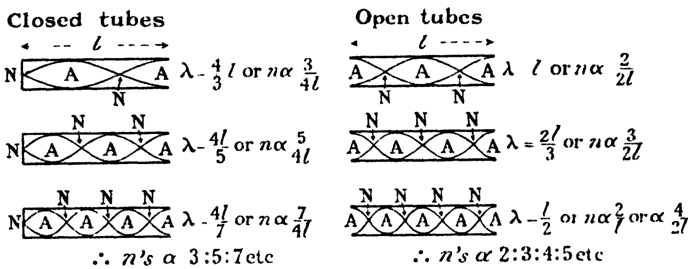


FIG. 247.

Thus it is seen, from these figures, that harmonics (or overtones) are produced by setting the air of the tube into vibration in other modes.

It also follows that in **the case of an open tube, or pipe, any harmonic may be produced; whilst in the case of a closed pipe only the odd-numbered harmonics are possible.** We shall mention later an effect on the quality of musical instruments which consist of air tubes or pipes.

The second harmonic (next above the fundamental) can be obtained with an ordinary open pipe by blowing hard. This is found in a tin whistle, for example, where ordinary blowing will enable the ordinary scale of notes to be obtained, and over-blowing will enable the octaves of these notes to be played. The bugle is another example of this.

To show the overtones, or harmonics, in a closed pipe, continue the experiment described on p. 444. Hold the tuning-fork ($n=256$) above the glass tube in the resonance position and gradually raise both. The sound will die away. Continue lifting both and eventually you will find another position of resonance (much weaker than the first), if the tube is long enough. To secure sufficient length, fit inside the glass tube one made of thin cardboard, or stiff paper, which can be pulled out on the telescopic principle. From Fig. 248 it is seen that the length of

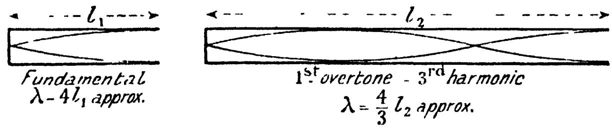


FIG. 248.

the tube for the second position of resonance is three times that for the fundamental, and the third harmonic is obtained (three loops instead of one loop).

From this, the wave-length $\lambda=2(l_2-l_1)$ and so $V=n\lambda=256\times 2(l_2-l_1)$, *thus eliminating the need for measuring the end-correction* (c).

It can, however, be measured if required, for in the case of the fundamental, $\lambda=4\times\text{length}=4(l_1+c)$, and in the case of the harmonic, $\lambda=\frac{4}{3}\times\text{length}=\frac{4}{3}(l_2+c)$.

These must be equal, and so $\dfrac{l_2+c}{3}=l_1+c$,

$$\text{or} \quad c=\frac{l_2-3l_1}{2}.$$

A similar experiment can be carried out for an open

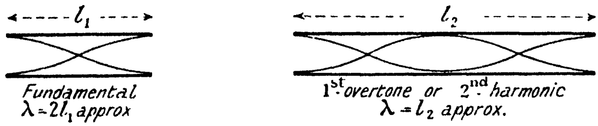


FIG. 249.

tube, by having a telescopic tube, adjusted till the positions of resonance are obtained. From Fig. 249 it is seen that

the first overtone (second harmonic) is obtained with a tube twice as long as for the fundamental. The student should calculate the velocity of sound without obtaining the end-correction. Further harmonics would, of course, be obtained with the open tube at lengths approximately proportional to 3, 4, 5, etc., times that of the fundamental length.

Experimental Investigation of Air Vibration in Pipes.—The conditions in a pipe or tube in which the air is set in vibration in these different modes can be investigated as follows:

1. *Savart's Method.*—For this it is preferable to use a glass tube set into vibration (see organ pipes, a little later) so that it is possible to see inside. Into the pipe are lowered, by light strings, small wooden rings on which parchment paper is stretched and fastened, having small particles of sand resting on the horizontal "drums" so formed. At positions of nodes there is no motion of the air, and hence of the sand, but at other positions the air in vibration strikes the parchment and so sets the sand jumping; at antinodes this movement is most rapid and intense.

2. *Koenig's Manometric Flame Method.*—Several small holes are bored at different positions along a wooden pipe which can be sounded, and then are covered with thin indiarubber. Fitted over each of these is a small air-tight box, which acts as a small gas-chamber, supplied with gas from outside and leading to a small gas-jet which can be lighted. Variations of pressure in the pipe are communicated to the indiarubber membranes so causing variations in the flames. A steady flame indicates no air movement (or node), a flickering flame indicates air movement. The rapid changes in the flames are observed by looking at their images in a set of mirrors on the four vertical sides of a box which is very rapidly rotated about a vertical axis. If the flame is steady a broad band of light is seen in the mirrors; if not, the band has a serrated edge like a saw.

3. *The Dust-Tube Method.*—If a glass pipe, or tube, is held horizontally, and it can be set into vibration or

"blown," the conditions inside can be seen if light particles of sand, or better still, cork dust or lycopodium powder, are lightly distributed along the bottom of the tube beforehand. The particles leave the antinodes and collect in ridges at the nodes. (See Kundt's dust-tube experiment on p. 452.)

Longitudinal Vibration in Rods.—Longitudinal vibrations of solids can be produced by rubbing rods of woods, or metals, with a resined cloth, or glass rods with a damp cloth. They give a musical note if the pressure exerted, a matter of experience, is correct. A great advantage of such a longitudinal vibration is that there is no end-correction to consider.

The case of a rod clamped at one end is analogous with that of a closed pipe. The fixed end must be a node and the other end an antinode. Thus, when vibrating with its fundamental note the wave-length λ must $=4\times$length of rod, and so $\underline{V=4nl}$ (usual symbols).

Thus the velocity of sound in the rod can be calculated, if the frequency, n, of the note emitted by it is known. This can be found by tuning to it a siren, Savart's wheel, or a sonometer wire.

E.g. suppose that a sonometer wire is 100 cms. long when giving a note of $n=256$, and, when a certain glass rod 200 cms. long is vibrating, the length of sonometer wire to give unison with its note is 40 cms.

Then frequency of the note $(n \propto \frac{1}{l}) = \frac{100}{40} \times 256 = 640$

But $V=4nl=4\times 640\times 200$ cms. per sec.

$\therefore$ Vel. of sound in the glass$=\underline{51{\cdot}2\times 10^4 \text{ cms. per sec.}}$

By holding such a rod at one-third its length from its free end, the note an octave above (*i.e.* the first overtone, or second harmonic), can be obtained. The mode of vibration is then analogous with that in Fig. 248.

A rod, clamped at its centre and rubbed, behaves like an open pipe, having an antinode at each end and a node in the middle. The wave-length of its fundamental mode of vibration is then twice its length (*i.e.* $2l$).

Thus the velocity of sound in the rod$=2nl$.

It follows then, that the rod when clamped at the centre must give a fundamental note which has twice the frequency of that emitted when it is clamped at one end. Moreover, with a rod clamped at the centre, all harmonics are possible, as with an open organ pipe. The second harmonic (first overtone) can be obtained by holding the rod one-quarter of its length from the end and by rubbing the shorter length.

By finding the velocity of sound, as above, in a rod of known density, its elasticity can be calculated from

$$\text{velocity of sound} = \sqrt{\frac{\text{Elasticity}}{\text{Density}}} \text{ (see p. 395).}$$

Kundt's Dust-Tube Method for finding the Velocity of Sound in Gases, etc.—The original form of Kundt's apparatus for measuring the distance between nodal points in the air vibrating in a glass tube is shown in Fig. 250. The tube, 4–6 ft. long and about $1\frac{1}{2}$–2 ins.

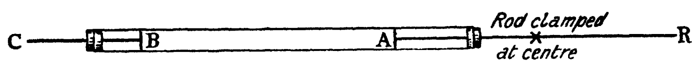


FIG. 250.—Simple Kundt's Dust Tube.

diameter, was fitted with rubber bungs. Through one was fitted a short rod BC, carrying a plate B, which could be moved part of the way along the glass tube. Through the other was a long glass or metal rod AR, clamped at its centre, carrying a similar plate A. These two plates, inside the tube, were slightly less in area than the internal cross-section of the tube. The long rod could be set into longitudinal vibration by being rubbed at the free end. Thus plate A was set vibrating and caused the air in the tube to vibrate. By adjusting the position of plate B, there was obtained a length of air column, AB, sufficient to be set into resonant vibration. Thus, the lycopodium powder, cork dust, or light sand lightly distributed along the bottom of the tube before starting the experiment, collected in distant ridges at nodes (points of no displacement). The wave-length of the vibration could be measured, since it equalled twice the

distance between adjacent nodes (the mean value being taken). The frequency of the note emitted was determined by sonometer or siren, and the velocity of sound in air was calculated. At the same time, by the method previously described, the velocity of sound in the rod could be determined.

The tube was then carefully filled with a gas, care being taken to drive out all the air, and the velocity of sound in the gas similarly found. Owing to the time elapsing between experiments, the temperature often changed, thus entailing a correction. Again, the pitch of the note emitted changed with temperature, and so it was necessary to determine the pitch each time. To avoid this, a combination form of apparatus was devised (Fig. 251) in which the rod, stroked down the middle,

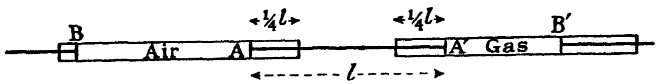


FIG. 251.—Kundt's Method for Determining Velocity of Sound in a Gas.

was clamped at two points each one-quarter of the length of the rod from each end. The stoppers, which acted as clamps, were made of layers of indiarubber tied with silk. Two tubes, one containing air and the other the gas under consideration, were used at once. In this case the rod vibrated longitudinally, with antinodes at the ends and in the middle, and nodes at the two positions of clamping. Thus the wave-length of vibration = the length of the rod (AA′). The experiment was carried out as with the earlier form except that the two experiments were amalgamated.

Also

$$\frac{\text{Vel. of sound in a gas}}{\text{Vel. of sound in air}} = \frac{\text{frequency } (n) \times \text{wave-length } (\lambda_1)}{\text{frequency } (n) \times \text{wave-length } (\lambda_2)}$$

$$= \frac{\lambda_1}{\lambda_2} = \frac{\text{distances between nodes in tube containing the gas}}{\text{distances between nodes in tube containing air}}.$$

Thus, when the velocity of sound in air was accurately measured, the velocity of sound in a gas could be quickly calculated from the above without making any corrections

for temperature, etc. This experiment has been very valuable, for it has given a method of determining for a gas the constant γ, the ratio

$$\frac{\text{specific heat of a gas at constant pressure}}{\text{specific heat of a gas at constant volume}}$$

for from the equation previously mentioned on p. 396, velocity of sound in a gas $=\sqrt{\frac{\gamma \cdot \text{Pressure}}{\text{Density}}}$. For some gases this has been the only convenient method of obtaining the required constant γ.

The apparatus has also been used to measure the velocity of sound in a liquid, using a tube of the liquid and having a layer of iron filings, or precipitated silica, as an indicator of nodes.

Organ Pipes and Wind Instruments.—Air columns set into vibration are used for many musical instruments, the organ being the most important. In this instrument many pipes, or tubes, are used and in them an air blast (from bellows) is produced. To set the air in a pipe in vibration, the blast is made discontinuous, and thus acts as a series of puffs,

(1) by making it impinge on an obstacle, in a *flute*, or *flue pipe*;
(2) by means of a vibrating tongue in a *reed pipe*.

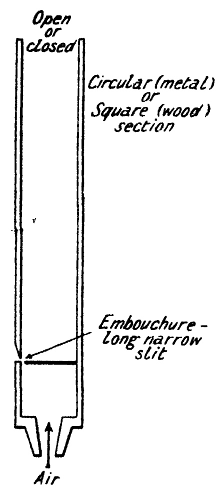


FIG. 252.—Flute Organ Pipe.

The first type is illustrated in Fig. 252. The blast of air strikes the thin edge of the pipe near the *embouchure*, the narrow slit which serves to keep the air column of the pipe open to the air (so making the embouchure end an open end). Thus the air blast travels out in puffs, and when the length of the organ pipe is a suitable value the air in it is set in resonant vibration. To tune an organ pipe, its length has to be adjusted to the correct

value, since that controls the pitch of the note emitted. Open pipes are tuned either by means of a telescopic upper part, or by means of a hole, with a movable door over it near the upper end. The plug at the end of a closed pipe is movable and so is used for altering the length in tuning it.

A reed is used in many pipes in organs, the reed being usually a narrow piece of thin metal, fixed at one end, which covers, or almost covers, the opening from the air-chamber into a pipe (or into the open air in the case of such instruments as the mouth-organ and concertina in which reeds are used). If the reed is larger than the opening it is called a *beating reed* (Fig. 253), if smaller it is called a *free reed*. The action of a free reed is practically to close the opening once in each of its vibrations to and fro. A beating reed, owing to its curved shape, gradually covers the hole, once in each vibration, and so there is less of the harshness usually produced by a sudden complete stoppage of the air blast. Beating reeds are always used with pipes; free reeds are used with pipes and also in instruments, such as the harmonium and concertina, where no pipes are used. A reed pipe is tuned by varying the degree of freedom of the movement of the reed, since this vibration settles the frequency, *i.e.* pitch, of the note emitted. The pipe merely acts as a resonator, and so has an effect on the quality of the note emitted. In reed pipes more harmonics are produced than in flute pipes.

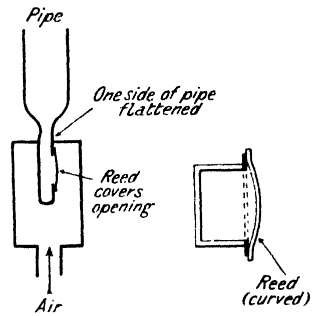


FIG. 253.—Beating Reed as used in Organ Pipes.

Wind instruments, such as the clarionet, have a single bamboo reed. The clarionet is an open pipe the length of which is altered by closing or opening various valves. The saxophone is a metal clarionet with a flat-shaped reed.

The oboe, English horn and bassoon have double reeds, the length of the tube in use fixing their pitch.

Brass instruments such as the cornet and French horn have no reed ; the lips of the player vibrate, acting as a double reed. The adjustment of the lips to the instrument, their tension, and the wind-pressure exerted decide which of the various notes natural to the instrument are sounded, but the length decides the exact pitch. The length of the cornet can be altered by a sliding pipe, but in the case of the horn and bugle, which are fixed in length, the vibration of the lips has to be altered to obtain the different notes. In ordinary whistling the two lips act as a double reed. Here the pitch-controlling agent is the tongue, by means of which the cavity of the mouth is varied in length. This cavity acts as a resonator.

In the case of large instruments variations in the temperature of the outside air affect their pitch For since $V=n\lambda$, $n\propto V$.

Also $V\propto\sqrt{\text{absolute temperature}}$,

and so $n\propto\sqrt{\text{absolute temperature}}$. This, of course, is why organ pipes need to be tuned.

The instruments in an orchestra have to be carefully tuned before a concert, and often during the concert as well owing to the rise in temperature in the room.

In the case of small instruments the pitch depends on the temperature of the breath of the player, and is hardly affected by the temperature of the outside air.

Determination of Frequency.—The following is a summary of the methods by which the frequency of a note can be determined:

(1) The *stroboscopic method* for the note given by a tuning-fork, using a revolving drum. This method is described on p. 415.

(2) The *method of beats*. This is explained fully on pp. 424, 425.

(3) The *sonometer method*. The length of sonometer wire, used at known tension, is adjusted by a bridge till it will give the same as that whose frequency is to be measured. The frequency is then calculated from the usual formula. (See pp. 433, 434.)

(4) The *resonance tube method*. The length of the column of air in a resonance tube is adjusted till the air

resounds to the note whose frequency is to be found. If the velocity of sound at the temperature is known, the frequency can be determined if the length of the column of air is measured. (See pp. 443, 444.)

The Recording and Reproduction of Sound.—This subject has become of great interest in recent years. The first instrument of real importance was the *phonograph*, invented by the famous scientist and inventor, Edison, in 1877. He possibly got the idea from a device of an Englishman, Scott, who about 1860 tightly stretched a membrane over the narrower end of a vessel similar to a Helmholtz resonator. To the centre of the membrane he fixed a light rod carrying a blunt pen. When sounds were made externally the air in the resonator, the membrane, the rod and the pen all vibrated. The latter was thus caused to indicate its movement in wavy lines on a sooty surface of a revolving cylinder pressing against it. Edison first formed a "record" of sounds and then afterwards reproduced them. For this a mica diaphragm, tightly held at the rim, carried a style, or pen, with a point which pressed on a tinfoil sheet held on a rotating drum. Sound waves caused the diaphragm to move to and fro, so tracing indentations in the tinfoil. By starting from the beginning and rotating the drum, the point was caused to move over these indentations and so caused the diaphragm to repeat its previous movements, thus reproducing the sounds. Very soon the tinfoil was replaced by a wax cylinder, and later ebonite and compositions were used. In the so-called *gramophone*, discs took the place of cylinders, the indentations being made sideways in spiral grooves. The pitch of the sound reproduced can be altered by varying the speed of movement of the disc. But this variation of the speed causing a variation in the pitch of the fundamental notes is accompanied by a change in the pitch of the harmonics ; hence the quality of the sounds is affected. Thus makers of gramophones and records state the frequency of rotation necessary to ensure correct quality of reproduction. The intensity of the reproduction is controlled by the pressure on the record —due to the kind of needle used, its length, etc. Many

improvements have been made in reproduction in recent years. These, which include the method of "electrical recording," are outside the scope of this book.

Such instruments as the telephone (Graham Bell, 1876), carbon transmitter (Edison, 1877), and the microphone (Hughes, 1878), are all important in this connection; they are dealt with in books on electricity.

A very recent development is the incorporation of sound in cinematograph films. This is based on the action of light producing an electrical effect which is then used to generate sound (and *vice versa*). Sounds acting on a microphone set up electric currents. These act upon an electrical instrument called the galvanometer, the movements of which are made to control the width of a band, or beam, of light which is photographed at the same time as the film is made. (This is mostly done on the edge of the film which is picturing the relevant scene; if not, the two separate films made have to be synchronised when displayed, a difficult process to carry out.) When the film is projected a beam of light is made to pass through the band (developed at the same time as the picture film), and so it varies in intensity. These variations in intensity set up varying currents in a "photo-electric cell" (see p. 163). These are amplified and then allowed to actuate a loud speaker, which thus reproduces the original sounds made. A single motor, suitably geared, drives the whole projecting mechanism, so that if a film is started aright the sound heard and the picture seen must keep together in correct sequence (*i.e.* in step). It is very interesting to note, however, that in printing a sound-film the sound band must be printed several inches in front of the correcting light-film portion. Any film passes in front of the projecting source of light in jerks (24 or so per second). The sound band must pass in front of the light beam, which affects the photo-electric cell, in a uniform non-jerky motion in order that sounds may be true and uniform. A small time-interval difference is necessary, therefore, and the distance between the light and sound portions depends on the type of reproducer used. Owing to the rapid development of "Talking Pictures,"

only a bare outline of one method has been given. Much progress is likely to be made in the near future which may possibly alter the whole system of recording and reproducing.

Finally, one hopes that by now the reader has realised how many of the modern devices for human welfare, as well as "popular crazes," are developments of inventions resulting from scientific research.

Exercises on Chapter XXIV

1. Describe a method of adjusting the length of a narrow column of air so as to obtain maximum resonance with a tuning-fork of known frequency, and show how the velocity of sound in air may be determined in this way. [*L.M.* 1920.]

2. Describe and explain the resonance tube method of determining the velocity of sound in air. Discuss whether the distance required should be measured from the water surface to the end of the tube or to the prong of the fork. [*L.M.* 1926.]

3. Explain, and carefully account for, the difference in the mode of vibration of the air at the middle point of an organ pipe sounding its fundamental note, (*a*) if the pipe is open, (*b*) if it is closed; and show how this affects the pitch of the note given out.

4. Describe the motions arising when a tuning-fork sets up a system of progressive sound waves in air; and explain how such motions differ from the stationary waves occurring in a resonance tube.

5. Explain how the velocity of sound in a gas depends upon the temperature. If the column of the air in a tube open at both ends has an effective length of 32 cms. and resounds most readily to a tuning-fork of frequency 520 when the temperature of the air is 15° C., what is the velocity of sound in air at 15° C. and at 0° C.?

6. Describe a siren and explain how it may be used to find the frequency of a tuning-fork. If there are 32 holes in the disc, which makes 1,050 revolutions per minute, what is the frequency of the note emitted by the siren? What would be the length of an open organ pipe which, sounding its fundamental, emitted the same note? (Assume the velocity of sound in air to be 1,120 f.s.)

7. Explain how to find the frequency of a tuning-fork by resonance. In what way will your result be affected (*a*) by a fall of the barometer from 30 ins. to 29 ins., (*b*) by a rise of temperature from 10° to 20° C.? [*L.M.* 1922.]

8. How may the pitch of an organ pipe be measured by a siren? A disc siren, 16 holes, revolving 1,500 times per minute, gives 9 beats in 4 seconds with an organ pipe. If the speed is increased to 1,515 times per minute, there are 7 beats in 4 seconds with the same pipe. Find the pitch of the note given by the organ pipe.

9. Give two methods of determining the frequency of a tuning-fork. [*L.G.S.* 1918.]

10. A glass tube is closed at one end by a piston whose position in

the tube can be altered. A tuning-fork is held near the mouth of the tube and resonance occurs when the piston is placed (*a*) 15 cms., (*b*) 46 cms. from the open end. Describe carefully how the air in the tube is vibrating in each case. [*L.M.* 1921.]

11. What is meant by an *overtone?* Give an account of the first three overtones which may be present in a vibrating air column closed at one end. How could you demonstrate the existence of overtones when a musical note is sounded (*e.g.* when a note is struck on a piano)? [*L.M.* 1920.]

12. What effect is produced on the frequency and quality of the note given by an open organ pipe if the top is suddenly closed? If the frequencies of the first overtones of the two notes so obtained differ by 440, what was the original frequency? [*L.M.* 1919.]

13. What are the causes of *pitch* and *tone* (*quality*) of a musical note? Two 4-ft. organ pipes, one closed at one end and the other open at both ends, are sounded on a day when the speed of sound in air is 1,120 ft. per second. Calculate the frequency of the fundamental note in each case. [*J.M.B.* 1927.]

14. Describe a method of finding the velocity of sound in air. What simple experiments could you make to test whether sound travels with the same velocity in a solid as in air? [*J.M.B.* 1928.]

15. An open organ pipe emits a fundamental note of frequency 256 when sounded in air. Assuming the velocities of sound in air and coal-gas to be 350 and 500 metres per sec. respectively, find the pitch and wave-length of the note emitted by an organ pipe when sounded in an atmosphere of coal-gas.

16. A vibrating tuning-fork is held near the open mouth of a glass tube in which a piston slides. Describe and explain the effect observed as the piston is moved gradually so as to increase the length of the air column. What would be the effect of replacing the air in the tube by coal-gas, a medium in which sound travels more rapidly? [*L.G.S.* 1922.]

17. Show clearly how it is possible for speech, with all its variations of pitch and vigour and tone, to be produced by a gramophone and transmitted to the ear of the hearer. [*J.M.B.* 1927.]

MISCELLANEOUS EXERCISES—III. SOUND

1. Draw a diagram showing the passage of transverse waves along a line of particles, and by the help of your diagram indicate the meaning of the words *amplitude*, *phase*, *period*, *wave-length*, *frequency*. [*L.M.* 1922.]

2. What is the connection between wave-length, frequency and speed of sound? Indicate briefly how you could demonstrate the truth of your statement by experiments carried out in the open country and in the laboratory. [*J.M.B.* 1926.]

3. What are the main characteristics of wave-motion? Point out the chief resemblances and differences between waves of sound and waves of light. [*L.M.* 1921.]

4. Define the terms frequency, amplitude and wave-length, and describe how the frequency of a tuning-fork may be found by experiment without assuming a knowledge of the velocity of sound in air. [*L.G.S.* 1919.]

5. Describe the changing conditions of a mass of air traversed by a succession of sound waves that proceed from a regularly vibrating source. How does the velocity of such waves depend upon the temperature, pressure,

and hygrometric condition of the air? Mention some observation that indicates that the velocity of a sound wave is independent of its length, and of its amplitude of vibration.

6. Explain how sound is propagated through air and describe how the velocity with which it travels may be measured. [*L.M.* 1927.]

7. An observer sets his watch by the sound of a gun fired at a fort 1 mile distant. If the temperature of the air at the time is 15° C., what will be the error? Mention other causes which are likely to lead to errors in the setting. (Velocity of sound in air at 0° C.=1,090 ft. per sec.) [*L.M.* 1928.]

8. Define *amplitude* and *wave-length*. How would you determine the wave-length in air of the note emitted by a tuning-fork? [*L.M.* 1921.]

9. Describe any two methods of measuring the velocity of sound in air.

10. Describe an experiment for showing a relation between the pitch of a note and the frequency of the vibration which causes the note. [*L.G.S.* 1926.]

11. Describe concisely two methods of measuring the velocity of sound in air; one a direct method suitable for carrying out in the school playing field, the other an indirect one to be carried out in the laboratory. What difficulty are you likely to meet in the first method? Suggest ways of overcoming it. [*L.M.* 1923.]

12. Describe an experiment to show that the difference in pitch between two notes depends only on the ratio of the frequencies of the vibrating sources. If you are given two notes whose vibration frequencies are 162 and 512, what is the vibration number of the note whose pitch is midway between them? [*L.M.* 1928.]

13. How do you explain why audible notes from different sources can generally be distinguished one from another, even when they have the same intensity of pitch? Describe the experiments you would perform in order to demonstrate the correctness of your answer.

14. Define the terms *wave-length* and *frequency* of a note, and state what relation exists between them. What is (approximately) the wave-length in air at 15° C. of the note emitted by a tuning-fork marked $n=256$? Indicate *briefly* an experimental method by which you could verify the result. (Velocity of sound at 0° C.=1,090 f.s.) [*L.G.S.* 1928.]

15. When the length of a stretched string is halved the note it gives on plucking is changed to the octave. Why do you conclude from this that the vibration frequency has been doubled? What alteration in tension will be necessary to regain the original note? [*L.M.* 1925.]

16. How would you verify with a sonometer the law connecting the frequency of a stretched string with its tension? If an additional 75-lb. wt. raises the pitch an octave, what was the original tension? [*L.M.* 1923.]

17. Explain how you would carry out, record and deduce a law from experiments on the relation between the length of a uniform wire stretched by a constant force and the note it emits when plucked. Two strings are in tune with one another. If the tension of one is increased to three times its original value, how must the length of the other be altered for them still to be in tune? [*L.M.* 1923.]

18. Describe an experiment to show that the frequency of transverse vibrations of a stretched string is proportional to the square root of the stretching force. When the tension of a string is increased by three kilograms weight, the frequency is altered in the ratio 9 : 8. What was the original tension? [*L.M.* 1921.]

19. Describe the experiments you would make to find out the relation between the length of a stretched string and the frequency of the note it gives when plucked or bowed. A stretched string 1 metre long is divided by two bridges into three parts so as to give the notes of the common chord whose frequencies are in the ratio 4 : 5 : 6. Find the distance between the bridges. [*L.G.S.* 1927.]

20. The frequency of a note given by a silver wire 1 mm. diameter and 1 metre in length is 256 per sec. Determine the tension of the wire. Density of silver is 10·5 grms. per c.c.

21. Define and explain the terms frequency, amplitude and wave-length as applied to sound waves in air. What are the differences in the sensations perceived which correspond to differences in these quantities? The shortest wave-length that is audible is about 1·8 cms. and the longest about 900 cms. What is the frequency in each case? What is approximately the number of octave intervals between them? (Velocity of sound in air=33,000 cms. per sec.)

22. Describe and *explain* the resonance tube method of finding the wave-length in air and the frequency of the note emitted by a tuning-fork. The velocity of sound in air may be assumed. [*L.G.S.* 1925.]

23. Describe a siren. If there are thirty-two holes in the disc of a siren which makes 1,575 revolutions per minute, what is the frequency of the note it emits? What would be the length of a closed organ pipe, which, sounding its fundamental, was in unison with it? (Velocity of sound in air=1,120 f.s.)

24. Explain how the positions of the nodes and antinodes in an organ pipe can be found experimentally. Describe the ways in which the air vibrates in pipes open at both ends.

25. Describe two distinct methods for *comparing* the frequencies of two tuning-forks. Show exactly how the numerical result is to be obtained, and point out, in one case only, what additional data would be needed in order to determine the actual frequency of each fork. [*L.G.S.* 1920.]

26. When a vibrating tuning-fork is held over the mouth of a tube which can be raised or lowered in water the sound of the fork appears to increase when the mouth of the tube is at a certain height above the water. Explain this, and show how this phenomenon may be used to determine the velocity of sound in air. [*L.G.S.* 1923.]

27. Describe a laboratory experiment to determine the velocity of sound in air, being given a tuning-fork of known frequency. How does *the velocity depend upon the temperature?* What effect would you expect change of temperature to produce on the pitch of musical instruments? [*L.M.* 1928.]

28. What is resonance? Explain in detail a resonance method of determining the velocity of sound in carbon dioxide. [*L.G.S.* 1919.]

29. If an organ pipe gives a note of frequency 256 when the temperature of the air is 30° C., what will be the frequency of the note when the temperature falls to 15° C.?

30. The note emitted by an organ pipe has the same frequency as that of a siren with a disc containing twelve holes and making 1,000 revolutions per min. If the velocity of sound in air is 1,100 ft. per sec. what is the length of the organ pipe? How nearly would you expect the measured length to agree with that calculated?

31. A motor-car travelling at 45 m.p.h. approaches a stationary observer. The horn, blown by the driver, has a frequency of 500 per sec. What is the pitch of the note of the horn heard by the observer?

32. From what familiar observations would you conclude (1) that light travels more rapidly than sound, (2) that sound travels more rapidly than an express train? Describe some method by which you might show, without the aid of any specially constructed apparatus, that light travels more than a mile in a second.

33. Compare and contrast in two parallel columns the chief facts concerning the propagation of light and sound. [*L.G.S.* 1920.]

34. What are the chief characteristics of a musical sound? How do these figure in a gramophone record? [*L.M.* 1924.]

35. Explain how the air moves when a sound wave is travelling through it. Describe an experiment to show that the wave-length of a musical note in air is inversely proportional to its frequency. [*L.M.* 1929.]

36. A long cylindrical tube open at one end is filled with water, a vibrating tuning-fork is held over the open end, and the level of the water is gradually lowered until the second level at which resonance occurs is reached. Describe the motion of the air in the tube when the water is at this level and the column of air is resounding. Explain how the frequency of the note given by the tuning-fork can be obtained from such an experiment. [*L.M.* 1930.]

ANSWERS

ANSWERS

SOUND

EXERCISES 21, p. 407.

4. 280 ft.
6. $\frac{10}{11}$ sec.
7. 1680 ft.
10. 19·8 ft. per sec. approx.; 17·1 ft. per sec. approx.
16. 0·633 mile.

EXERCISES 22, p. 427.

2. 360.
4. 560.
8. 576; A major tone above the octave of 256.
11. 520; 367·7.
12. 626; $533\cdot\dot{3}$.
13. A fall in frequency of 9·62 per sec.

EXERCISES 23, p. 439.

6. Reduce to $\frac{\sqrt{2}}{2}$ (or 0·707) of its previous length.
7. $1\frac{2}{3}$ kilograms wt.
8. 2 kilograms wt.
9. Make tension $2\frac{1}{4}$ times as great.
10. 9, 12 and 18 ins.
11. 0·81 of the former tension.
13. (1) Reduce length to two-thirds its value ($33\cdot\dot{3}$ cms.). (2) Increase tension to $2\frac{1}{4}$ times, *i.e.* to 22·5 kilograms wt.
14. 49·5.
16. Fork 288; wire 290 and 286.
17. Diameters, A : B $= 1 : \sqrt{3}$; 1·195 : 1.
18. Make it half as long again.
19. 1·2 : 1.
20. $22\cdot\dot{2}$ lbs. wt.
22. $1 : 2\sqrt{2}$.
24. $\frac{20}{47}$ and $\frac{12}{47}$ of the length from the ends.

EXERCISES 24, p. 459.

5. 332·8 m. per sec.; 324·0 m. per sec.
6. 560; 1 ft.
7. (*a*) No effect. (*b*) Increased about 1·76%.
8. $402\frac{1}{4}$.
12. 110.
13. 140; 70.
15. $365\frac{5}{7}$; 68·36 cms. approx.

MISCELLANEOUS EXERCISES, p. 460.

7. 4·667 secs.
12. 288.
14. 4·42 approx.
15. Reduce to one-quarter.
16. 25 lbs. wt.
17. Reduce to $\frac{\sqrt{3}}{3}$ (or 0·577) of its original length.
18. 11·29 kilograms wt.
19. $32\cdot4\dot{3}\dot{2}$ cms.
20. 220·4 kilograms wt.
21. $18{,}333\frac{1}{3}$ and $36\frac{2}{3}$; 9.
23. 840; $\frac{1}{3}$ foot.
29. 249·5 approx.
30. $2\frac{3}{4}$ ft.
31. Frequency 531·9 approx.

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